

2-3

Solving Multi-Step Equations

Common Core State Standards

A-CED.A.1 Create equations . . . in one variable and use them to solve problems. *Include equations arising from linear . . . functions.* Also A-REI.A.1, A-REI.B.3
MP 1, MP 2, MP 3, MP 4

Objective To solve multi-step equations in one variable



Try some numbers to help you construct a pattern for this problem.



MATHEMATICAL PRACTICES

Getting Ready!

You are buying movie tickets online for a group of your friends. You have to enter the number of tickets you want on the screen shown. You are using a debit card to pay for the tickets and have a total of \$45 to spend. How many tickets can you buy? Explain your answer.

Number of tickets you would like to purchase:

Ticket price	Processing fee	Service charge	Total
$\$9.00 \times$ <input style="width: 40px;" type="text"/>	$+$ $\$1.00 \times$ <input style="width: 40px;" type="text"/>	$+$ $\$5.00$	$=$ $\$$ <input style="width: 40px;" type="text"/>

In this lesson, you will learn to write and solve multi-step equations.

Essential Understanding To solve multi-step equations, you form a series of simpler equivalent equations. To do this, use the properties of equality, inverse operations, and properties of real numbers. You use the properties until you isolate the variable.

Think

How is this equation different from equations you've seen before?

The variable occurs in two terms. You can simplify the equation by grouping like terms and combining them.



Problem 1 Combining Like Terms

What is the solution of $5 = 5m - 23 + 2m$?

$$5 = 5m - 23 + 2m$$

$$5 = 5m + 2m - 23 \quad \text{Commutative Property of Addition}$$

$$5 = 7m - 23 \quad \text{Combine like terms.}$$

$$5 + 23 = 7m - 23 + 23 \quad \text{Add 23 to each side.}$$

$$28 = 7m \quad \text{Simplify.}$$

$$\frac{28}{7} = \frac{7m}{7} \quad \text{Divide each side by 7.}$$

$$4 = m \quad \text{Simplify.}$$

Check $5 = 5m - 23 + 2m$

$$5 \stackrel{?}{=} 5(4) - 23 + 2(4) \quad \text{Substitute 4 for } m.$$

$$5 = 5 \quad \checkmark \quad \text{Simplify. The solution checks.}$$



Got It? 1. What is the solution of each equation? Check each answer.

a. $11m - 8 - 6m = 22$

b. $-2y + 5 + 5y = 14$



Problem 2 Solving a Multi-Step Equation

Concert Merchandise Martha takes her niece and nephew to a concert. She buys T-shirts and bumper stickers for them. The bumper stickers cost \$1 each. Martha's niece wants 1 shirt and 4 bumper stickers, and her nephew wants 2 shirts but no bumper stickers. If Martha's total is \$67, what is the cost of one shirt?

Know

- Bumper stickers cost \$1
- Niece's items: 1 shirt, 4 bumper stickers
- Nephew's items: 2 shirts
- Total spent: \$67

Need

The cost of one shirt

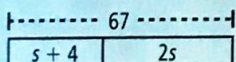
Plan

Write and solve an equation that models the situation.

Think

How can a model help you write the equation?

The model shows that the cost of the niece's items plus the cost of the nephew's items is \$67.



Relate cost of niece's items (1 shirt and 4 stickers) plus cost of nephew's items (2 shirts) is total Martha spent

Define Let s = the cost of one shirt.

Write $(s + 4)$ + $2s$ = 67

$$(s + 4) + 2s = 67$$

$$s + 2s + 4 = 67 \quad \text{Commutative Property of Addition}$$

$$3s + 4 = 67 \quad \text{Combine like terms.}$$

$$3s + 4 - 4 = 67 - 4 \quad \text{Subtract 4 from each side.}$$

$$3s = 63 \quad \text{Simplify.}$$

$$\frac{3s}{3} = \frac{63}{3} \quad \text{Divide each side by 3.}$$

$$s = 21 \quad \text{Simplify.}$$

One shirt costs \$21.



Got It? 2. Noah and Kate are shopping for new guitar strings in a music store.

Noah buys 2 packs of strings. Kate buys 2 packs of strings and a music book. The book costs \$16. Their total cost is \$72. How much is one pack of strings?



