10-6 Trigonometric Ratios

Common Core State Standards

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Also G-SRT.C.6**

MP 1, MP 3, MP 4, MP 5, MP 5, MP 6

Objective To find and use trigonometric ratios

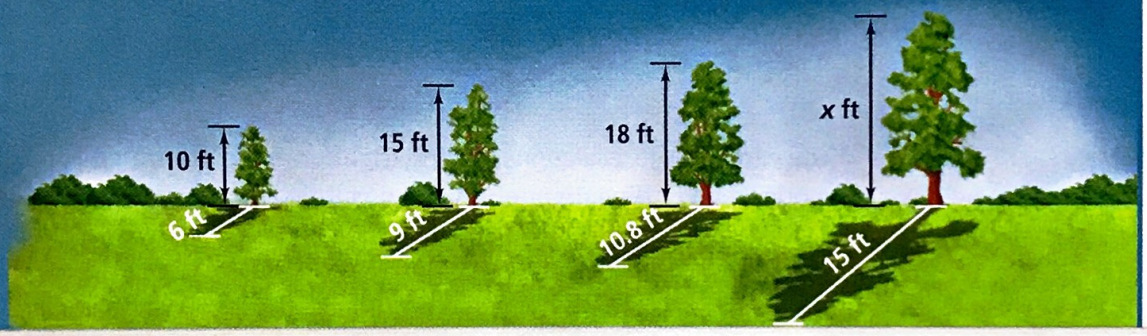
There is a lot of information here. Choose what you need to solve the problem.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

The sun strikes the four trees shown below at the same angle. You know the heights of three of the trees. How tall is the fourth tree? How do you know?



Lesson Vocabulary

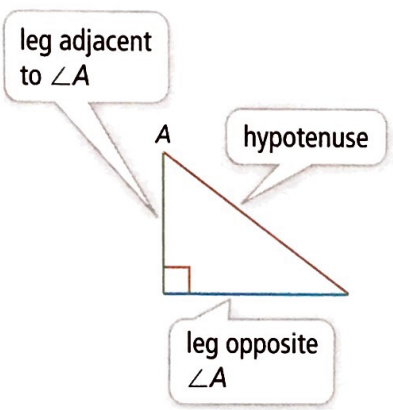
- trigonometric ratios
- sine
- cosine
- tangent
- angle of elevation
- angle of depression

The Solve It involves ratios and right triangles. Ratios of the side lengths of a right triangle are called **trigonometric ratios.** Below are the definitions of three trigonometric ratios.

take note

Key Concept Trigonometric Ratios

Name	Written	Definition
sine of $\angle A$	$\sin A$	$\frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$
cosine of $\angle A$	$\cos A$	$\frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$
tangent of $\angle A$	$\tan A$	$\frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$



Essential Understanding You can use the sine, cosine, and tangent ratios to find the measurements of sides and angles of right triangles.

Problem 1 Finding Trigonometric Ratios

Plan

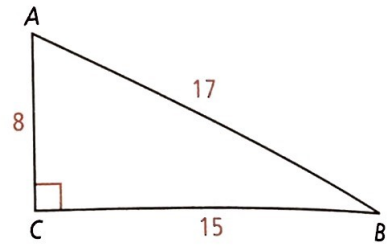
How do you calculate trigonometric ratios? Calculate a trigonometric ratio by substituting the lengths of the appropriate sides into the ratio.

What are sin A, cos A, and tan A for the triangle shown?

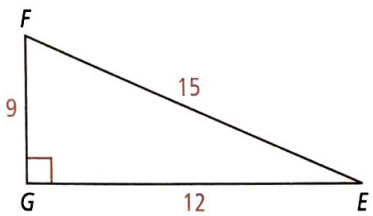
$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{8}{17}$$

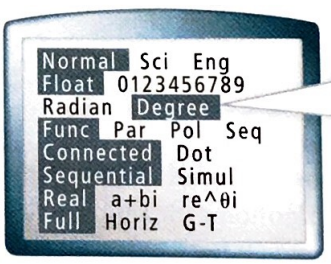
$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{15}{8}$$



Got It? 1. What are sin E, cos E, and tan E for the triangle below?



You can also use a calculator to find trigonometric ratios. In this chapter, use Degree mode when finding trigonometric ratios. That allows you to enter angles in degrees.



Set your calculator to Degree mode.

