3-1 Inequalities and Their Graphs

Common Core State Standards

Prepares for A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

MP 1, MP 2, MP 3, MP 4, MP 6

Objective To write, graph, and identify solutions of inequalities

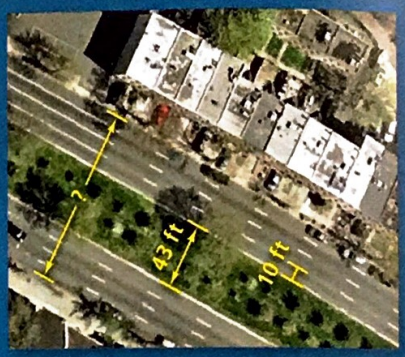
A sketch might help.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

By law, the height of a newly constructed building in Washington, D.C., can be no greater than the width of the adjacent street, plus 20 ft. Pennsylvania Avenue, shown at the right, is the widest street in Washington, D.C. What is the maximum allowable height of a new building? Explain your reasoning.



VOCABULARY

Lesson Vocabulary

- solution of an inequality

The Solve It involves comparing two quantities—the height of a building and the width of the street adjacent to it. You can use an inequality to compare such quantities.

Essential Understanding An inequality is a mathematical sentence that uses an inequality symbol to compare the values of two expressions. You can use a number line to visually represent the values that satisfy an inequality.

Problem 1 Writing Inequalities

What inequality represents the verbal expression?

A all real numbers x less than or equal to -7

all real numbers x	less than or equal to	-7
x	$\leq$	-7

The inequality $x \leq -7$ represents the verbal expression.

B 6 less than a number k is greater than 13.

6 less than a number k	is greater than	13
$k - 6$	$>$	13

The inequality $k - 6 > 13$ represents the verbal expression.

Think

Less than and *is less than* have different meanings. For example, "6 less than k" means $k - 6$, while "6 is less than k" means $6 < k$.

Got It? 1. What is an inequality that represents the verbal expression?

a. all real numbers p greater than or equal to 1.5

b. The sum of t and 7 is less than -3.

A **solution of an inequality** is any number that makes the inequality true. The solutions of the inequality $x < 5$ are all real numbers x that are less than 5. You can evaluate an expression to determine whether a value is a solution of an inequality.

Problem 2 Identifying Solutions by Evaluating

Is the number a solution of $2x + 1 > -3$?

A -3

$$\begin{aligned} 2x + 1 &> -3 \\ 2(-3) + 1 &\stackrel{?}{>} -3 \quad \leftarrow \text{Substitute for } x. \rightarrow \\ -6 + 1 &\stackrel{?}{>} -3 \quad \leftarrow \text{Simplify.} \rightarrow \\ -5 &\not> -3 \quad \leftarrow \text{Compare.} \rightarrow \end{aligned}$$

-3 does not make the original inequality true, so -3 is *not* a solution.

B -1

$$\begin{aligned} 2x + 1 &> -3 \\ 2(-1) + 1 &\stackrel{?}{>} -3 \\ -2 + 1 &\stackrel{?}{>} -3 \\ -1 &> -3 \end{aligned}$$

-1 does make the original inequality true, so -1 is a solution.

Think

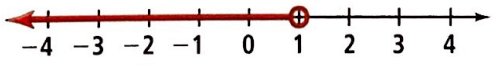
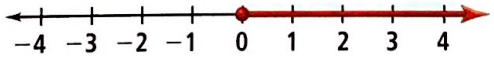
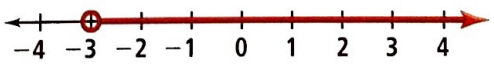
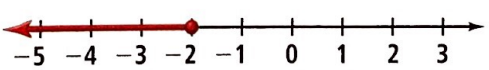
Is -1 the *only* solution to the inequality?

No. *Any* number that makes the original inequality true is a solution of the inequality. The solution -1 is one of an infinite number of solutions.

Got It? 2. **a.** Consider the numbers -1, 0, 1, and 3. Which are solutions of $13 - 7y \leq 6$?

b. Reasoning In Problem 2, how is the solution of the related equation $2x + 1 = -3$ related to the solutions of the inequality?

You can use a graph to indicate all of the solutions of an inequality.

