10-5 Graphing Square Root Functions

Common Core State Standards

F-IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **Also A-CED.A.2**

MP 1, MP 2, MP 3, MP 4, MP 6

Objectives To graph square root functions
To translate graphs of square root functions

Look at your graph. How is it similar to and different from a parabola?

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

A landscaper is planning to build a square yard with a wall on one side. The size of the yard will determine the project's cost. Graph the length of the wall as a function of the area of the yard. What is an equation of this graph? Explain your reasoning.

The Solve It involves a *square root function.* Square root functions are examples of radical functions.

Lesson Vocabulary
- square root function

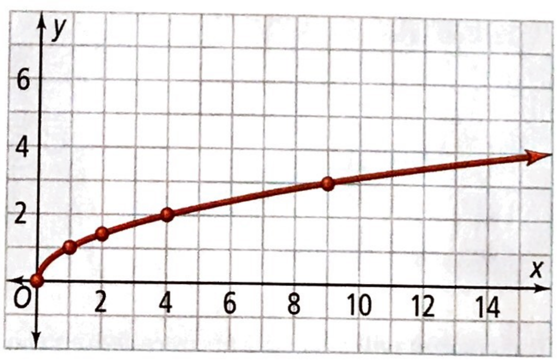
take note

Key Concept Square Root Functions

A **square root function** is a function containing a square root with the independent variable in the radicand. The parent square root function is $y = \sqrt{x}$.

The table and graph below show the parent square root function.

x	y
0	0
1	1
2	1.4
4	2
9	3



Essential Understanding You can graph a square root function by plotting points or using a translation of the parent square root function.

For real numbers, the value of the radicand cannot be negative. So the domain of a square root function is limited to values of x for which the radicand is greater than or equal to 0.

Problem 1 Finding the Domain of a Square Root Function

What is the domain of the function $y = 2\sqrt{3x - 9}$?

Think	Write
The radicand cannot be negative.	$3x - 9 \geq 0$
Solve for *x*.	$3x \geq 9$ $x \geq 3$ The domain of the function is the set of real numbers greater than or equal to 3.

Got It? **1.** What is the domain of $y = \sqrt{-2x + 5}$?

Problem 2 Graphing a Square Root Function STEM

Plan

How can you solve this problem?
Make a chart of ordered pairs that satisfy the equation. Then plot the ordered pairs on a graph.

Engineering **Graph the function $I = \frac{1}{5}\sqrt{P}$, which gives the current I in amperes for a certain circuit with P watts of power. When will the current exceed 2 amperes?**

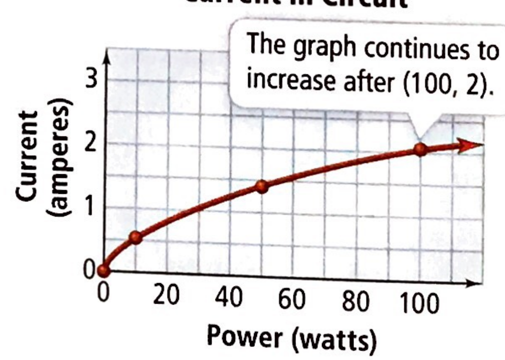
Step 1 Make a table.

Current in Circuit

Power (watts)	Current (amperes)
0	0
10	0.6
50	1.4
100	2

Step 2 Plot the points on a graph.

Current in Circuit



The current will exceed 2 amperes when the power is more than 100 watts.

Got It? **2.** **a.** When will the current in Problem 2 exceed 1.5 amperes?
b. **Reasoning** By how many times must you increase the power to double the current?

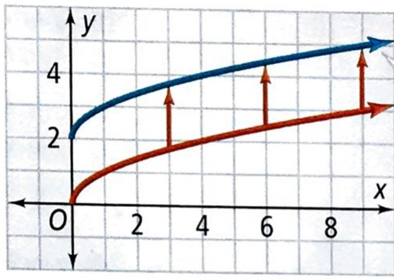
For any positive number k, graphing $y = \sqrt{x} + k$ translates the graph of $y = \sqrt{x}$ up k units. Graphing $y = \sqrt{x} - k$ translates the graph of $y = \sqrt{x}$ down k units.

Problem 3 Graphing a Vertical Translation

Think

Is this similar to a problem you've seen before?
Yes. You have graphed functions of the form $y = |x| + k$ by translating the graph of $y = |x|$.

What is the graph of $y = \sqrt{x} + 2$?



For the graph of $y = \sqrt{x} + 2$, the graph of $y = \sqrt{x}$ is shifted 2 units up.

Got It? **3.** What is the graph of $y = \sqrt{x} - 3$?