Problem 2 Finding a Trigonometric Ratio

GRIDDED RESPONSE

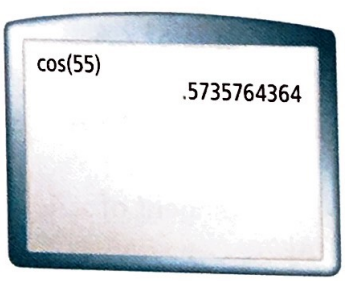
Think

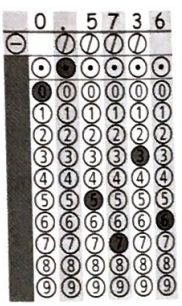
What is an upper limit on the value of the cosine? The cosine is the ratio $\frac{\text{adjacent leg}}{\text{hypotenuse}}$ in a right triangle. The hypotenuse is always the longest side. The cosine of an acute angle is always less than 1.

What is the value of cos 55° to the nearest ten-thousandth?

To find cos 55°, press [cos] 55 [)] [enter].

The cosine of 55° is approximately 0.5736.





Got It? 2. What is the value of each expression in parts (a)–(d)?

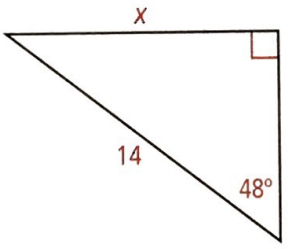
a. sin 80° **b.** tan 45° **c.** cos 15° **d.** sin 9°

e. Reasoning Describe the relationship between sin 45° and cos 45°. Explain why this is true.

You can use trigonometry to find missing lengths in a right triangle when you know the length of one side and the measure of an acute angle.

Problem 3 Finding a Missing Side Length

To the nearest tenth, what is the value of x in the triangle at the right?



Know
- The measure of an acute angle
- The length of the hypotenuse

Need The length of the opposite leg

Plan Use the sine ratio.

Write an equation and solve.

$\sin 48° = \frac{\text{opposite leg}}{\text{hypotenuse}}$ Use the definition of sine.

$\sin 48° = \frac{x}{14}$ Substitute x and 14 from the diagram.

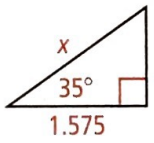
$x = 14(\sin 48°)$ Solve for x.

$x \approx 10.40402756$ Use a calculator.

$x \approx 10.4$ Round to the nearest tenth.

The value of x is about 10.4.

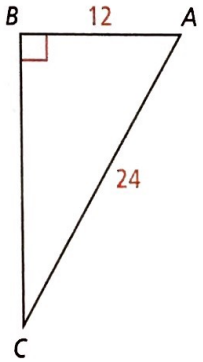
Got It? 3. To the nearest tenth, what is the value of x in the triangle at the right?



If you know the lengths of two sides of a right triangle, you can find a trigonometric ratio for each acute angle of the triangle. If you know a trigonometric ratio for an angle, you can use the inverse of the trigonometric ratio to find the measure of the angle. Use the $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ feature on your calculator.

Problem 4 Finding the Measures of Angles

What is the measure of each angle in the triangle at the right?



Step 1 Since you know the length of the side adjacent to $\angle A$ and the length of the hypotenuse, use the cosine ratio.

Step 2 Write an equation and solve.

$\cos A = \frac{12}{24}$ Use the definition of cosine.

$\cos A = 0.5$ Divide.

measure of $\angle A = \cos^{-1}(0.5)$ Use the inverse of cosine.

measure of $\angle A = 60°$ Use a calculator.

$\angle A$ measures 60°. The right angle B measures 90°. $\angle C$ measures $180° - 90° - 60° = 30°$.

Think

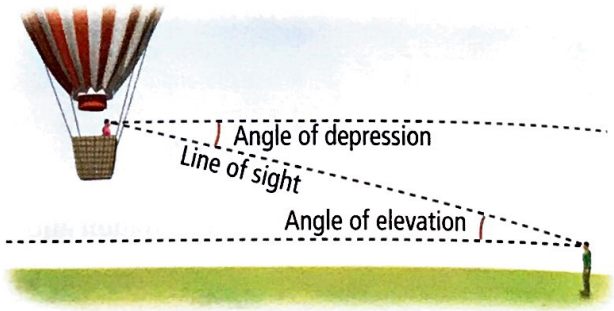
How can you find the measure of the third angle of the triangle?
The sum of the measures of the angles of a triangle is 180°. You can subtract the measures of the two known angles from 180° to find the third angle's measure.

