2-4 Solving Equations With Variables on Both Sides

Common Core State Standards

A-CED.A.1 Create equations . . . in one variable and use them to solve problems. *Include equations arising from linear . . . functions.* **Also A-REI.A.1, A-REI.B.3**

MP 1, MP 2, MP 3, MP 4

Objectives To solve equations with variables on both sides

To identify equations that are identities or have no solution

SOLVE IT! Getting Ready!

You could make a table to help you model a solution to this problem.

MATHEMATICAL PRACTICES

The diagram gives information about the populations of two towns. After how many years will the populations be equal? How do you know?

Lesson Vocabulary
- identity

The problem in the Solve It can be modeled by an equation that has variables on *both* sides.

Essential Understanding To solve equations with variables on both sides, you can use the properties of equality and inverse operations to write a series of simpler equivalent equations.

Problem 1 Solving an Equation With Variables on Both Sides

Plan

How do you get started?
There are variable terms on both sides of the equation. Decide which variable term to add or subtract to get the variable on one side only.

What is the solution of $5x + 2 = 2x + 14$?

$$5x + 2 = 2x + 14$$

$$5x + 2 - 2x = 2x + 14 - 2x \quad \text{Subtract } 2x \text{ from each side.}$$

$$3x + 2 = 14 \quad \text{Simplify.}$$

$$3x + 2 - 2 = 14 - 2 \quad \text{Subtract 2 from each side.}$$

$$3x = 12 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{12}{3} \quad \text{Divide each side by 3.}$$

$$x = 4 \quad \text{Simplify.}$$

Check $5x + 2 = 2x + 14$

$$5(4) + 2 \stackrel{?}{=} 2(4) + 14 \quad \text{Substitute 4 for } x.$$

$$22 = 22 \checkmark \quad \text{Simplify. The solution checks.}$$

Got It? 1. a. What is the solution of $7k + 2 = 4k - 10$?

b. **Reasoning** Solve the equation in Problem 1 by subtracting $5x$ from each side instead of $2x$. Compare and contrast your solution with the solution in Problem 1.

Problem 2 Using an Equation With Variables on Both Sides

Graphic Design **It takes a graphic designer 1.5 h to make one page of a Web site. Using new software, the designer could complete each page in 1.25 h, but it takes 8 h to learn the software. How many Web pages would the designer have to make in order to save time using the new software?**

Know
- Current design time: 1.5 h per page
- Time with new software: 1.25 h per page
- Time to learn software: 8 h

Need
The number of pages the designer needs to make for the new software to save time

Plan
Write and solve an equation that models the situation.

Think
How can a model help you write the equation?
The model shows that the current design time is equal to the new design time plus the 8 h needed to learn the new software.

1.5p	
1.25p	8

Relate current design time = design time with new software + time to learn software

Define Let p = the number of pages the designer needs to make.

Write $1.5p \quad = \quad 1.25p \quad + \quad 8$

$$1.5p = 1.25p + 8$$

$$1.5p - 1.25p = 1.25p + 8 - 1.25p \quad \text{Subtract } 1.25p \text{ from each side.}$$

$$0.25p = 8 \quad \text{Simplify.}$$

$$\frac{0.25p}{0.25} = \frac{8}{0.25} \quad \text{Divide each side by 0.25.}$$

$$p = 32 \quad \text{Simplify.}$$

It will take the designer the same amount of time to make 32 Web pages using either software. The designer must make 33 pages or more in order to save time using the new software.

Got It? 2. An office manager spent $650 on a new energy-saving copier that will reduce the monthly electric bill for the office from $112 to $88. In how many months will the copier pay for itself?

Problem 3 Solving an Equation With Grouping Symbols

Plan

How do you get started?
There are parentheses on both sides of the equation. So, remove the parentheses using the Distributive Property.

What is the solution of $2(5x - 1) = 3(x + 11)$?

