2-9 Percents

Common Core State Standards

Prepares for N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

MP 1, MP 2, MP 3, MP 4

Objectives To solve percent problems using proportions
To solve percent problems using the percent equation

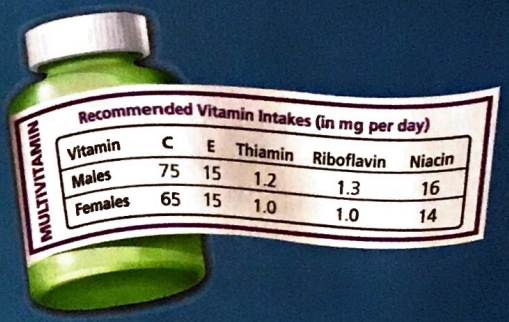
Make sense of the problem first. Develop a plan for solving it.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

Some recommended daily intakes of vitamins are shown. Carrie consumed 10 mg of niacin today. Her brother consumed 11 mg of niacin. Who consumed the greater percent of their recommended intake? How do you know?



Recommended Vitamin Intakes (in mg per day)

Vitamin	C	E	Thiamin	Riboflavin	Niacin
Males	75	15	1.2	1.3	16
Females	65	15	1.0	1.0	14

The problem in the Solve It involves percents. Percents are useful because they standardize comparisons to a common base of 100. In this lesson, you will solve percent problems in a variety of ways.

Essential Understanding You can solve problems involving percents using either proportions or the percent equation, which are closely related. If you write a percent as a fraction, you can use a proportion to solve a percent problem.

take note

Key Concept The Percent Proportion

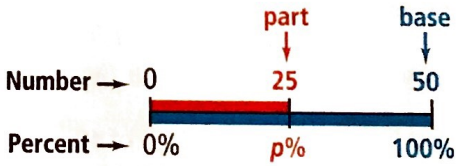
You can represent "a is p percent of b" using the percent proportion shown below. In the proportion, b is the base, and a is a *part* of base b.

Algebra $\frac{a}{b} = \frac{p}{100}$, where $b \neq 0$

Example What percent of 50 is 25?

$$\frac{25}{50} = \frac{p}{100}$$

Model



Problem 1 Finding a Percent Using the Percent Proportion

What percent of 56 is 42?

$\frac{a}{b} = \frac{p}{100}$ Write the percent proportion.

$\frac{42}{56} = \frac{p}{100}$ Substitute 42 for a and 56 for b.

$42(100) = 56p$ Cross Products Property

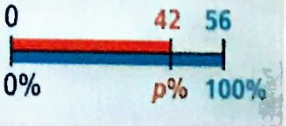
$4200 = 56p$ Multiply.

$75 = p$ Divide each side by 56.

42 is 75% of 56.

Think

How can a model help you visualize the proportion?
Use a model like the one below to visualize any percent problem. A model for the proportion $\frac{42}{56} = \frac{p}{100}$ is



Got It? 1. What percent of 90 is 54?

In Problem 1, you used the percent proportion $\frac{a}{b} = \frac{p}{100}$ to find a percent. When you write $\frac{p}{100}$ as $p\%$ and solve for a, you get the equation $a = p\% \cdot b$. This equation is called the percent equation. You can use either the percent equation or the percent proportion to solve any percent problem.

take note

Key Concept The Percent Equation

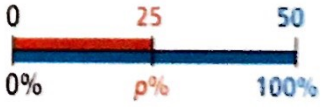
You can represent "a is p percent of b" using the percent equation shown below. In the equation, a is a part of the base b.

Algebra $a = p\% \cdot b$, where $b \neq 0$

Example What percent of 50 is 25?

$25 = p\% \cdot 50$

Model



Problem 2 Finding a Percent Using the Percent Equation

What percent of 40 is 2.5?

$a = p\% \cdot b$ Write the percent equation.

$2.5 = p\% \cdot 40$ Substitute 2.5 for a and 40 for b.

$0.0625 = p\%$ Divide each side by 40.

$6.25\% = p\%$ Write the decimal as a percent.

2.5 is 6.25% of 40.

Think

Is this problem related to other problems you've seen?
Yes. This problem is related to Problem 1. Both involve finding a percent, but they use different methods.

Got It? 2. **Reasoning** What percent of 84 is 63? Use the percent equation to solve. Then use the percent proportion. Compare your answers.

Problem 3 Finding a Part

Shopping **A dress shirt that normally costs $38.50 is on sale for 30% off. What is the sale price of the shirt?**

Step 1 Use the percent equation to find the amount of discount.

$a = p\% \cdot b$ Write the percent equation.

$= 30\% \cdot 38.50$ Substitute 30 for p and 38.50 for b.

$= 0.30 \cdot 38.50$ Write the percent as a decimal.

$= 11.55$ Multiply.

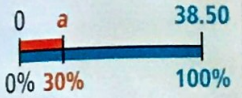
Step 2 Find the sale price.

$\$38.50 - \$11.55 = \$26.95$

The sale price of the shirt is $26.95.