Problem 3 Solving an Equation Using the Distributive Property

What is the solution of $-8(2x - 1) = 36$?

$$-8(2x - 1) = 36$$

$$-16x + 8 = 36 \quad \text{Distributive Property}$$

$$-16x + 8 - 8 = 36 - 8 \quad \text{Subtract 8 from each side.}$$

$$-16x = 28 \quad \text{Simplify.}$$

$$\frac{-16x}{-16} = \frac{28}{-16} \quad \text{Divide each side by } -16.$$

$$x = -\frac{7}{4} \quad \text{Simplify.}$$

Think

How can you make the equation easier to solve?

Remove the grouping symbols by using the Distributive Property.



- Got It?** 3. a. What is the solution of $18 = 3(2x - 6)$? Check your answer.
 b. **Reasoning** Can you solve the equation in part (a) by using the Division Property of Equality instead of the Distributive Property? Explain.

You can use different methods to solve equations that contain fractions.



Problem 4 Solving an Equation That Contains Fractions

What is the solution of $\frac{3x}{4} - \frac{x}{3} = 10$?

Method 1 Write the like terms using a common denominator and solve.

$$\frac{3}{4}x - \frac{1}{3}x = 10 \quad \text{Rewrite the fractions.}$$

$$\frac{9}{12}x - \frac{4}{12}x = 10 \quad \text{Write the fractions using a common denominator, 12.}$$

$$\frac{5}{12}x = 10 \quad \text{Combine like terms.}$$

$$\frac{12}{5} \left(\frac{5}{12}x \right) = \frac{12}{5}(10) \quad \text{Multiply each side by } \frac{12}{5}, \text{ the reciprocal of } \frac{5}{12}.$$

$$x = 24 \quad \text{Simplify.}$$

Method 2 Clear the fractions from the equation.

$$12 \left(\frac{3x}{4} - \frac{x}{3} \right) = 12(10) \quad \text{Multiply each side by a common denominator, 12.}$$

$$12 \left(\frac{3x}{4} \right) - 12 \left(\frac{x}{3} \right) = 12(10) \quad \text{Distributive Property}$$

$$9x - 4x = 120 \quad \text{Multiply.}$$


$$5x = 120 \quad \text{Combine like terms.}$$

$$x = 24 \quad \text{Divide each side by 5 and simplify.}$$

Plan

How do you get started?

You can either combine like terms by writing the fractions with a common denominator, or you can clear the fractions from the equation.

 **Got It?** 4. What is the solution of each equation? Why did you choose the method you used to solve each equation?

a. $\frac{2b}{5} + \frac{3b}{4} = 3$

b. $\frac{1}{9} = \frac{5}{6} - \frac{m}{3}$

You can clear decimals from an equation by multiplying by a power of 10. First, find the greatest number of digits to the right of any decimal point, and then multiply by 10 raised to that power.

 **Problem 5 Solving an Equation That Contains Decimals**

What is the solution of $3.5 - 0.02x = 1.24$?

Plan

The equation contains tenths (3.5) and hundredths (0.02 and 1.24). The greatest number of digits to the right of any decimal point is 2. So, multiply each side of the equation by 10^2 , or 100, to clear the decimals.

$$3.5 - 0.02x = 1.24$$

$$100(3.5 - 0.02x) = 100(1.24) \quad \text{Multiply each side by } 10^2, \text{ or } 100.$$


$$350 - 2x = 124 \quad \text{Distributive Property}$$

$$350 - 2x - 350 = 124 - 350 \quad \text{Subtract 350 from each side.}$$

$$-2x = -226 \quad \text{Simplify.}$$

$$\frac{-2x}{-2} = \frac{-226}{-2} \quad \text{Divide each side by } -2.$$

$$x = 113 \quad \text{Simplify.}$$

 **Got It?** 5. What is the solution of $0.5x - 2.325 = 3.95$? Check your answer.

Think

When you multiply a decimal by 10^n , where n is a positive integer, you can move the decimal point n places to the right. For example, $100(3.5) = 350$.

 **Lesson Check**

Do you know HOW?

Solve each equation. Check your answer.

1. $7p + 8p - 12 = 59$

2. $-2(3x + 9) = 24$

3. $\frac{2m}{7} + \frac{3m}{14} = 1$

4. $1.2 = 2.4 - 0.6x$

5. **Gardening** There is a 12-ft fence on one side of a rectangular garden. The gardener has 44 ft of fencing to enclose the other three sides. What is the length of the garden's longer dimension?


