

## 2-5

## Literal Equations and Formulas

## Common Core State Standards

**A-CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations . . . Also **A-CED.A.1**, **A-REI.A.1**

**MP 1**, **MP 2**, **MP 3**, **MP 4**, **MP 8**

**Objective** To rewrite and use literal equations and formulas



What happens to the number of sandwiches as the number of pizzas increases?



## Getting Ready!

You are ordering pizzas and sandwiches. You have a budget of \$80. How many sandwiches can you buy if you buy 4 pizzas? 5 pizzas? Explain your answer.



**MATHEMATICAL PRACTICES**

In this lesson, you will learn to solve problems using equations in more than one variable. A **literal equation** is an equation that involves two or more variables.

**Essential Understanding** When you work with literal equations, you can use the methods you have learned in this chapter to isolate any particular variable.



## Lesson Vocabulary

- literal equation
- formula

## Think

**Why should you rewrite the equation?**

If you rewrite the equation, you have to isolate  $y$  only once. Then substitute for  $x$ . If you substitute for  $x$  first, you must isolate  $y$  twice (once for each  $x$ -value).



## Problem 1 Rewriting a Literal Equation

The equation  $10x + 5y = 80$ , where  $x$  is the number of pizzas and  $y$  is the number of sandwiches, models the problem in the Solve It. How many sandwiches can you buy if you buy 3 pizzas? 6 pizzas?

**Step 1** Solve the equation  $10x + 5y = 80$  for  $y$ .

$$10x + 5y = 80$$

$$10x + 5y - 10x = 80 - 10x \quad \text{Subtract } 10x \text{ from each side.}$$

$$5y = 80 - 10x \quad \text{Simplify.}$$

$$\frac{5y}{5} = \frac{80 - 10x}{5} \quad \text{Divide each side by 5.}$$

$$y = 16 - 2x \quad \text{Simplify.}$$

**Step 2** Use the rewritten equation to find  $y$  when  $x = 3$  and when  $x = 6$ .

$$y = 16 - 2x \qquad y = 16 - 2x$$

$$y = 16 - 2(3) \quad \text{Substitute for } x. \quad y = 16 - 2(6)$$

$$y = 10 \qquad \text{Simplify.} \qquad y = 4$$

If you buy 3 pizzas, you can buy 10 sandwiches. If you buy 6 pizzas, you can buy 4 sandwiches.



- Got It?** 1. a. Solve the equation  $4 = 2m - 5n$  for  $m$ . What are the values of  $m$  when  $n = -2, 0,$  and  $2$ ?
- b. **Reasoning** Solve Problem 1 by substituting  $x = 3$  and  $x = 6$  into the equation  $10x + 5y = 80$  and then solving for  $y$  in each case. Do you prefer this method or the method shown in Problem 1? Explain.

When you rewrite literal equations, you may have to divide by a variable or variable expression. When you do so in this lesson, assume that the variable or variable expression is not equal to zero because division by zero is not defined.



### Problem 2 Rewriting a Literal Equation With Only Variables

What equation do you get when you solve  $ax - bx = c$  for  $x$ ?

$$ax - bx = c$$

$$x(a - b) = c \quad \text{Distributive Property}$$

$$\frac{x(a - b)}{a - b} = \frac{c}{a - b} \quad \text{Divide each side by } a - b, \text{ where } a - b \neq 0.$$

$$x = \frac{c}{a - b} \quad \text{Simplify.}$$

### Think

**How can you solve a literal equation for a variable?**

When a literal equation contains only variables, treat the variables you are *not* solving for as constants.



- Got It?** 2. What equation do you get when you solve  $-t = r + px$  for  $x$ ?

A **formula** is an equation that states a relationship among quantities. Formulas are special types of literal equations. Some common formulas are given below. Notice that some of the formulas use the same variables, but the definitions of the variables are different.

Formula Name	Formula	Definitions of Variables
Perimeter of a rectangle	$P = 2\ell + 2w$	$P$ = perimeter, $\ell$ = length, $w$ = width
Circumference of a circle	$C = 2\pi r$	$C$ = circumference, $r$ = radius
Area of a rectangle	$A = \ell w$	$A$ = area, $\ell$ = length, $w$ = width
Area of a triangle	$A = \frac{1}{2}bh$	$A$ = area, $b$ = base, $h$ = height
Area of a circle	$A = \pi r^2$	$A$ = area, $r$ = radius
Distance traveled	$d = rt$	$d$ = distance, $r$ = rate, $t$ = time
Temperature	$C = \frac{5}{9}(F - 32)$	$C$ = degrees Celsius, $F$ = degrees Fahrenheit

## Plan

Choose an appropriate formula and solve it for the variable you need to find. Substitute what you know into the rewritten formula. Simplify.

