

Objectives To find percent change
To find the relative error in linear and nonlinear measurements



You have to understand percents to shop for a good deal.



Getting Ready!

Two stores advertise sales on their Web sites, as shown below. Before the sale, each sweater cost \$35. Which store offers the better deal? How do you know?

Men's Sweater

\$5 off

Colors

Size: Qty.:

ADD TO CART

15% off

Men's Sweater

Color:

Size:

Qty.:

BUY NOW

In the Solve It, the 15% discount is a percent change in the price of the sweater. A **percent change** expresses an amount of change as a percent of an original amount. In this lesson, you will learn how to calculate a percent change.

Essential Understanding You can find a percent change when you know the original amount and how much it has changed.

If a new amount is greater than the original amount, the percent change is called a **percent increase**. If the new amount is less than the original amount, the percent change is called a **percent decrease**.



Key Concept Percent Change

Percent change is the ratio of the amount of change to the original amount.

$$\text{percent change, } p\% = \frac{\text{amount of increase or decrease}}{\text{original amount}}$$

- amount of increase = new amount – original amount
- amount of decrease = original amount – new amount



Lesson Vocabulary

- percent change
- percent increase
- percent decrease
- relative error
- percent error

A common example of finding a percent decrease is finding a percent discount. In this lesson, round your answers to the nearest percent, if necessary.

Problem 1 Finding a Percent Decrease

Clothing A coat is on sale. The original price of the coat is \$82. The sale price is \$74.50. What is the discount expressed as a percent change?

$$\text{percent change} = \frac{\text{amount of increase or decrease}}{\text{original amount}}$$

$$= \frac{\text{original amount} - \text{new amount}}{\text{original amount}}$$

$$= \frac{82 - 74.50}{82}$$

$$= \frac{7.5}{82}$$

$$\approx 0.09, \text{ or } 9\%$$


This is a percent decrease. Write the appropriate ratio.

Substitute.

Simplify.

Write the result as a percent.

The price of the coat decreased by about 9%.

-  **Got It?** 1. The average monthly precipitation for Chicago, Illinois, peaks in June at 4.1 in. The average monthly precipitation in December is 2.8 in. What is the percent decrease from June to December?

A common example of finding a percent increase is finding a percent markup.

Problem 2 Finding a Percent Increase

Music A store buys an electric guitar for \$295. The store then marks up the price of the guitar to \$340. What is the markup expressed as a percent change?

$$\text{percent change} = \frac{\text{amount of increase or decrease}}{\text{original amount}}$$

$$= \frac{\text{new amount} - \text{original amount}}{\text{original amount}}$$

$$= \frac{340 - 295}{295}$$

$$= \frac{45}{295}$$

$$\approx 0.15 \text{ or } 15\%$$


This is a percent increase. Write the appropriate ratio.

Substitute.

Simplify.

Write the result as a percent.

The price of the guitar increased by about 15%.

-  **Got It?** 2. In one year, the toll for passenger cars to use a tunnel rose from \$3 to \$3.50. What was the percent increase?

Think

Does this problem involve a percent decrease or a percent increase?

The new amount is less than the original amount, so the problem involves a percent decrease.

Think

Have you seen a problem like this one?

Yes. Finding percent increase is like finding percent decrease.

The difference is in calculating the amount of increase or decrease.

Essential Understanding You can use percents to compare estimated or measured values to actual or exact values.

Take note

Key Concept Relative Error

Relative error is the ratio of the absolute value of the difference of a measured (or estimated) value and an actual value compared to the actual value.

$$\text{relative error} = \frac{|\text{measured or estimated value} - \text{actual value}|}{\text{actual value}}$$

When relative error is expressed as a percent, it is called **percent error**.



Problem 3 Finding Percent Error

Multiple Choice A decorator estimates that a rectangular rug is 5 ft by 8 ft. The rug is actually 4 ft by 8 ft. What is the percent error in the estimated area?

- (A) 0.25% (B) 20% (C) 25% (D) 80%

$$\begin{aligned} \text{percent error} &= \frac{|\text{estimated value} - \text{actual value}|}{\text{actual value}} && \text{Write the ratio.} \\ &= \frac{|5(8) - 4(8)|}{4(8)} && \text{Substitute.} \\ &= \frac{|40 - 32|}{32} && \text{Multiply.} \\ &= \frac{8}{32} && \text{Simplify.} \\ &= 0.25, \text{ or } 25\% && \text{Write the result as a percent.} \end{aligned}$$

The estimated area is off by 25%. The correct answer is C.



Got It? 3. You think that the distance between your house and a friend's house is 5.5 mi. The actual distance is 4.75 mi. What is the percent error in your estimation?

In Problem 3, the actual measurements were known. Often you don't know actual measurements, but you know how precise your measurements can be.

Think about the last time you used a ruler. Because the precision of a ruler is limited, you measured to the nearest unit or fraction of a unit, such as centimeters or quarter inches. The most any measurement can be off by is one half of the unit used in measuring.

Think

What does the percent error tell you?

