

# Proportions and Similar Figures

**Objectives** To find missing lengths in similar figures  
To use similar figures when measuring indirectly

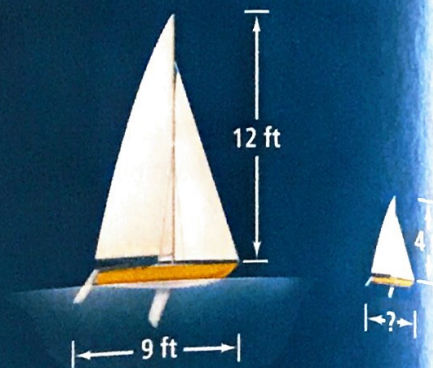


A good model is an exact copy of the thing that it represents. It's just a different size.



## Getting Ready!

A sailboat and an accurate model of the boat are shown. What is the length of the model sailboat? Explain your reasoning.

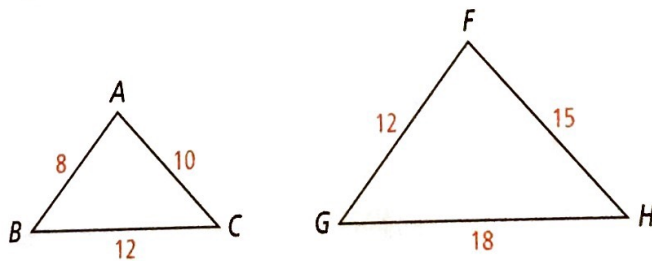


MATHEMATICAL PRACTICES

In the Solve It, the sailboat and its model have the same shape but they are different sizes. **Similar figures** have the same shape but not necessarily the same size.

**Essential Understanding** You can use proportions to find missing side lengths in similar figures. Such figures can help you measure real-world distances indirectly.

The symbol  $\sim$  means "is similar to." In the diagram,  $\triangle ABC \sim \triangle FGH$ .



In similar figures, the measures of corresponding angles are equal, and corresponding side lengths are in proportion. The order of the letters when you name similar figures is important because it tells which parts of the figures are corresponding parts. So, because  $\triangle ABC \sim \triangle FGH$ , the following is true.

$$\angle A \cong \angle F \quad \angle B \cong \angle G \quad \angle C \cong \angle H \quad \text{and} \quad \frac{AB}{FG} = \frac{AC}{FH} = \frac{BC}{GH}$$

The symbol  $\cong$  means "is congruent to."  
Congruent angles have the same measure.

The ratios are equal.



### Lesson Vocabulary

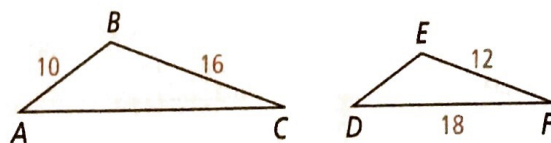
- similar figures
- scale drawing
- scale
- scale model



## Problem 1 Finding the Length of a Side

**Multiple Choice** In the diagram,  $\triangle ABC \sim \triangle DEF$ . What is  $DE$ ?

- (A) 7.5                      (C) 21.3  
 (B) 9.5                      (D) 24



### Know

- The length of  $\overline{AB}$ , which corresponds to  $\overline{DE}$
- The lengths of two other corresponding sides,  $\overline{BC}$  and  $\overline{EF}$
- The triangles are similar.

### Need

The length of  $\overline{DE}$

### Plan

Write a proportion involving two pairs of corresponding sides:  $\overline{AB}$  and  $\overline{DE}$ , and  $\overline{BC}$  and  $\overline{EF}$ . The length of  $\overline{DE}$  is the only unknown, so you can solve for it.

$$\frac{BC}{EF} = \frac{AB}{DE} \quad \text{Write a proportion.}$$

$$\frac{16}{12} = \frac{10}{DE} \quad \text{Substitute lengths.}$$

$$16(DE) = 12(10) \quad \text{Cross Products Property}$$

$$16DE = 120 \quad \text{Multiply.}$$

$$DE = 7.5 \quad \text{Divide each side by 16 and simplify.}$$

$DE$  is 7.5. The correct answer is A.