### Problem 3 Rewriting a Geometric Formula

What is the radius of a circle with circumference 64 ft? Round to the nearest tenth.  
Use 3.14 for  $\pi$ .

$$C = 2\pi r \quad \text{Write the appropriate formula.}$$


$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide each side by } 2\pi.$$

$$\frac{C}{2\pi} = r \quad \text{Simplify.}$$

$$\frac{64}{2\pi} = r \quad \text{Substitute 64 for } C.$$

$$10.2 \approx r \quad \text{Simplify. Use 3.14 for } \pi.$$

The radius of the circle is about 10.2 ft.

 **Got It?** 3. What is the height of a triangle that has an area of 24 in.<sup>2</sup> and a base with a length of 8 in.?

### Problem 4 Rewriting a Formula **STEM**

**Biology** The monarch butterfly is the only butterfly that migrates annually north and south. The distance that a particular group of monarch butterflies travels is shown. It takes a typical butterfly about 120 days to travel one way. What is the average rate at which a butterfly travels in miles per day? Round to the nearest mile per day.

$$d = rt \quad \text{Write the appropriate formula.}$$


$$\frac{d}{t} = \frac{rt}{t} \quad \text{Divide each side by } t.$$

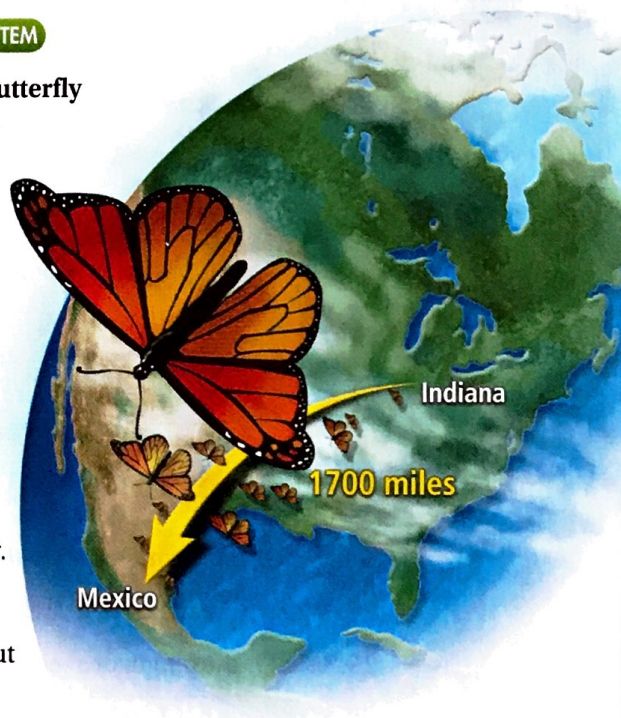
$$\frac{d}{t} = r \quad \text{Simplify.}$$

$$\frac{1700}{120} = r \quad \text{Substitute 1700 for } d \text{ and 120 for } t.$$

$$14 \approx r \quad \text{Simplify.}$$

The butterflies travel at an average rate of about 14 mi per day.

 **Got It?** 4. Pacific gray whales migrate annually from the waters near Alaska to the waters near Baja California, Mexico, and back. The whales travel a distance of about 5000 mi each way at an average rate of 91 mi per day. About how many days does it take the whales to migrate one way?



## Think

**How do you know which formula to use?**  
Read the information given in the problem. This problem gives you a measure of time and a distance. You need to find the rate, so use  $d = rt$ .



## Lesson Check

### Do you know HOW?

Solve each equation for the given variable.

1.  $-2x + 5y = 12$  for  $y$       2.  $a - 2b = -10$  for  $b$

3.  $mx + 2nx = p$  for  $x$       4.  $C = \frac{5}{9}(F - 32)$  for  $F$

5. **Gardening** Jonah is planting a rectangular garden. The perimeter of the garden is 120 yd, and the width is 20 yd. What is the length of the garden?

### Do you UNDERSTAND?



- © **Vocabulary** Classify each equation below as a formula, a literal equation, or both.