The percent error tells how accurate a measurement or estimate is.

Think

Can a measurement be off by more than one half of the unit that is used?

No, not if you measure correctly. Suppose you measure a length as 8 in. If you were off by 0.5 in. or more, you would have rounded to either 7 in. or 9 in.



Problem 4 Finding Minimum and Maximum Dimensions

Posters You are framing a poster and measure the length of the poster as 18.5 in., to the nearest half inch. What are the minimum and maximum possible lengths of the poster?

You measured to the nearest 0.5 in., so the greatest possible error is 0.25 in.

$$\text{Minimum length} = \text{measured value} - \text{possible error} = 18.5 - 0.25 = 18.25$$

$$\text{Maximum length} = \text{measured value} + \text{possible error} = 18.5 + 0.25 = 18.75$$

The minimum possible length is 18.25 in. The maximum is 18.75 in.



Got It? 4. A student's height is measured as 66 in. to the nearest inch. What are the student's minimum and maximum possible heights?



Problem 5 Finding the Greatest Possible Percent Error

Crafts The diagram at the right shows the dimensions of a gift box to the nearest inch. What is the greatest possible percent error in calculating the volume of the gift box?



Know

- The dimensions of the gift box to the nearest inch
- The formula for volume:
 $V = \ell wh$
- The greatest possible error in each dimension is 0.5 in.

Need

The greatest possible percent error in calculating the volume

Plan

Find the minimum and maximum volumes. Find the differences between the possible volumes and the measured volume. Use the greater difference to find the percent error.

Measured volume

$$\begin{aligned} V &= \ell wh \\ &= (12)(6)(5) \\ &= 360 \end{aligned}$$

Minimum volume

$$\begin{aligned} V &= \ell wh \\ &= (11.5)(5.5)(4.5) \\ &= 284.625 \end{aligned}$$

Maximum volume

$$\begin{aligned} V &= \ell wh \\ &= (12.5)(6.5)(5.5) \\ &= 446.875 \end{aligned}$$

Find the differences.

$$|\text{minimum volume} - \text{measured volume}| = |284.625 - 360| = 75.375$$

$$|\text{maximum volume} - \text{measured volume}| = |446.875 - 360| = 86.875$$

Use the greater difference to find the greatest possible percent error.

$$\text{greatest possible percent error} = \frac{\text{greater difference in volume}}{\text{measured volume}}$$

$$= \frac{86.875}{360} \quad \text{Substitute.}$$



$$\approx 0.24 \text{ or } 24\% \quad \text{Write the result as a percent.}$$

The greatest possible percent error in the volume, based on measurements to the nearest inch, is about 24%.

Think

Which difference should you use?

The greater difference will result in the greatest possible percent error, so use the difference that has the greatest absolute value.

-   **Got It?** 5. **Reasoning** If the gift box's dimensions in Problem 5 were measured to the nearest half inch, how would the greatest possible error be affected?



Lesson Check

Do you know HOW?

- Running** Last year, an athlete's average time to run a mile was 6 min 13 s. This year, the athlete's average time is 6 min 5 s. What is the percent decrease?
- Cars** A used-car dealership buys a car for \$2800 and then sells it for \$4500. What is the percent increase?
- Horses** A veterinarian measures a horse to be 7.5 ft tall at the shoulder to the nearest half foot. What are the minimum and maximum possible heights of the horse?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

- Vocabulary** Determine whether each situation involves a percent increase or a percent decrease.
 - A hat that originally costs \$12 sold for \$9.50.
 - You buy a CD for \$10 and sell it for \$8.
 - A store buys glasses wholesale for \$2 per glass. The store sells them for \$4.50.
- Reasoning** What is the greatest possible error of a measurement taken to the nearest tenth of a meter?
- Writing** How is calculating percent increase different from calculating percent decrease?



Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

Practice

Tell whether each percent change is an increase or decrease. Then find the percent change. Round to the nearest percent.

 See Problems 1 and 2

- | | | |
|---|---|---|
| 7. original amount: 12
new amount: 18 | 8. original amount: 9
new amount: 6 | 9. original amount: 15
new amount: 14 |
| 10. original amount: 7.5
new amount: 9.5 | 11. original amount: 40.2
new amount: 38.6 | 12. original amount: 2008
new amount: 1975 |
| 13. original amount: 14,500
new amount: 22,320 | 14. original amount: 195.50
new amount: 215.25 | 15. original amount: 1325.60
new amount: 1685.60 |
16. **Employment** An employee was hired at a wage of \$8 per hour. After a raise, the employee earned \$8.75 per hour. What was the percent increase?
17. **Climate** On June 1, 2007, there were about 18.75 h of daylight in Anchorage, Alaska. On November 1, 2007, there were about 8.5 h of daylight. What was the percent decrease?

Find the percent error in each estimation. Round to the nearest percent.

 See Problem 3

- You estimate that your friend's little brother is about 8 years old. He is actually 6.5 years old.
- You estimate that your school is about 45 ft tall. Your school is actually 52 ft tall.

A measurement is given. Find the minimum and maximum possible measurements.