Think

How can a model help you visualize finding a part or base?
Use the model below to help you visualize finding the part in the equation $a = 30\% \cdot 38.50$.



Got It? **3.** A family sells a car to a dealership for 60% less than they paid for it. They paid $9000 for the car. For what price did they sell the car?

Problem 4 Finding a Base

125% of what number is 17.5?

$a = p\% \cdot b$ Write the percent equation.

$17.5 = 125\% \cdot b$ Substitute 17.5 for a and 125 for p.

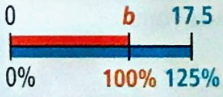
$17.5 = 1.25 \cdot b$ Write the percent as a decimal.

$14 = b$ Divide each side by 1.25.

125% of 14 is 17.5.

Think

Will 125% of a number be greater than the number?
Yes. When you multiply a positive number by a percent greater than 100%, the part will be greater than the base, as shown in the model below.



Got It? **4.** 30% of what number is 12.5? Solve the problem using the percent equation. Then solve the problem using the percent proportion.

A common application of percents is simple interest, which is interest you earn on only the principal in an account.

Take note

Key Concept Simple Interest Formula

The simple interest formula is given below, where I is the interest, P is the principal, r is the annual interest rate written as a decimal, and t is the time in years.

Algebra $I = Prt$

Example If you invest $50 at a simple interest rate of 3.5% per year for 3 years, the interest you earn is $I = 50(0.035)(3) = \$5.25$.

When you solve problems involving percents, it is helpful to know fraction equivalents for common percents. You can use the fractions to check your answers for reasonableness. Here are some common percents represented as fractions.

$1\% = \frac{1}{100}$ $\quad 5\% = \frac{1}{20}$ $\quad 10\% = \frac{1}{10}$ $\quad 20\% = \frac{1}{5}$ $\quad 25\% = \frac{1}{4}$

$33.\overline{3}\% = \frac{1}{3}$ $\quad 50\% = \frac{1}{2}$ $\quad 66.\overline{6}\% = \frac{2}{3}$ $\quad 75\% = \frac{3}{4}$ $\quad 100\% = 1$

Problem 5 Using the Simple Interest Formula

Finance **You deposited $840 in a savings account that earns a simple interest rate of 4.5% per year. You want to keep the money in the account for 4 years. How much interest will you earn? Check your answer for reasonableness.**

Think	Write
This is a simple interest problem, so use the formula for simple interest.	$I = Prt$
Identify what you know from the problem: $P = 840$, $r = 0.045$, and $t = 4$.	$= 840(0.045)(4)$ $= 151.2$
	The account will earn $151.20.
Check for reasonableness by using a common percent. Since 4.5% is about 5%, use 5%.	$840 \cdot \frac{1}{20} \cdot 4 = 42 \cdot 4 = \$168$
	So, $151.20 is reasonable. ✔

Got It? **5.** You deposited $125 in a savings account that earns a simple interest rate of 1.75% per year. You earned a total of $8.75 in interest. For how long was your money in the account?

take note

Concept Summary Solving Percent Problems

Problem Type	Example	Proportion	Equation
Find a percent.	What percent of 6.3 is 3.5?	$\frac{3.5}{6.3} = \frac{p}{100}$	$3.5 = p\% \cdot 6.3$
Find a part.	What is 32% of 125?	$\frac{a}{125} = \frac{32}{100}$	$a = 32\% \cdot 125$
Find a base.	25% of what number is 11?	$\frac{11}{b} = \frac{25}{100}$	$11 = 25\% \cdot b$

Lesson Check

Do you know HOW?

1. What percent of 70 is 21?
2. What percent of 50 is 60?
3. What is 35% of 80?
4. 75% of what number is 36?
5. **Finance** How much interest will you earn by investing $1200 at a simple interest rate of 2.5% per year for 6 years?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** Complete: p% is equivalent to a fraction with a numerator of p and a denominator of ___?___.
7. **Reasoning** You deposited money in a savings account paying 4% simple interest per year. The first year, you earned $75 in interest. How much interest will you earn during the following year?
8. **Open-Ended** Give an example of a percent problem where the part is greater than the base.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find each percent. See Problems 1 and 2.

9. What percent of 75 is 15?
10. What percent of 15 is 75?
11. What percent of 16 is 10?
12. What percent of 32 is 40?
13. What percent of 48 is 20?
14. What percent of 88 is 88?

Find each part. See Problem 3.

15. What is 25% of 144?
16. What is 63% of 150?
17. What is 12.5% of 104?
18. What is 150% of 63?
19. What is 125% of 12.8?
20. What is 1% of 1?

21. **Shopping** A tennis racket normally costs $65. The tennis racket is on sale for 20% off. What is the sale price of the tennis racket?
22. **Hair Care** A beauty salon buys bottles of styling gel for $4.50 per bottle and marks up the price by 40%. For what price does the salon sell each bottle?

Find each base. See Problem 4.

23. 20% of what number is 80?
24. 80% of what number is 20?
25. 60% of what number is 13.5?
26. 160% of what number is 200?
27. 150% of what number is 34?
28. 1% of what number is 1?