Do you UNDERSTAND?  **MATHEMATICAL PRACTICES**

Explain how you would solve each equation.

6. $1.3 + 0.5x = -3.41$

7. $7(3x - 4) = 49$

8. $-\frac{2}{9}x - 4 = \frac{7}{18}$

 9. **Reasoning** Ben solves the equation $-24 = 5(g + 3)$ by first dividing each side by 5. Amelia solves the equation by using the Distributive Property. Whose method do you prefer? Explain.

A Practice

Solve each equation. Check your answer.

10. $7 - y - y = -1$

11. $72 + 4 - 14d = 36$

12. $13 = 5 + 3b - 13$

13. $6p - 2 - 3p = 16$

14. $x + 2 + x = 22$

15. $b - 9 + 6b = 30$

16. $9t - 6 - 6t = 6$

17. $17 = p - 3 - 3p$

18. $-23 = -2a - 10 + a$

← See Problem 1.

Write an equation to model each situation. Then solve the equation.

← See Problem 2.

19. **Employment** You have a part-time job. You work for 3 h on Friday and 6 h on Saturday. You also receive an allowance of \$20 per week. You earn \$92 per week. How much do you earn per hour at your part-time job?

20. **Travel** A family buys airline tickets online. Each ticket costs \$167. The family buys travel insurance with each ticket that costs \$19 per ticket. The Web site charges a fee of \$16 for the entire purchase. The family is charged a total of \$1132. How many tickets did the family buy?

Solve each equation. Check your answer.

← See Problem 3.

21. $64 = 8(r + 2)$

22. $5(2x - 3) = 15$

23. $5(2 + 4z) = 85$

24. $2(8 + 4c) = 32$

25. $7(f - 1) = 45$

26. $15 = -2(2t - 1)$

27. $26 = 6(5 - 4f)$

28. $n + 5(n - 1) = 7$

29. $-4(r + 6) = -63$

Solve each equation. Choose the method you prefer to use. Check your answer.

← See Problem 4.

30. $\frac{b}{13} - \frac{3b}{13} = \frac{8}{13}$

31. $5y - \frac{3}{5} = \frac{4}{5}$

32. $\frac{n}{5} - \frac{3n}{10} = \frac{1}{5}$

33. $\frac{2}{3} + \frac{3m}{5} = \frac{31}{15}$

34. $\frac{n}{2} - \frac{2n}{16} = \frac{3}{8}$

35. $\frac{b}{3} + \frac{1}{8} = 19$

36. $\frac{1}{4} + \frac{4x}{5} = \frac{11}{20}$

37. $\frac{11z}{16} + \frac{7z}{8} = \frac{5}{16}$

38. $\frac{x}{3} - \frac{7x}{12} = \frac{2}{3}$

Solve each equation. Check your answer.

← See Problem 5.

39. $1.06g - 3 = 0.71$

40. $0.11k + 1.5 = 2.49$

41. $1.025v + 2.458 = 7.583$

42. $1.12 + 1.25g = 8.62$

43. $25.24 = 5g + 3.89$

44. $0.25n + 0.1n = 9.8$

B Apply

Solve each equation.

45. $6 + \frac{v}{-8} = \frac{4}{7}$

46. $\frac{2}{3}(c - 18) = 7$

47. $3d + d - 7 = \frac{25}{4}$

48. $0.25(d - 12) = 4$

49. $8n - (2n - 3) = 12$

50. $\frac{2}{3} + n + 6 = \frac{3}{4}$

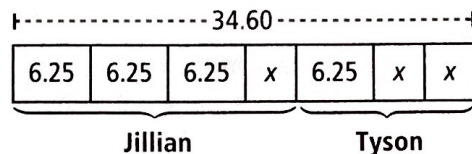
51. $0.5d - 3d + 5 = 0$

52. $-(w + 5) = -14$

53. $\frac{a}{20} + \frac{4}{15} = \frac{9}{15}$

- © 54. **Think About a Plan** Jillian and Tyson are shopping for knitting supplies. Jillian wants 3 balls of yarn and 1 set of knitting needles. Tyson wants 1 ball of yarn and 2 sets of knitting needles. Each ball of yarn costs \$6.25. If their total cost is \$34.60, what is the cost of 1 set of knitting needles?