Got It? **4.** In a right triangle, the side opposite $\angle A$ is 8 mm long and the hypotenuse is 12 mm long. What is the measure of $\angle A$?

You can use trigonometric ratios to measure some distances indirectly. To measure such distances, it is often convenient to use an *angle of elevation* or an *angle of depression.*

An **angle of elevation** is an angle from the horizontal up to a line of sight.

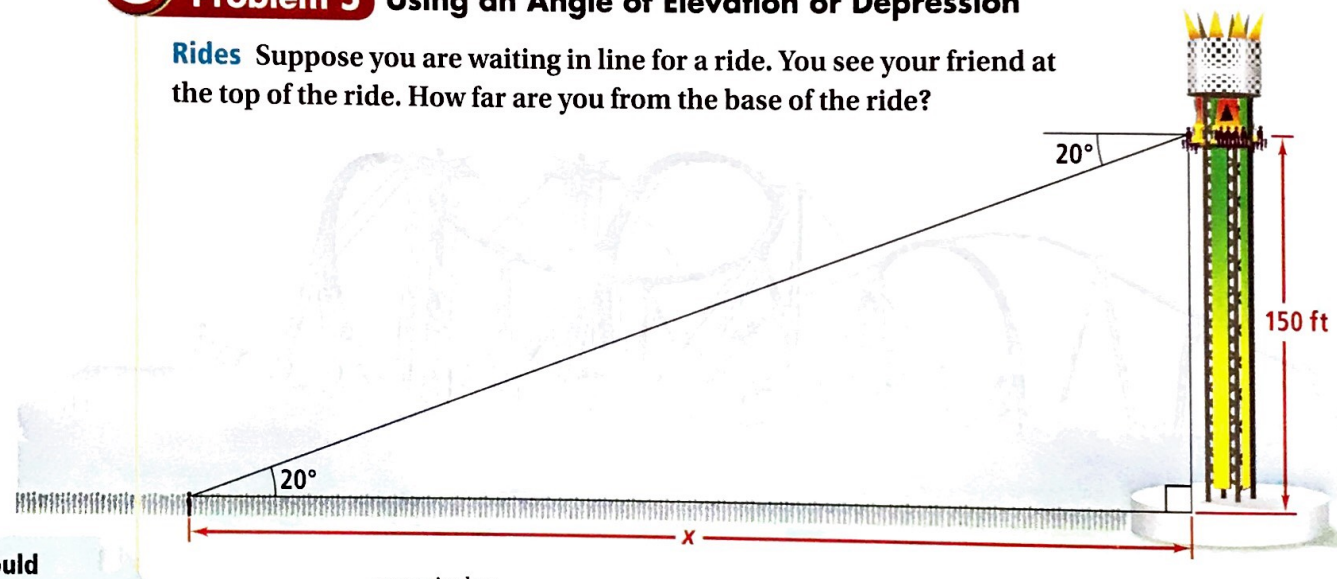
An **angle of depression** is an angle from the horizontal down to a line of sight.



When you solve real-world problems using trigonometric ratios, you often need to round your answers. The problem may tell you how to round. Otherwise, round your answers to the precision of the measurements used in the problem. For instance, if the problem has measurements to the nearest 10 ft, round your answer to the nearest 10 ft.

Problem 5 Using an Angle of Elevation or Depression

Rides **Suppose you are waiting in line for a ride. You see your friend at the top of the ride. How far are you from the base of the ride?**



Think

Which ratio should you use?
You know the length of the opposite leg and want to find the length of the adjacent leg. Use the tangent ratio.

$$\tan 20° = \frac{\text{opposite leg}}{\text{adjacent leg}}$$ Use the tangent ratio.

$$\tan 20° = \frac{150}{x}$$ Substitute 150 and x from the diagram.

$$x(\tan 20°) = 150$$ Multiply each side by x.

$$x = \frac{150}{\tan 20°}$$ Divide each side by $\tan 20°$.

$$x \approx 412.1216129$$ Use a calculator.

$$x \approx 410$$ Round to the nearest 10 ft.

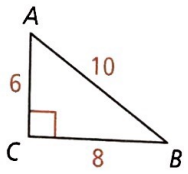
You are about 410 ft from the base of the ride.

Got It? **5.** After you move forward in the line, the angle of elevation to the top of the ride becomes 50°. How far are you from the base of the ride now?