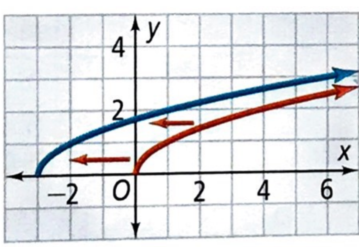
For any positive number h, graphing $y = \sqrt{x + h}$ translates the graph of $y = \sqrt{x}$ to the left h units. Graphing $y = \sqrt{x - h}$ translates the graph of $y = \sqrt{x}$ to the right h units.

Problem 4 Graphing a Horizontal Translation

Think

Is there another way to solve this problem?
Yes. You could make a table of ordered pairs that satisfy the equation and then plot them.

What is the graph of $y = \sqrt{x + 3}$?



For the graph of $y = \sqrt{x + 3}$, the graph of $y = \sqrt{x}$ is shifted 3 units to the left.

Got It? **4.** What is the graph of $y = \sqrt{x - 3}$?

Lesson Check

Do you know HOW?

1. What is the domain of the function $y = \sqrt{x + 3}$?

Graph each function.

2. $y = 2\sqrt{x}$
3. $y = \sqrt{x} - 6$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Is $y = x\sqrt{5}$ a square root function? Explain.
5. **Writing** Explain how the graph of $y = \sqrt{x - 1}$ is related to the graph of $y = \sqrt{x}$.
6. **Reasoning** Can the domain of a square root function include negative numbers? Explain.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Find the domain of each function. See Problem 1.

7. $y = \frac{1}{2}\sqrt{x}$
8. $y = \sqrt{x} + 2$
9. $y = \sqrt{x - 7}$
10. $y = 3\sqrt{\frac{x}{3}}$
11. $y = 2.7\sqrt{x + 2} + 11$
12. $y = \sqrt{4x - 13}$
13. $y = \frac{4}{7}\sqrt{18 - x}$
14. $y = \sqrt{3x + 9} - 6$
15. $y = \sqrt{3(x - 4)}$

Make a table of values and graph each function. See Problem 2.

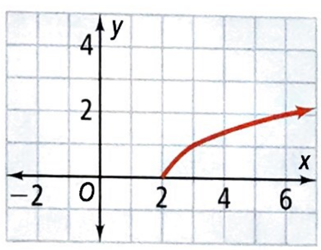
16. $y = \sqrt{2x}$
17. $f(x) = 4\sqrt{x}$
18. $y = \sqrt{4x - 8}$
19. $y = \sqrt{3x}$
20. $f(x) = 3\sqrt{x}$
21. $y = -3\sqrt{x}$
22. $f(x) = \frac{1}{3}\sqrt{x}$
23. $y = \sqrt{\frac{x}{2}}$
24. $y = 2\sqrt{x - 3}$

STEM 25. **Physics** The function $v = \sqrt{19.6h}$ models an object's velocity v in meters per second after it has fallen h meters, ignoring the effects of air resistance. Make a table and graph the function. For what values of h will the object's velocity be more than 10 m/s?

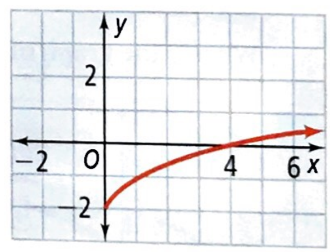
Match each function with its graph. See Problems 3 and 4.

26. $y = \sqrt{x + 4}$
27. $y = \sqrt{x - 2}$
28. $y = \sqrt{x} + 4$
29. $y = \sqrt{x} - 2$

A.



B.



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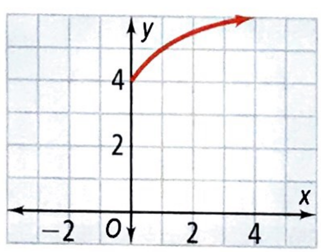
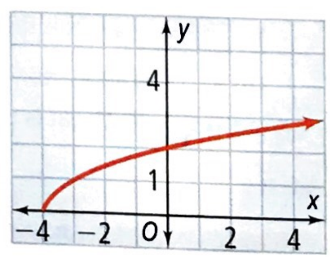


D.



Graph each function by translating the graph of $y = \sqrt{x}$.

30. $y = \sqrt{x} + 5$
31. $y = \sqrt{x} - 5$
32. $y = \sqrt{x} - 1$
33. $y = \sqrt{x + 2}$
34. $f(x) = \sqrt{x - 5}$
35. $f(x) = \sqrt{x - 4}$
36. $y = \sqrt{x} + 1$
37. $y = \sqrt{x + 1}$
38. $y = \sqrt{x - 1}$

B Apply

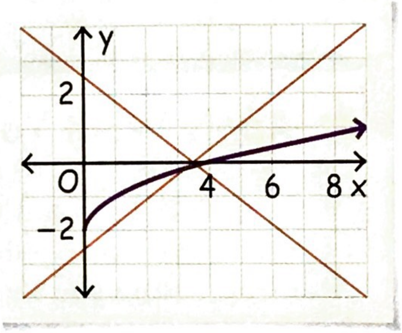
39. What are the domain and the range of the function $y = \sqrt{2x - 8}$?
40. What are the domain and the range of the function $y = \sqrt{8 - 2x}$?