$2(5x - 1) = 3(x + 11)$	
$10x - 2 = 3x + 33$	Distributive Property
$10x - 2 - 3x = 3x + 33 - 3x$	Subtract $3x$ from each side.
$7x - 2 = 33$	Simplify.
$7x - 2 + 2 = 33 + 2$	Add 2 to each side.
$7x = 35$	Simplify.
$\frac{7x}{7} = \frac{35}{7}$	Divide each side by 7.
$x = 5$	Simplify.

Got It? 3. What is the solution of each equation?

a. $4(2y + 1) = 2(y - 13)$ **b.** $7(4 - a) = 3(a - 4)$

An equation that is true for every possible value of the variable is an **identity.** For example, $x + 1 = x + 1$ is an identity. An equation has no solution if there is no value of the variable that makes the equation true. The equation $x + 1 = x + 2$ has no solution.

Problem 4 Identities and Equations With No Solution

Think

How can you tell how many solutions an equation has?
If you eliminate the variable in the process of solving, the equation is either an identity with infinitely many solutions or an equation with no solution.

What is the solution of each equation?

A $10x + 12 = 2(5x + 6)$

$10x + 12 = 2(5x + 6)$	
$10x + 12 = 10x + 12$	Distributive Property

Because $10x + 12 = 10x + 12$ is always true, there are infinitely many solutions of the equation. The original equation is an identity.

B $9m - 4 = -3m + 5 + 12m$

$9m - 4 = -3m + 5 + 12m$	
$9m - 4 = 9m + 5$	Combine like terms.
$9m - 4 - 9m = 9m + 5 - 9m$	Subtract $9m$ from each side.
$-4 = 5$ ✗	Simplify.

Because $-4 \neq 5$, the original equation has no solution.

Got It? 4. What is the solution of each equation?

a. $3(4b - 2) = -6 + 12b$ **b.** $2x + 7 = -1(3 - 2x)$

When you solve an equation, you use reasoning to select properties of equality that produce simpler equivalent equations until you find a solution. The steps below provide a general guideline for solving equations.

take note

Concept Summary Solving Equations

Step 1 Use the Distributive Property to remove any grouping symbols. Use properties of equality to clear decimals and fractions.

Step 2 Combine like terms on each side of the equation.

Step 3 Use the properties of equality to get the variable terms on one side of the equation and the constants on the other.

Step 4 Use the properties of equality to solve for the variable.

Step 5 Check your solution in the original equation.

Lesson Check

Do you know HOW?

Solve each equation. Check your answer.

1. $3x + 4 = 5x - 10$

2. $5(y - 4) = 7(2y + 1)$

3. $2a + 3 = \frac{1}{2}(6 + 4a)$

4. $4x - 5 = 2(2x + 1)$

5. Printing Pristine Printing will print business cards for \$.10 each plus a setup charge of \$15. The Printing Place offers business cards for \$.15 each with a setup charge of \$10. What number of business cards costs the same from either printer?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Vocabulary Match each equation with the appropriate number of solutions.

6. $3y - 5 = y + 2y - 9$ **A.** infinitely many

7. $2y + 4 = 2(y + 2)$ **B.** one solution

8. $2y - 4 = 3y - 5$ **C.** no solution

9. Writing A student solved an equation and found that the variable was eliminated in the process of solving the equation. How would the student know whether the equation is an identity or an equation with no solution?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice Solve each equation. Check your answer. See Problem 1.

10. $5x - 1 = x + 15$

11. $4p + 2 = 3p - 7$

12. $6m - 2 = 2m + 6$

13. $3 + 5q = 9 + 4q$

14. $8 - 2y = 3y - 2$

15. $3n - 5 = 7n + 11$

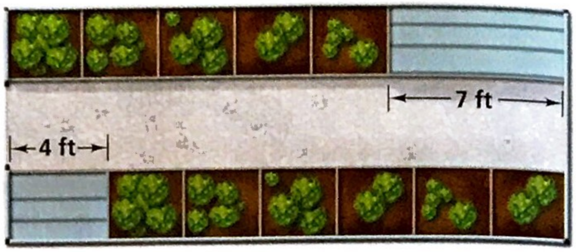
16. $2b + 4 = -18 - 9b$

17. $-3c - 12 = -5 + c$

18. $-n - 24 = 5 - n$

Write and solve an equation for each situation. Check your solution. See Problem 2.