6.  $c = 2d$

7.  $y = 2x - 1$

8.  $A = \frac{1}{2}bh$

9.  $P = 2\ell + 2w$

- © **10. Compare and Contrast** How is the process of rewriting literal equations similar to the process of solving equations in one variable? How is it different?



## Practice and Problem-Solving Exercises



### A Practice

Solve each equation for  $y$ . Then find the value of  $y$  for each value of  $x$ .

← See Problem 1.

11.  $y + 2x = 5$ ;  $x = -1, 0, 3$

12.  $2y + 4x = 8$ ;  $x = -2, 1, 3$

13.  $3x - 5y = 9$ ;  $x = -1, 0, 1$

14.  $4x = 3y - 7$ ;  $x = 4, 5, 6$

15.  $5x = -4y + 4$ ;  $x = 1, 2, 3$

16.  $2y + 7x = 4$ ;  $x = 5, 10, 15$

17.  $x - 4y = -4$ ;  $x = -2, 4, 6$

18.  $6x = 7 - 4y$ ;  $x = -2, -1, 0$

Solve each equation for  $x$ .

← See Problem 2.

19.  $mx + nx = p$

20.  $ax - x = c$

21.  $\frac{rx + sx}{t} = 1$

22.  $y = \frac{x - v}{b}$

23.  $S = C + xC$

24.  $\frac{x}{a} = \frac{y}{b}$

25.  $A = Bxt + C$

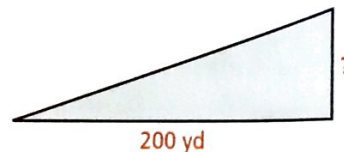
26.  $4(x - b) = x$

27.  $\frac{x + 2}{y - 1} = 2$

Solve each problem. Round to the nearest tenth, if necessary. Use 3.14 for  $\pi$ .

← See Problem 3.

28. What is the radius of a circle with circumference 22 m?
29. What is the length of a rectangle with width 10 in. and area 45 in.<sup>2</sup>?
30. A triangle has height 4 ft and area 32 ft<sup>2</sup>. What is the length of its base?
31. A rectangle has perimeter 84 cm and length 35 cm. What is its width?
32. **Parks** A public park is in the shape of a triangle. The side of the park that forms the base of the triangle is 200 yd long, and the area of the park is 7500 yd<sup>2</sup>. What is the length of the side of the park that forms the height of the triangle?



Solve each problem. Round to the nearest tenth, if necessary.

◀ See Problem 4.

**33. Travel** A vehicle travels on a highway at a rate of 65 mi/h. How long does it take the vehicle to travel 25 mi?

**34. Baseball** You can use the formula  $a = \frac{h}{n}$  to find the batting average  $a$  of a batter who has  $h$  hits in  $n$  times at bat. Solve the formula for  $h$ . If a batter has a batting average of .290 and has been at bat 300 times, how many hits does the batter have?

**STEM 35. Construction** Bricklayers use the formula  $n = 7\ell h$  to estimate the number  $n$  of bricks needed to build a wall of length  $\ell$  and height  $h$ , where  $\ell$  and  $h$  are in feet. Solve the formula for  $h$ . Estimate the height of a wall 28 ft long that requires 1568 bricks.

**B Apply**

Solve each equation for the given variable.

36.  $2m - nx = x + 4$  for  $x$

37.  $\frac{x}{a} - 1 = \frac{y}{b}$  for  $x$

38.  $ax + 2xy = 14$  for  $y$

39.  $V = \frac{1}{3}\pi r^2 h$  for  $h$

40.  $A = \left(\frac{f+g}{2}\right)h$  for  $g$

41.  $2(x + a) = 4b$  for  $a$

**© 42. Think About a Plan** The interior angles of a polygon are the angles formed inside a polygon by two adjacent sides. The sum  $S$  of the measures of the interior angles of a polygon with  $n$  sides can be found using the formula  $S = 180(n - 2)$ . The sum of a polygon's interior angle measures is  $1260^\circ$ . How many sides does the polygon have?

- What information are you given in the problem?
- What variable do you need to solve for in the formula?

**STEM 43. Weather** Polar stratospheric clouds are colorful clouds that form when temperatures fall below  $-78^\circ\text{C}$ . What is this temperature in degrees Fahrenheit?