29. **Finance** You deposit $1200 in a savings account that earns simple interest at a rate of 3% per year. How much interest will you have earned after 3 years? See Problem 5.
30. **Finance** You deposit $150 in a savings account that earns simple interest at a rate of 5.5% per year. How much interest will you have earned after 4 years?

B Apply

Tell whether you are finding a *percent*, a *part*, or a *base*. Then solve.

31. What is 9% of 56?

32. What percent of 36 is 96?

33. What is 95% of 150?

34. What is 175% of 64?

35. What percent of 30 is 400?

36. 60 is 250% of what num

37. Geography Water covers approximately 11,800 mi^2 of Florida, which is about 18% of Florida's area. What is the total area of Florida to the nearest thousand square miles?

38. Finance A student has $1500 to deposit in a savings account. What is the lowest rate that would allow the student to earn $95 in simple interest in a year?

(A) 5.5%

(B) 6.25%

(C) 6.33%

(D) 7%

Solve using mental math.

39. 20% of 80 is ?.

40. 120 is 200% of ?.

41. 30 is ? % of 40.

Tell which is greater, *A* or *B*. Assume *A* and *B* are positive numbers.

42. *A* is 20% of *B*.

43. 150% of *A* is *B*.

44. *B* is 90% of *A*.

45. Think About a Plan The United States Mint reported at the end of 2006 that the unit cost of producing and distributing a penny was 1.21¢. What percent of the value of a penny is this cost? What can you conclude about the cost of making pennies?

- How can a model help you to visualize the problem?
- How can you use a proportion or the percent equation to solve the problem?

46. Economics Would you produce an item if the cost of producing and distributing the item were more than 100% of its value? Explain your answer.

47. Error Analysis A student was asked to make up and solve a percent problem, so the student wrote, "What percent of 1.5 is 3?" and solved it as shown at the right. Describe and correct the error in the student's solution.

$$\frac{a}{b} = \frac{p}{100}$$
$$\frac{1.5}{3} = \frac{p}{100}$$
$$150 = 3p$$
$$50 = p$$

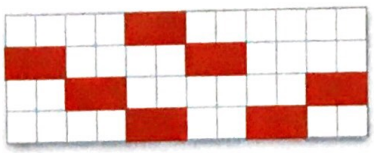
3 is 50% of 1.5

48. Writing Part of a bottle of water has been consumed. Write the steps needed to determine the percent of water that has been consumed.

49. Finance A savings account earns simple interest at a rate of 6% per year. Last year the account earned $10.86 in interest. What was the balance in the account at the beginning of last year?

50. Furniture A furniture store offers a set of furniture for $990. You can also purchase the set on an installment plan for 24 payments of $45 each. If you choose the installment plan, what percent of the original price will you have paid when you finish? Round to the nearest percent.

51. Geometry Each grid square in the figure at the right is the same size. What percent of the figure is red?



Challenge

52. Shopping Marcia buys a dress that is on sale for 15% off its original price. She uses a store coupon to obtain an additional 10% off the sale price. Marcia pays $91.80 for the dress. What was the original price of the dress?

53. Public Transportation In Mr. Ferreira's class, 80% of the students live more than half a mile from school. Of those students, 80% come to school by public transportation. Of the students using public transportation, 75% take the bus, and 75% of those students buy monthly bus passes. Nine students buy monthly bus passes. How many students are in Mr. Ferreira's class?

Standardized Test Prep

SAT/ACT

54. A rare disease has been discovered that affects 2 out of every 10,000 trees in a forest. What percent of trees are affected?

(A) 0.0002% (B) 0.002% (C) 0.02% (D) 0.2%

55. One kilometer equals about $\frac{5}{8}$ mi. A European racecar driver is driving at 120 km/h. Approximately what is this speed in miles per hour?

(F) 75 mi/h (G) 100 mi/h (H) 160 mi/h (I) 192 mi/h

56. What is the solution of $\frac{x}{2} + \frac{x}{3} - 15 = 0$?

(A) 12 (B) 18 (C) 24 (D) 30

Mixed Review

57. Art A painting 36 cm wide and 22.5 cm tall is going to be reproduced on a postcard. The image on the postcard will be 9 cm tall. How wide will the image be?

See Lesson 2-8.

58. Cats Alexis's cat eats 3 cans of cat food every 5 days. Alexis is going away for 30 days. A friend has offered to feed her cat. How many cans of cat food must Alexis leave for her cat while she is away?

See Lesson 2-7.

59. Taxis A taxi charges $1.75 for the first $\frac{1}{8}$ mi and $.30 for each additional $\frac{1}{8}$ mi. Write an equation that gives the cost c of a taxi ride in terms of the number of miles m. How many miles did you travel if a ride cost $7.75?

See Lesson 2-7.

Get Ready! To prepare for Lesson 2-10, do Exercises 60–62.

Solve each percent problem.

See Lesson 2-9.

60. What percent of 8 is 100?

61. What is 20% of 3?

62. 35 is what percent of 20?