STEM **19. Architecture** An architect is designing a rectangular greenhouse. Along one wall is a 7-ft storage area and 5 sections for different kinds of plants. On the opposite wall is a 4-ft storage area and 6 sections for plants. All of the sections for plants are of equal length. What is the length of each wall?



20. Business A hairdresser is deciding where to open her own studio. If the hairdresser chooses Location A, she will pay \$1200 per month in rent and will charge \$45 per haircut. If she chooses Location B, she will pay \$1800 per month in rent and will charge \$60 per haircut. How many haircuts would she have to give in one month to make the same profit at either location?

Solve each equation. Check your answer. See Problem 3.

21. $3(q - 5) = 2(q + 5)$

22. $8 - (3 + b) = b - 9$

23. $7(6 - 2a) = 5(-3a + 1)$

24. $(g + 4) - 3g = 1 + g$

25. $2r - (5 - r) = 13 + 2r$

26. $5g + 4(-5 + 3g) = 1 - g$

Determine whether each equation is an *identity* or whether it has *no solution*. See Problem 4.

27. $2(a - 4) = 4a - (2a + 4)$

28. $5y + 2 = \frac{1}{2}(10y + 4)$

29. $k - 3k = 6k + 5 - 8k$

30. $2(2k - 1) = 4(k - 2)$

31. $-6a + 3 = -3(2a - 1)$

32. $4 - d = -(d - 4)$

B Apply

Solve each equation. If the equation is an identity, write *identity*. If it has no solution, write *no solution*.

33. $3.2 - 4d = 2.3d + 3$

34. $3d + 4 = 2 + 3d - \frac{1}{2}$

35. $2.25(4x - 4) = -2 + 10x + 12$

36. $3a + 1 = -3.6(a - 1)$

37. $\frac{1}{2}h + \frac{1}{3}(h - 6) = \frac{5}{6}h + 2$

38. $0.5b + 4 = 2(b + 2)$

39. $-2(-c - 12) = -2c - 12$

40. $3(m + 1.5) = 1.5(2m + 3)$

41. Travel Suppose a family drives at an average rate of 60 mi/h on the way to visit relatives and then at an average rate of 40 mi/h on the way back. The return trip takes 1 h longer than the trip there.

a. Let d be the distance in miles the family traveled to visit their relatives. How many hours did it take to drive there?

b. In terms of d, how many hours did it take to make the return trip?

c. Write and solve an equation to determine the distance the family drove to see their relatives. What was the average rate for the entire trip?

42. **Think About a Plan** Each morning, a deli worker has to make several pies and peel a bucket of potatoes. On Monday, it took the worker 2 h to make the pies and an average of 1.5 min to peel each potato. On Tuesday, the worker finished the work in the same amount of time, but it took 2.5 h to make the pies and an average of 1 min to peel each potato. About how many potatoes are in a bucket?

- What quantities do you know and how are they related to each other?
- How can you use the known and unknown quantities to write an equation for this situation?

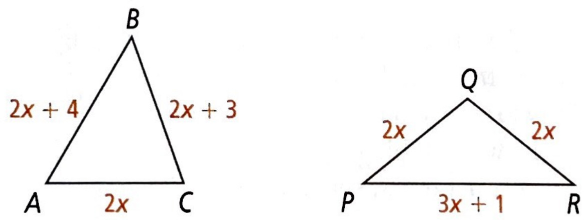
43. **Error Analysis** Describe and correct the error in finding the solution of the equation $2x = 6x$.

$$2x = 6x$$
$$\frac{2x}{x} = \frac{6x}{x}$$
$$2 \neq 6$$
The equation has no solution.