**Got It?** 1. Use the figures in Problem 1. What is  $AC$ ?

You can also use proportions to solve indirect measurement problems like finding a distance using a map. You can use similar figures and proportions to find lengths that you cannot measure directly.

## Problem 2 Applying Similarity

**Indirect Measurement** The sun's rays strike the building and the girl at the same angle, forming the two similar triangles shown. How tall is the building?

$$\frac{\text{girl's shadow}}{\text{building's shadow}} = \frac{\text{girl's height}}{\text{building's height}} \quad \text{Write a proportion.}$$

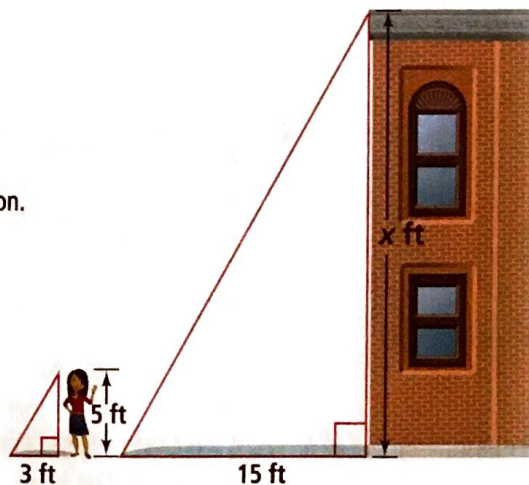
$$\frac{3}{15} = \frac{5}{x} \quad \text{Substitute.}$$

$$3x = 15(5) \quad \text{Cross Products Property}$$

$$3x = 75 \quad \text{Multiply.}$$

$$x = 25 \quad \text{Divide each side by 3.}$$

The building is 25 ft tall.



### Think

Is there only one way to write a proportion?

No. You can write different proportions to find the height. For example, the following proportion also works.

$$\frac{\text{building's shadow}}{\text{girl's shadow}} = \frac{\text{building's height}}{\text{girl's height}}$$



**Got It?** 2. A man who is 6 ft tall is standing next to a flagpole. The shadow of the man is 3.5 ft and the shadow of the flagpole is 17.5 ft. What is the height of the flagpole?

A **scale drawing** is a drawing that is similar to an actual object or place. Floor plans, blueprints, and maps are all examples of scale drawings. In a scale drawing, the ratio of any length on the drawing to the actual length is always the same. This ratio is called the **scale** of the drawing.



### Problem 3 Interpreting Scale Drawings

**Maps** What is the actual distance from Jacksonville to Orlando? Use the ruler to measure the distance from Jacksonville to Orlando on the map below.



#### Think

**What does the scale of the map tell you?**  
The scale tells you that each inch on the map represents 110 mi of actual distance.

**Relate**  $\text{map scale} = \frac{\text{map distance}}{\text{actual distance}}$

**Define** Let  $x$  = the total distance from Jacksonville to Orlando.

**Write**  $\frac{1}{110} = \frac{1.25}{x}$

$$1(x) = 110(1.25) \quad \text{Cross Products Property}$$

$$x = 137.5 \quad \text{Multiply.}$$

The actual distance from Jacksonville to Orlando is 137.5 mi.



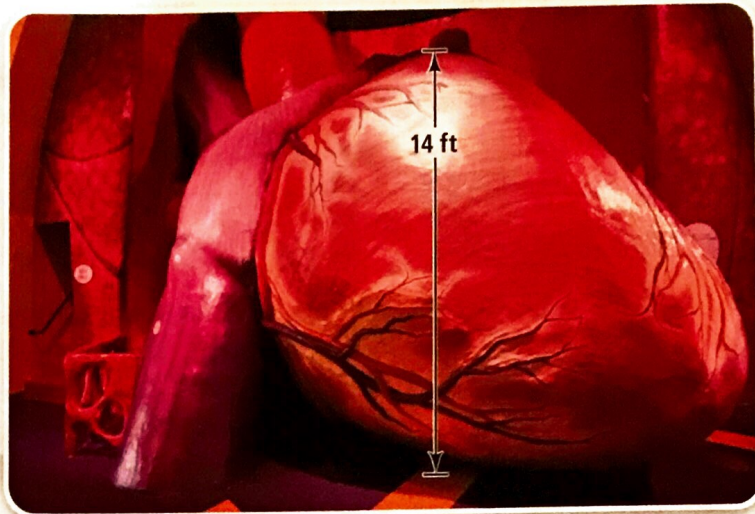
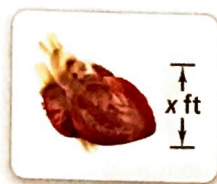
**Got It?** 3. a. The distance from Jacksonville to Gainesville on the map is about 0.6 in. What is the actual distance from Jacksonville to Gainesville?  
b. **Reasoning** If you know that the actual distance between two cities is 250 mi and that the cities are 2 in. apart on a map, how can you find the scale of the map?