Lesson Check

Do you know HOW?

Find each trigonometric ratio for angle A in the triangle at the right.



1. $\sin A$
2. $\cos A$
3. $\tan A$

Solve using trigonometric ratios.

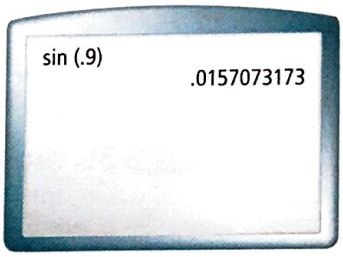
4. A right triangle has a 40° angle. The hypotenuse is 10 cm long. What is the length of the side opposite the 40° angle?

5. A right triangle's legs are 7 in. and 24 in. long. What is the measure of the angle opposite the 24-in. leg?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** Describe the difference between finding the sine of an angle and the cosine of an angle.

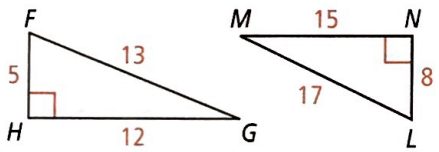
7. **Error Analysis** In a right triangle, the hypotenuse is 5 in. long, and the side opposite $\angle A$ is 4.5 in. long. A student found the measure of $\angle A$ as shown on the calculator screen at the right. Describe and correct the student's error.



Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

For $\triangle FGH$ and $\triangle LMN$, find the value of each expression. See Problem 1.



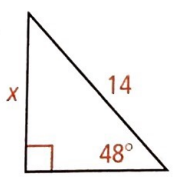
8. $\sin F$
9. $\cos F$
10. $\tan G$
11. $\cos L$
12. $\tan M$
13. $\sin M$
14. $\tan F$
15. $\sin G$
16. $\tan L$

Find the value of each expression. Round to the nearest ten-thousandth. See Problem 2.

17. $\sin 10°$
18. $\tan 25°$
19. $\cos 85°$
20. $\tan 12°$
21. $\sin 70°$
22. $\cos 22°$
23. $\sin 71°$
24. $\tan 30°$

Find the value of x to the nearest tenth. See Problem 3.

25.


26.


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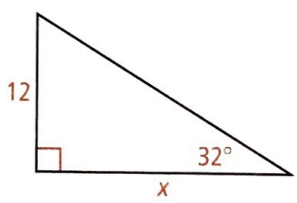

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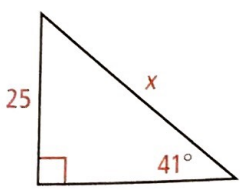

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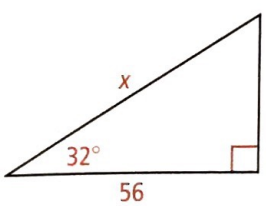

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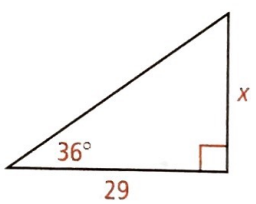
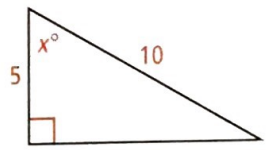


Find the value of x to the nearest degree.

See Problem 4.

31.



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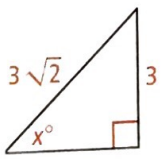

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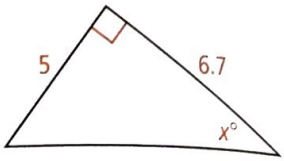
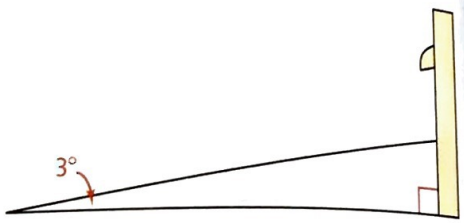


See Problem 5.

STEM **34. Geology** From an observation point 20 ft from the base of a geyser, the angle of elevation to the top of the geyser is 50°. How tall is the geyser?

STEM **35. Architecture** The wheelchair ramp shown is being planned for a new building. The ramp will rise a total of 2.5 ft and form a 3° angle with the ground. How far from the base of the building should the wheelchair ramp start?