44. **Skiing** A skier is trying to decide whether or not to buy a season ski pass. A daily pass costs \$67. A season ski pass costs \$350. The skier would have to rent skis with either pass for \$25 per day. How many days would the skier have to go skiing in order to make the season pass less expensive than daily passes?

45. **Health Clubs** One health club charges a \$50 sign-up fee and \$65 per month. Another club charges a \$90 sign-up fee and \$45 per month. For what number of months is the cost of the clubs equal?

46. **Geometry** The perimeters of the triangles shown are equal. Find the side lengths of each triangle.



47. **Business** A small juice company spends \$1200 per day on business expenses plus \$1.10 per bottle of juice they make. They charge \$2.50 for each bottle of juice they produce. How many bottles of juice must the company sell in one day in order to equal its daily costs?

48. **Spreadsheet** You set up a spreadsheet to solve $7(x + 1) = 3(x - 1)$.

a. Does your spreadsheet show the solution of the equation?

b. Between which two values of x is the solution of the equation? How do you know?

c. For what spreadsheet values of x is $7(x + 1)$ less than $3(x - 1)$?

	A	B	C
1	x	$7(x + 1)$	$3(x - 1)$
2	−5	−28	−18
3	−3	−14	−12
4	−1	0	−6
5	1	14	0
6	3	28	6

49. **Reasoning** Determine whether each statement is *always*, *sometimes*, or *never* true.
  a. An equation of the form $ax + 1 = ax$ has no solution.
  b. An equation in one variable has at least one solution.
  c. An equation of the form $\frac{x}{a} = \frac{x}{b}$ has infinitely many solutions.

Challenge

Open-Ended Write an equation with a variable on both sides such that you get each solution.

50. $x = 5$

51. $x = 0$

52. x can be any number.

53. No values of x are solutions.

54. x is a negative number.

55. x is a fraction.

56. Suppose you have three consecutive integers. The greatest of the three integers is twice as great as the sum of the first two. What are the integers?

Standardized Test Prep

SAT/ACT

57. What is the solution of $-2(3x - 4) = -2x + 2$?

Ⓐ $-\frac{2}{3}$ Ⓑ $\frac{3}{2}$ Ⓒ 2 Ⓓ 24

58. Two times a number plus three equals one half of the number plus 12. What is the number?

Ⓕ 3.6 Ⓖ 6 Ⓗ 8 Ⓘ 10

59. Josie's goal is to run 30 mi each week. This week she has already run the distances shown in the table. She wants to have one day of rest and to spread out the remaining miles evenly over the rest of the week. Which equation can she use to find how many miles m per day she must run?

Miles per Day

M	T	W	T	F	S	S
4	4.5	3.5	■	■	■	■

Ⓐ $4 + 4.5 + 3.5 + 3m = 30$

Ⓑ $4 + 4.5 + 3.5 + 4m = 30$

Ⓒ $30 - (4 + 4.5 + 3.5) = m$

Ⓓ $4 + 4.5 + 3.5 + m = 30$

Mixed Review

Solve each equation.

See Lesson 2-3.

60. $-2a + 5a - 4 = 11$

61. $6 = -3(x + 4)$

62. $3\left(c + \frac{1}{3}\right) = 4$

63. A carpenter is filling in an open entranceway with a door and two side panels of the same width. The entranceway is 3 m wide. The door will be 1.2 m wide. How wide should the carpenter make the panels on either side of the door so that the two panels and the door will fill the entranceway exactly?

See Lesson 2-2.

Get Ready! **To prepare for Lesson 2-5, do Exercises 64–66.**

Evaluate each expression for the given values of the variables.

See Lesson 1-2.

64. $n + 2m$; $m = 12$, $n = -2$

65. $3b \div c$; $b = 12$, $c = 4$

66. xy^2; $x = 2.8$, $y = 2$