A **scale model** is a three-dimensional model that is similar to a three-dimensional object. The ratio of a linear measurement of a model to the corresponding linear measurement of the actual object is always the same. This ratio is called the **scale** of the model.



**Problem 4** Using Scale Models **STEM**

**Science** A giant model heart is shown below. The heart is the ideal size for a person who is 170 ft tall. About what size would you expect the heart of a man who is 6 ft tall to be?



**Think**

Is this problem like ones you have seen?

Yes. Scale model problems are like scale drawing problems, so you can write a proportion like you did to find the height of the building in Problem 2.

$$\frac{\text{height of giant heart}}{\text{height of man's heart}} = \frac{\text{height of giant person}}{\text{height of man}} \quad \text{Write a proportion.}$$

$$\frac{14}{x} = \frac{170}{6} \quad \text{Substitute.}$$

$$14(6) = 170x \quad \text{Cross Products Property}$$

$$0.49 \approx x \quad \text{Divide each side by 170 and simplify.}$$

The size of the man's heart would be about 0.49 ft, or 5.9 in.

- Got It?** 4. A scale model of a building is 6 in. tall. The scale of the model is 1 in. : 50 ft. How tall is the actual building?

**Lesson Check**

**Do you know HOW?**

- Photocopies** You use a photocopier to enlarge a drawing of a right triangle with a base of 13 cm and a height of 7 cm. The enlarged triangle has a height of 17.5 cm.
  - What is the base of the enlarged triangle?
  - What is the scale of the enlargement?
- Maps** The scale of a map is 1 cm : 75 km. What is the actual distance between two towns that are 3 cm apart on the map?

**Do you UNDERSTAND?** **MATHEMATICAL PRACTICES**

- Vocabulary** Suppose  $\triangle MNP \sim \triangle RST$ . How can you identify corresponding parts?
- Reasoning** Suppose  $\triangle ABC \sim \triangle TUV$ . Determine whether each pair of measures is equal.
  - the measures of  $\angle A$  and  $\angle T$
  - the perimeters of the two triangles
  - the ratios of the sides  $\frac{BC}{UV}$  and  $\frac{AC}{TV}$
- Reasoning** The scale of a map is 1 in. : 100 mi. Is the actual distance between two towns 100 times the map distance between the two towns? Explain.

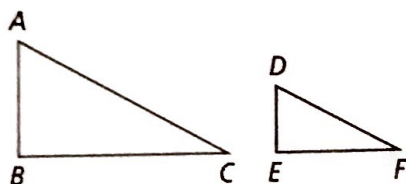


**A** Practice

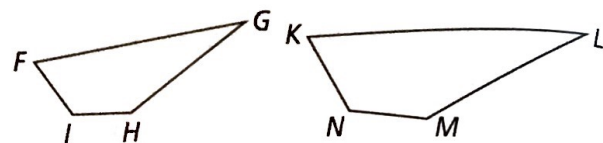
The figures in each pair are similar. Identify the corresponding sides and angles.