Inequality	Graph	
$n < 1$		The open dot shows that 1 is *not* a solution. Shade to the left of 1.
$a \geq 0$		The closed dot shows that 0 is a solution. Shade to the right of 0.
$f > -3$		The open dot shows that -3 is *not* a solution. Shade to the right of -3.
$-2 \geq x$		The closed dot shows that -2 is a solution. Shade to the left of -2.

You can also write $-2 \geq x$ as $x \leq -2$.

Problem 3 Graphing an Inequality

What is the graph of $2 \geq a$?

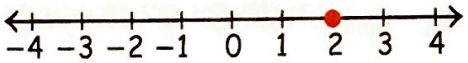
Think

If 2 is greater than or equal to a, then a must be less than or equal to 2.

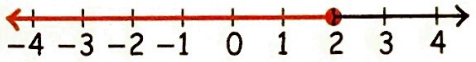
Write

$a \leq 2$

$a \leq 2$ means all real numbers a that are less than or equal to 2. Since 2 is a solution, draw a closed dot at 2.



The numbers less than 2 are to the left of 2 on the number line. Shade to the left of 2.



Got It? **3.** What is the graph of each inequality?

a. $x > -4$ **b.** $c < 0$ **c.** $3 \leq n$

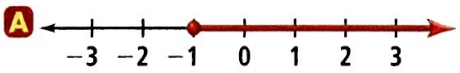
Problem 4 Writing an Inequality From a Graph

What inequality represents the graph?

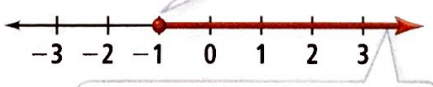
Plan

How do you know which inequality symbol to use?
Look at the arrow to see whether the solution is for quantities greater than or less than the endpoint. Look at the endpoint to see whether "equal to" is included in the solution.

A



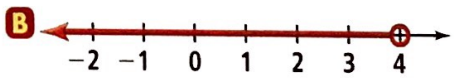
The closed dot means that −1 is a solution.



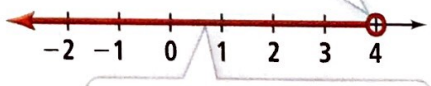
The number line is shaded to the right of −1, so all numbers greater than −1 are solutions.

The inequality $x \geq -1$ represents the graph.

B



The open dot means that 4 is *not* a solution.

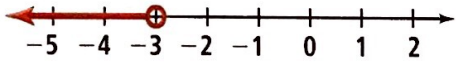


The number line is shaded to the left of 4, so all numbers less than 4 are solutions.

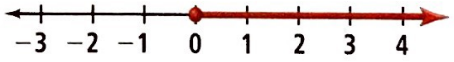
The inequality $x < 4$ represents the graph.

Got It? **4.** What inequality represents each graph?

a.



b.



Problem 5 Writing Real-World Inequalities

What inequality describes the situation? Be sure to define a variable.

Plan

How do you know which inequality symbol to use?
The phrase "starting at $19.99" implies that the cost of a trail ride starts at $19.99 and goes up. So the cost is greater than or equal to 19.99.

A

Let c = the cost of a trail ride in dollars.

The sign indicates that $c \geq 19.99$.

B

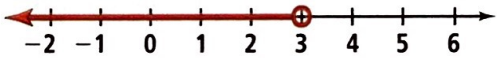
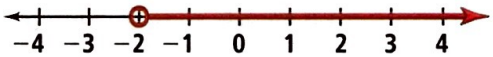
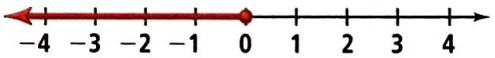
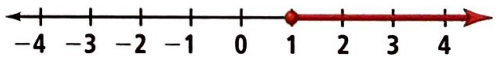
Let s = a legal speed in miles per hour.

The sign indicates that $s \leq 8$.

Got It? **5. Reasoning** In part (B) of Problem 5, can the speed be *all* real numbers less than or equal to 8? Explain.

take note

Concept Summary Representing Inequalities

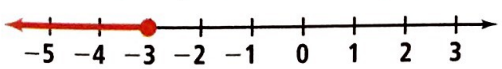
Words	Symbols	Graph
x is less than 3.	$x < 3$	
x is greater than -2.	$x > -2$	
x is less than or equal to 0.	$x \leq 0$	
x is greater than or equal to 1.	$x \geq 1$	

Lesson Check

Do you know HOW?