**STEM 44. Science** The energy  $E$  of a moving object is called its *kinetic energy*. It is calculated using the formula  $E = \frac{1}{2}mv^2$ , where  $m$  is the object's mass in kilograms and  $v$  is its speed in meters per second. The units of kinetic energy are  $\frac{\text{kilograms} \cdot \text{meters}^2}{\text{second}^2}$ , abbreviated as  $\text{kg} \cdot \text{m}^2/\text{s}^2$ .

- Solve the given formula for  $m$ .
- What is the mass of an object moving at 10 m/s with a kinetic energy of  $2500 \text{ kg} \cdot \text{m}^2/\text{s}^2$ ?

**© 45. Error Analysis** Describe and correct the error made in solving the literal equation at the right for  $n$ .

**46. Geometry** The formula for the volume of a cylinder is  $V = \pi r^2 h$ , where  $r$  is the cylinder's radius and  $h$  is its height. Solve the equation for  $h$ . What is the height of a cylinder with volume  $502.4 \text{ cm}^3$  and radius 4 cm? Use 3.14 for  $\pi$ .

**47. Density** The density of an object is calculated using the formula  $D = \frac{m}{V}$ , where  $m$  is the object's mass and  $V$  is its volume. Gold has a density of  $19.3 \text{ g/cm}^3$ . What is the volume of an amount of gold that has a mass of 96.5 g?



Polar stratospheric clouds

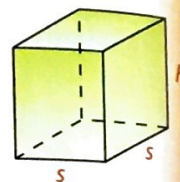
~~$$\begin{aligned} 2m &= -6n + 3 \\ 2m + 3 &= -6n \\ \frac{2m + 3}{-6} &= n \end{aligned}$$~~

48. **Open-Ended** Write an equation in three variables. Solve the equation for each variable. Show all your steps.



**Challenge**

49. **Surface Area** A rectangular prism with height  $h$  and with square bases with side length  $s$  is shown.



- Write a formula for the surface area  $A$  of the prism.
  - Rewrite the formula to find  $h$  in terms of  $A$  and  $s$ . If  $s$  is 10 cm and  $A$  is  $760 \text{ cm}^2$ , what is the height of the prism?
50. **Midpoints** Suppose a segment on a number line has endpoints with coordinates  $a$  and  $b$ . The coordinate of the segment's midpoint  $m$  is given by the formula  $m = \frac{a+b}{2}$ .
- Find the midpoint of a segment with endpoints at 9.3 and 2.1.
  - Rewrite the given formula to find  $b$  in terms of  $a$  and  $m$ .
  - The midpoint of a segment is at 3.5. One endpoint is at 8.9. Find the other endpoint.

**Standardized Test Prep**

**GRIDDED RESPONSE**

**SAT/ACT**

- What is the value of the expression  $-\frac{3}{4}m + 15$  when  $m = 12$ ?
- What is the solution of  $9p + 6 - 3p = 45$ ?
- The formula  $F = \frac{n}{4} + 37$  relates the number of chirps  $n$  a cricket makes in 1 min to the outside temperature  $F$  in degrees Fahrenheit. How many chirps can you expect a cricket to make in 1 min when the outside temperature is  $60^\circ\text{F}$ ?

**Mixed Review**

Solve each equation. If the equation is an *identity*, write *identity*. If it has no solution, write *no solution*.

See Lesson 2-4.

- |                            |                           |
|----------------------------|---------------------------|
| 54. $3x - 3 = x + 7$       | 55. $2b - 10 = -3b + 5$   |
| 56. $4 + 12a = -2(6 - 4a)$ | 57. $2(y - 4) = -4y + 10$ |
| 58. $4c - 10 = 2(2c - 5)$  | 59. $5 + 4p = 2(2p + 1)$  |

Evaluate each expression for  $b = 3$  and  $c = 7$ .

See Lesson 1-2.

- |            |                 |               |                 |
|------------|-----------------|---------------|-----------------|
| 60. $bc^2$ | 61. $b^2 - c^2$ | 62. $(3b)^2c$ | 63. $(b + c)^2$ |
|------------|-----------------|---------------|-----------------|

**Get Ready!** To prepare for Lesson 2-6, do Exercises 64–66.

Simplify each product.

See p. 804.

- |  |   |   |
|--|---|---|
| 64. $\frac{35}{25} \times \frac{30}{14}$ | 65. $\frac{99}{108} \times \frac{96}{55}$ | 66. $\frac{21}{81} \times \frac{63}{105}$ |
|--|---|---|