← See Problem 1

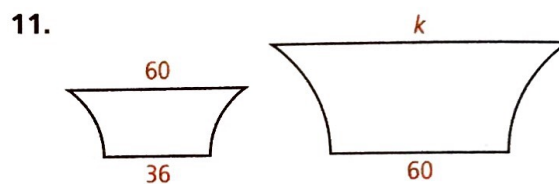
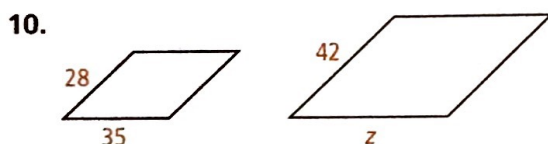
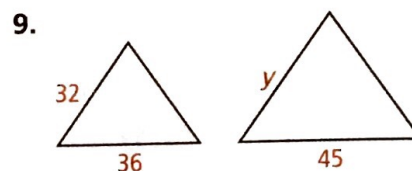
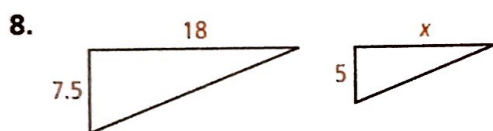
6.  $\triangle ABC \sim \triangle DEF$



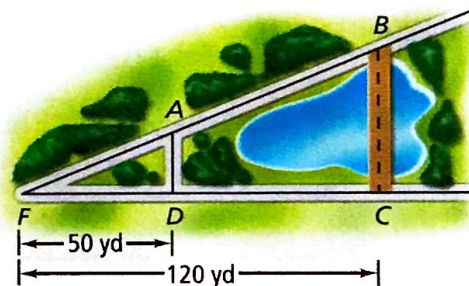
7.  $FGHI \sim KLMN$



The figures in each pair are similar. Find the missing length.



12. **Bridges** In the diagram of the park,  $\triangle ADF \sim \triangle BCF$ . The crosswalk at point A is about 20 yd long. A bridge across the pond will be built, from point B to point C. What will the length of the bridge be?



← See Problem 2.

The scale of a map is 1 cm : 15 km. Find the actual distance corresponding to each map distance.

← See Problem 3.

13. 2.5 cm

14. 0.2 cm

15. 15 cm

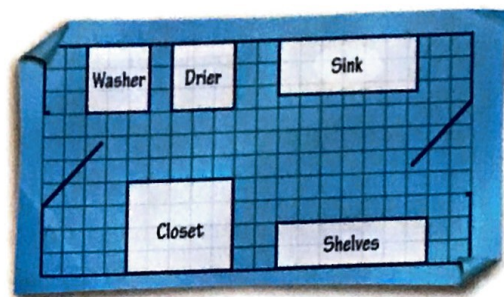
16. 4.6 cm

17. **Movies** A professional model-maker is building a giant scale model of a house fly to be used in a science fiction film. An actual fly is about 0.2 in. long with a wingspan of about 0.5 in. The model fly for the movie will be 27 ft long. What will its wingspan be?

← See Problem 4.

18. **Maps** Abbottsville and Broken Branch are 175 mi apart. On a map, the distance between the two towns is 2.5 in. What is the scale of the map?

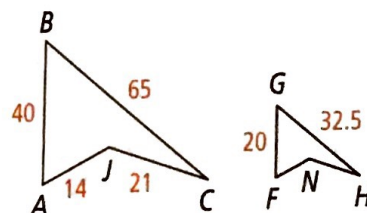
**B Apply STEM** **Architecture** An architect is using the blueprint below to remodel a laundry room. The side length of each grid square represents 12 in.



19. Find the actual length and width of the sink.
20. Find the total length and width of the actual room.
21. Will it be possible to wheel a laundry cart that is  $3\frac{1}{2}$  ft wide through the room from the doorway at the left to the doorway at the right?

22. **Model Rockets** A particular model rocket kit uses the scale 1 : 144. The actual rocket is 168 ft tall. How tall will the model rocket be when completed?

23. **Error Analysis** The two figures at the right are similar. A student uses the proportion  $\frac{BC}{CJ} = \frac{GH}{FN}$  to find  $FN$ .
- a. What mistake did the student make?
  - b. What proportion should the student have used instead?