41. **Firefighting** When firefighters are trying to put out a fire, the rate at which they can spray water on the fire depends on the nozzle pressure. You can find the flow rate f in gallons per minute using the function $f = 120\sqrt{p}$, where p is the nozzle pressure in pounds per square inch.

a. Graph the function.

b. What nozzle pressure gives a flow rate of 800 gal/min?

42. **Error Analysis** A student graphed the function $y = \sqrt{x-2}$ at the right. What mistake did the student make? Draw the correct graph.



43. **Think About a Plan** The velocity v in meters per second of a 2,000,000-kg rocket is given by the function $v = \sqrt{E}$, where E is the rocket's kinetic energy in megajoules (MJ). When the rocket's kinetic energy is 8,000,000 MJ, what is its velocity?

- How can you use a graph to solve the problem?
- How can you check your answer?

Make a table of values and graph each function.

44. $y = \sqrt{x - 2.5}$

45. $f(x) = 4\sqrt{x}$

46. $y = \sqrt{x+6}$

47. $y = \sqrt{0.5x}$

48. $y = \sqrt{x-2} + 3$

49. $f(x) = \sqrt{x+2} - 4$

50. $y = \sqrt{2x} + 3$

51. $y = \sqrt{2x+6} + 1$

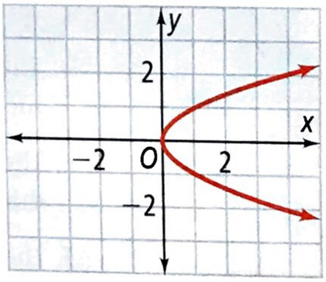
52. $y = \sqrt{3x-3} - 2$

53. The graph of $x = y^2$ is shown at the right.

a. Is this the graph of a function?

b. How does $x = y^2$ relate to the square root function $y = \sqrt{x}$?

c. **Reasoning** What is a function for the part of the graph that is shown in Quadrant IV? Explain.



54. **Reasoning** Without graphing, determine which graph rises more steeply, $y = \sqrt{3x}$ or $y = 3\sqrt{x}$. Explain your answer.

Graph each function by translating the graph of $y = \sqrt{x}$.

55. $y = \sqrt{x+4} - 1$

56. $y = \sqrt{x+1} + 5$

57. $y = \sqrt{x-3} - 2$

58. $y = \sqrt{x-6} + 3$

59. $y = \sqrt{x+2.5} - 1$

60. $y = \sqrt{x-4.5} + 1.5$

Challenge

61. **a.** Graph $y = \sqrt{x^2} + 5$.

b. Write a function for the graph you drew that does not require a radical.

62. In parts (a)–(d), graph each function.

a. $y = \sqrt{4x}$

b. $y = \sqrt{5x}$

c. $y = \sqrt{6x}$

d. $y = \sqrt{-6x}$

e. **Reasoning** Describe how the graph of $y = \sqrt{nx}$ changes as the value of n varies.

63. **Data Collection** Mark at least 6 places on a ramp that is at least 6 ft long. For each mark, measure the distance d from the mark to the bottom of the ramp. Measure the time t it takes a ball to roll from each mark to the bottom of the ramp.

a. Graph the data points (d, t). Connect the points with a smooth curve.

b. Describe your graph. What function does it resemble?

c. Is the graph linear? Why or why not?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 5

In the Apply What You've Learned in Lesson 10-2, you wrote a formula that gives the speed s of the car described on page 613, in terms of the length d of the skid marks. The formula is a square root function, $s = \sqrt{21d}$. Use a graphing calculator to graph this function. Then select all of the following that are true. Explain your reasoning.

A. The domain of the function $s = \sqrt{21d}$ is the set of all real numbers greater than or equal to 0.

B. The graph of $s = \sqrt{21d}$ lies entirely in Quadrant I.

C. The graph of $s = \sqrt{21d}$ is a vertical translation of the graph of the parent square root function.

D. The graph of $s = \sqrt{21d}$ is a horizontal translation of the graph of the parent square root function.

E. The graph of $s = \sqrt{21d}$ shows that the speed of the car increases as the length of the skid marks increases.

F. The graph of $s = \sqrt{21d}$ intersects the horizontal line $s = 50$.