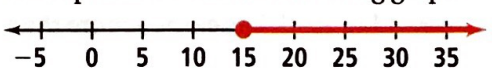
1. What algebraic inequality represents all real numbers y that are greater than or equal to 12?
2. Is the number a solution of $6x - 3 \geq 10$?

   a. -1 **b.** 0 **c.** 3 **d.** 4
3. What is the graph of $2 > p$?
4. What inequality represents the graph?



Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** How do you decide whether a number is a solution of an inequality?
6. **Compare and Contrast** What are some situations you could model with $x \geq 0$? How do they differ from situations you could model with $x > 0$?
7. **Open-Ended** What is a real-world situation that you can represent with the following graph?



Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Write an inequality that represents each verbal expression. See Problem 1.

8. v is greater than or equal to 5.

9. b is less than 4.

10. 3 less than g is less than or equal to 17.

11. The quotient of k and 9 is greater than $\frac{1}{3}$.

Determine whether each number is a solution of the given inequality. See Problem 2.

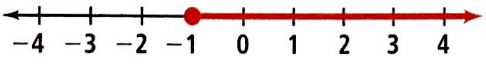
12. $3y - 8 > 22$	**a.** 2	**b.** 0	**c.** 5
13. $8m - 6 \leq 10$	**a.** 2	**b.** 3	**c.** -1
14. $4x + 2 < -6$	**a.** 0	**b.** -2	**c.** 1
15. $\frac{6 - n}{n} \geq 11$	**a.** 0.5	**b.** 2	**c.** 4
16. $m(m - 3) < 54$	**a.** -10	**b.** 0	**c.** 9

Match each inequality with its graph. See Problem 3.

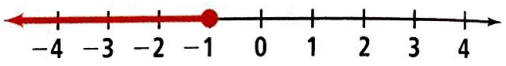
17. $x < -1$ **18.** $x \geq -1$ **19.** $-1 < x$ **20.** $-1 \geq x$

A.

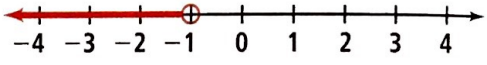
B.



C.



D.

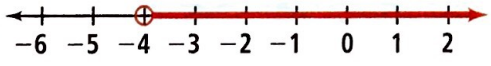


Graph each inequality.

21. $y > 2$	**22.** $t < -4$	**23.** $z \leq -5$	**24.** $v \geq -2$
25. $-3 < f$	**26.** $-\frac{9}{4} \leq c$	**27.** $8 \geq b$	**28.** $5.75 > d$

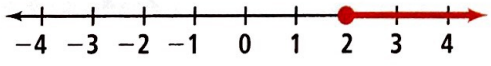
Write an inequality for each graph. See Problem 4.

29.

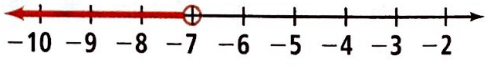


30.

31.



32.



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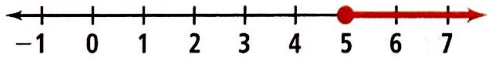


34.

Define a variable and write an inequality to model each situation. See Problem 5.

35. The restaurant can seat at most 172 people.

36. A person must be at least 35 years old to be elected President of the United States.

37. A light bulb can be no more than 75 watts to be safely used in this light fixture.

38. At least 475 students attended the orchestra concert Thursday night.

39. A law clerk has earned more than $20,000 since being hired.

B Apply

40. Error Analysis A student claims that the inequality $3x + 1 > 0$ is always true because multiplying a number by 3 and then adding 1 to the result always produces a number greater than 0. Explain the student's error.

41. Open-Ended Describe a situation that you can model with $x \geq 25$.

42. Ticket Sales Suppose your school plans a musical. The director's goal is ticket sales of at least \$4500. Adult tickets are \$7.50 and student tickets are \$5.00. Let a represent the number of adult tickets and s represent the number of student tickets. Which inequality represents the director's goal?

(A) $5a + 7.5s < 4500$

(B) $7.5a + 5s > 4500$

(C) $7.5a + 5s \leq 4500$

(D) $7.5a + 5s \geq 4500$

STEM 43. Physics According to Albert Einstein's special theory of relativity, no object can travel faster than the speed of light, which is approximately 186,000 mi/s. What is an inequality that represents this information?