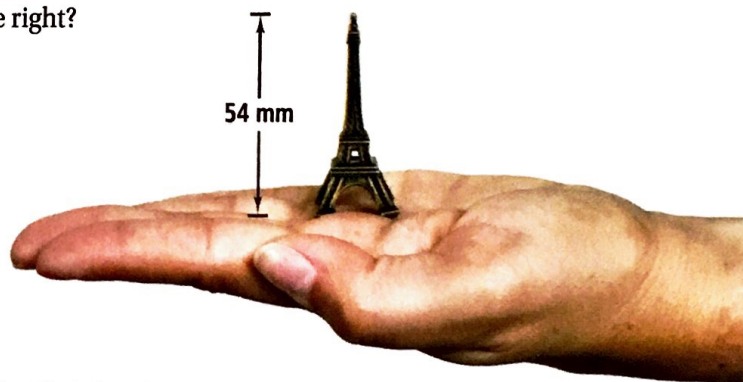


24. **Think About a Plan** An interior designer sketches a design for a rectangular rug. The dimensions of the sketch are 4 in. by 7.5 in. The dimensions of the actual rug will be ten times the dimensions of the drawing, so the scale of the drawing is 1 : 10. How many times the area of the sketch is the area of the actual rug?
- Which figures in the problem are similar? What are their dimensions?
  - How can proportions help you find the dimensions of the actual rug?

25. **Trucks** A model of a tractor-trailer is shaped like a rectangular prism and has a width of 2 in., a length of 9 in., and a height of 4 in. The scale of the model is 1 : 34. How many times the volume of the model is the volume of the actual tractor-trailer?

26. **Eiffel Tower** The height of the Eiffel Tower is 324 m. Which scale was used to make the model of the Eiffel Tower shown at the right?

- (A) 1 mm : 0.9 m
- (B) 1 mm : 6 m
- (C) 1 mm : 30 m
- (D) 1 mm : 324 m



27. **Writing** Are all squares similar? Explain your answer.

28. **Reasoning** A boat maker wanted to build a canoe 6 ft long and  $2\frac{1}{2}$  ft wide but decided that those dimensions were too small. The boat maker wants to add 2 ft to both the length and the width but also wants to keep the canoe the same shape. Explain why this will not work.



**Challenge**

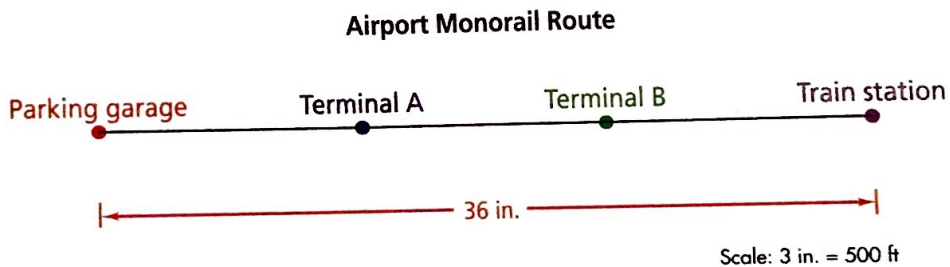
29. **Carpentry** A carpenter is building a tabletop from a sketch. The sketch shows a parallelogram with side lengths 2 in. and 3 in. Also, the sketch specifies that the sides of the finished tabletop should be 4 ft and 6 ft. Can the carpenter be certain that the finished tabletop will be a similar parallelogram? Explain.
30. **Painting** You have a painting that is 30 in. wide and 22.5 in. tall. You would like to reproduce it on a sheet of paper that measures  $8\frac{1}{2}$  in. by 11 in., leaving at least a 1-in. margin on all four sides.
- What scale should you use if you keep the sheet of paper in the normal upright orientation? Assume that the reproduction will be as large as possible.
  - What scale should you use if you turn the paper on its side?



## Apply What You've Learned



Look back at the information about the monorail on page 79. The scale drawing of the monorail route is shown again below.



- Write a proportion that you can use to find the total distance the monorail travels from the parking garage to the train station.
- Determine the total distance the monorail travels from the parking garage to the train station. Explain how you know that your answer is reasonable.
- What is the distance the monorail travels from Terminal B to the train station? Find the length that represents this distance in the scale drawing on the airport's information board.