See Problem 4.

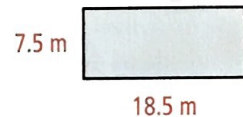
20. A doctor measures a patient's weight as 162 lb to the nearest pound.
 21. An ostrich egg has a mass of 1.1 kg to the nearest tenth of a kilogram.
 22. The length of an onion cell is 0.4 mm to the nearest tenth of a millimeter.

23. **Geometry** The table below shows the measured dimensions of a prism and the minimum and maximum possible dimensions based on the greatest possible error. What is the greatest possible percent error in finding the volume of the prism?

See Problem 5.

Dimensions	Length	Width	Height
Measured	10	6	4
Minimum	9.5	5.5	3.5
Maximum	10.5	6.5	4.5

24. **Geometry** The side lengths of the rectangle at the right have been measured to the nearest half of a meter, as shown. What is the greatest possible percent error in finding the area of the rectangle?



B Apply

Find the percent change. Round to the nearest percent.

25. 2 ft to $5\frac{1}{2}$ ft 26. 18 lb to $22\frac{1}{4}$ lb 27. $140\frac{1}{4}$ g to $80\frac{3}{4}$ g
 28. \$8.99 to \$15.99 29. \$168.45 to \$234.56 30. \$4023.52 to \$982.13

The measured dimensions of a rectangle are given to the nearest whole unit. Find the minimum and maximum possible areas of each rectangle.

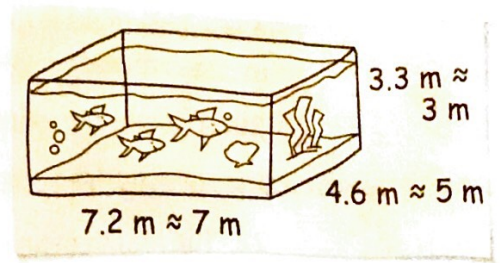
31. 7 m by 8 m 32. 18 in. by 15 in. 33. 24 ft by 22 ft

34. **Writing** How are percent change and percent error similar?
 35. **Open-Ended** Write a percent change problem that you recently experienced.
 36. **Think About a Plan** In one season, an average of 6500 fans attended each home game played by the basketball team at a college. In the next season, the average number of fans per game increased by about 12%. What was the average number of fans per game for that season?
 • What is missing—the new amount or the original amount?
 • How can a percent change help you find the missing amount?

37. **Error Analysis** A student is trying to find the percent of change when an amount increases from 12 to 18, as shown. Describe and correct the student's error.

$$\begin{aligned}
 & \frac{\text{new amount} - \text{original amount}}{\text{original amount}} \\
 &= \frac{18 - 12}{18} \\
 &= \frac{6}{18} \approx 0.33, \text{ or } 33\%
 \end{aligned}$$

38. **Rounding Error** Your science class visits an aquarium. In a report on your class's visit, you sketch one of the fish tanks and round the dimensions as shown in the diagram at the right. You use the rounded dimensions to state the tank's volume is approximately $(7\text{ m})(5\text{ m})(3\text{ m}) = 105\text{ m}^3$. What is the percent error in your volume calculation due to rounding?



39. **Student Discounts** You show your student identification at a local restaurant in order to receive a 5% discount. You spend \$12 for your meal at the restaurant. How much would your meal cost without the discount?



40. **Geometry** The height of a cylinder is 2 ft. The diameter of the base is 5 ft. Each dimension is accurate to the nearest foot. What is the greatest possible error in calculating the volume of the cylinder? Use 3.14 for π .
41. a. The sides of a square that measures 4 m by 4 m increased in length by 10%. Find the percent of increase in the area.
 b. The sides of a square that measures 6 m by 6 m increased in length by 10%. Find the percent of increase in the area.
 c. **Reasoning** Predict the percent of increase in the area of a square that measures 8 m by 8 m if the side lengths increase by 10%. Explain and check your answer.

Standardized Test Prep

SAT/ACT

42. Marcus bought a shirt that was marked \$28, but it was on sale for 15% off the marked price. What was the price of the shirt after the discount?
 (A) \$4.20 (B) \$23.80 (C) \$24.80 (D) \$32.20
43. What equation do you get when you solve $ax + bx = c$ for x ?
 (F) $x = c - ab$ (G) $x = c - a - b$ (H) $x = \frac{c}{a - b}$ (I) $x = \frac{c}{a + b}$
44. A teacher wants to give each student 2 pencils. A store is selling pencils in boxes of 24. If the teacher has a total of 125 students, how many boxes of pencils should he buy?
 (A) 5 (B) 6 (C) 10 (D) 11

Mixed Review

Solve each percent problem.

See Lesson 2-9.

45. What percent of 12 is 8? 46. What is 35% of 185? 47. 20% of what number is 4.2?

Get Ready! To prepare for Lesson 3-1, do Exercises 48-51.

Graph the numbers on the same number line. Then order them from least to greatest.

See Lesson 1-3.

48. -3 49. $\frac{1}{2}$ 50. 2 51. -2.8