Write each inequality in words.

44. $n < 5$

45. $b > 0$

46. $7 \geq x$

47. $z \geq 25.6$

48. $4 > q$

49. $21 \geq m$

50. $35 \geq w$

51. $g - 2 < 7$

52. $a \leq 3$

53. $6 + r > -2$

54. $8 \leq h$

55. $1.2 > k$

56. Class Party You are making muffins for a class party. You need 2 cups of flour to make a pan of 12 muffins. You have a 5-lb bag of flour, which contains 18 cups. What is an inequality that represents the possible numbers of muffins you can make?

57. Writing Explain what the phrases *no more than* and *no less than* mean when writing inequalities that model real-world situations.

Use the map at the right for Exercises 58 and 59.

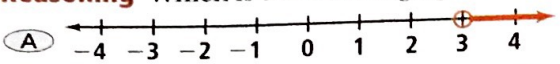
58. Think About a Plan You plan to go from Portland to Tucson. Let x be the distance in miles of any flight between Portland and Tucson. What is a true statement about the mileage of any route from Portland to Tucson? Assume that no route visits the same city more than once and that each route has no more than one layover.

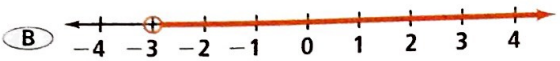
- How many routes exist between Portland and Tucson? What are they? Which route is the shortest?
- Can you write an inequality that represents the mileage of any route from Portland to Tucson?

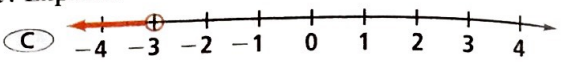
59. Air Travel Your travel agent is making plans for you to go from San Diego to Seattle. A direct flight is not available. Option A consists of flights from San Diego to Boise to Seattle. Option B consists of flights from San Diego to Las Vegas to Seattle. What inequality compares the flight distances of these two options?

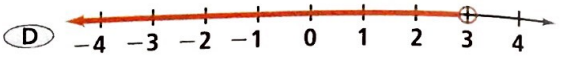
Challenge

60. Reasoning Which is the correct graph of $-3 < -x$? Explain.

(A) 

(B) 

(C) 

(D) 

61. Reasoning Give a counterexample for this statement: If $x > y$, then $x^2 > y^2$.

62. Reasoning Describe the numbers x and y such that if $x > y$, then $x^2 = y^2$.

Graph on a number line.

63. all values of p such that $p > -3$ and $p \leq 3$

64. all values of q such that $q < -2$ or $q > 5$

Standardized Test Prep

SAT/ACT

65. Which inequality has the same solutions as $k > 6$?

(A) $k < -6$ (B) $k < 6$ (C) $6 < k$ (D) $-k > -6$

66. What is the value of the expression $\frac{2^3 \cdot 4 - (-3)^2}{(-3)^2 + 4 \cdot 5}$?

(F) $\frac{23}{29}$ (G) $\frac{41}{29}$ (H) $\frac{23}{11}$ (I) $\frac{41}{11}$

67. Last season, Betsy scored 36 points. This is 8 less than twice the number of points that Amy scored. How many points did Amy score?

(A) 22 (B) 36 (C) 44 (D) 72

Short Response

68. At an airport, a runway 1263 ft long is being repaired. The project foreman reports that less than one third of the job is complete. Draw a diagram of the runway that shows how much of it has been repaired. What is an inequality that represents the number of feet f that still need to be repaired?

Mixed Review

Tell whether each percent change is an *increase* or *decrease*. Then find the percent change. Round to the nearest percent. See Lesson 2-10.

69. original amount: \$10
new amount: \$12

70. original amount: 20 in.
new amount: 18 in.

71. original amount: 36°
new amount: 12°

Find each product or quotient. See Lesson 1-6.

72. $-4(-11)$ **73.** $\frac{5}{6} \cdot \left(-\frac{1}{4}\right)$ **74.** $-3.9 \div 1.3$ **75.** $\frac{4}{7} \div \left(-\frac{2}{5}\right)$

Get Ready! To prepare for Lesson 3-2, do Exercises 76–79.

Solve each equation. See Lesson 2-1.

76. $y - 5 = 6$ **77.** $p - 4 = -6$ **78.** $v + 5 = -6$ **79.** $k + \frac{2}{3} = \frac{5}{9}$