- How can the model at the right help you solve the problem?
- How does the model tell you which operations to use in the equation?



55. **Online Video Games** Angie and Kenny play online video games. Angie buys 1 software package and 3 months of game play. Kenny buys 1 software package and 2 months of game play. Each software package costs \$20. If their total cost is \$115, what is the cost of one month of game play?

- © 56. **Error Analysis** Describe and correct the error in solving the equation at the right.

- © 57. **Reasoning** Suppose you want to solve $-4m + 5 + 6m = -3$. What would you do as your first step? Explain.

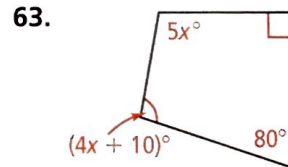
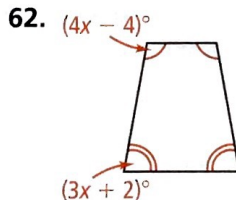
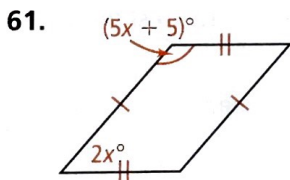
- © 58. **Writing** Describe two ways in which you can solve $-\frac{1}{2}(5x - 9) = 17$.

$$\begin{aligned} \frac{3x}{8} - 1 &= \frac{5}{8} \\ 8\left(\frac{3x}{8} - 1\right) &= 8\left(\frac{5}{8}\right) \\ 3x - 1 &= 5 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

59. **Bowling** Three friends go bowling. The cost per person per game is \$5.30. The cost to rent shoes is \$2.50 per person. Their total cost is \$55.20. How many games did they play?

60. **Moving Expenses** A college student is moving into a campus dormitory. The student rents a moving truck for \$19.95 plus \$.99 per mile. Before returning the truck, the student fills the tank with gasoline, which costs \$65.32. The total cost is \$144.67. How many miles did the student drive the truck?

Geometry Find the value of x . (*Hint: The sum of the angle measures of a quadrilateral is 360° .*)



64. **Dining Out** You are ordering a meal and have \$15 to spend. The restaurant charges 6% sales tax. You plan to leave a 15% tip. The equation $c = x + 0.06x + 0.15x$ gives the total cost c of your meal, where x is the cost before tax and tip. What is the maximum amount you can spend before tax and tip?

65. **Savings** You have \$85 in your bank account. Each week you plan to deposit \$8 from your allowance and \$15 from your paycheck. The equation $b = 85 + (15 + 8)w$ gives the amount b in your bank account after w weeks. How many weeks from now will you have \$175 in your bank account?



66. **Open-Ended** Find three consecutive integers with a sum of 45. Show your work.
67. **Cooking** A cook buys two identical bags of rice and uses some of the rice in each bag so that one bag is half full and the other is one-third full. The cook combines them into one bag, which then contains $3\frac{1}{3}$ cups of rice. How much rice was in a full bag?
68. **Painting** Tim can paint a house in 6 days. Tara can paint the same house in 3 days.
- What fraction of the house can Tim paint in one day? What fraction of the house can Tara paint in one day?
 - What fraction of the house can Tim paint in d days? What fraction of the house can Tara paint in d days?
 - What fraction of the house can Tim and Tara together paint in one day? What fraction of the house can Tim and Tara together paint in d days?
 - Write and solve an equation to find the number of days it will take Tim and Tara to paint the whole house working together.



Apply What You've Learned



Look back at the information given about the monorail on page 79. Let r represent the average speed of the monorail, in feet per second, between the parking garage and Terminal A.

- Write an expression for the distance the monorail travels between the parking garage and Terminal A. Then write an expression for the distance the monorail travels between Terminal A and Terminal B. (*Hint: Use the relationship $distance = rate \times time$.)*)
- Write an equation that relates the total distance the monorail travels between the parking garage and Terminal B to the two expressions you wrote in part (a).
- Solve the equation you wrote in part (b). Round your answer to the nearest tenth. Interpret your solution in terms of the situation.
- Use your result from part (c) to find the rate of the monorail between Terminal A and Terminal B.
- Do the rates you found in parts (c) and (d) make sense in terms of the diagram and the times given on page 79? Explain.