B Apply

36. a. Find the values of each pair of expressions.

i. sin 80°, cos 10° ii. cos 25°, sin 65°

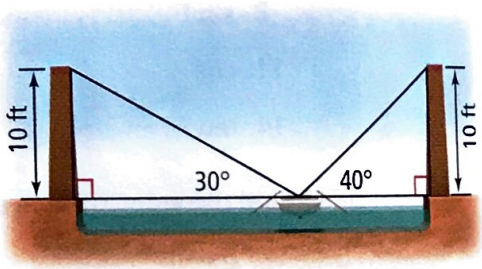
b. What do you notice about your values and the angles in each pair?

c. Reasoning Explain why your results make sense.

37. Writing Describe how you can find the length of the hypotenuse of a right triangle if you know the measure of one of the acute angles and the length of the leg adjacent to that angle.

38. Think About a Plan A boat is passing between two towers, as shown in the diagram. How far does the boat need to move to be in the middle of the channel?

- How far is the center of the boat from each tower?
- What is the distance from the base of a tower to the middle of the channel?



Find the value of each variable in each figure to the nearest tenth.

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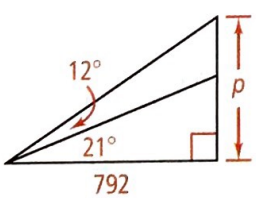

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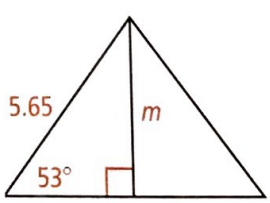

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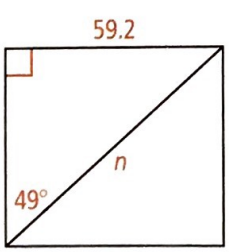

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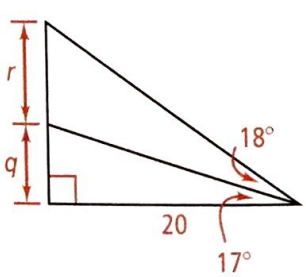

43. a. Aviation A pilot is flying a plane at an altitude of 30,000 ft. The angle of depression from the plane to the start of an airport runway is 1°. How far is the plane from the start of the runway, in horizontal distance along the ground?

b. What is your answer to part (a) in miles?

44. Hobbies Suppose you are flying a kite. The kite string is 60 m long, and the angle of elevation of the string is 65° from your hand. Your hand is 1 m above the ground. How high above the ground is the kite?

Challenge

45. At a certain point in a large, level park, the angle of elevation to the top of an office building is 30°. If you move 400 ft closer to the building, the angle of elevation is 45°. To the nearest 10 ft, how tall is the building?

46. A line passes through the origin of a coordinate plane and forms a 14° angle with the positive x-axis. What is the slope of the line? Round to the nearest hundredth.

47. Reasoning Use the definitions of sine, cosine, and tangent to simplify each expression.

a. $\cos A \cdot \tan A$ **b.** $\sin A \div \tan A$ **c.** $\sin A \div \cos A$

Standardized Test Prep

SAT/ACT

48. What is the value of b in the proportion $\frac{3}{7} = \frac{2b}{4b+2}$?

(A) 3 (C) 12

(B) 6 (D) 28

49. The profits of a large corporation can be graphed as a line that passes through $(-3, 6)$ and $(4, -1)$. Which equation represents the line?

(F) $y = 3 - x$ (H) $y = -3x + 1$

(G) $y = 3x + 1$ (I) $y = x + 3$

50. Which expression is equivalent to $3\sqrt{12} + 2\sqrt{3}$?

(A) $5\sqrt{3}$ (C) $5\sqrt{15}$

(B) $8\sqrt{3}$ (D) $8\sqrt{12}$

Short Response

51. Graph the solutions of the inequality $-2x \geq 1$ on a number line.

Mixed Review

Graph each function. See Lesson 10-5.

52. $y = \sqrt{x} + 8$ **53.** $y = \sqrt{x - 6}$ **54.** $y = 4\sqrt{x}$

Determine whether the given lengths can be side lengths of a right triangle. See Lesson 10-1.

55. 15, 36, 39 **56.** $\frac{7}{9}, \frac{24}{9}, \frac{25}{9}$ **57.** 12, 35, 36

Get Ready! To prepare for Lesson 11-1, do Exercises 58–61.

Factor each expression. See Lesson 8-5.

58. $x^2 + x - 12$ **59.** $x^2 + 6x + 8$ **60.** $x^2 - 2x - 15$ **61.** $x^2 + 9x + 18$