3-2 Solving Inequalities Using Addition or Subtraction

Common Core State Standards

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Also A-CED.A.1**

MP 1, MP 2, MP 3, MP 4

Objective To use addition or subtraction to solve inequalities

Make sure you understand the problem. Which numbers do you need to use?

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

In a U.S. presidential election, a candidate must win at least 270 out of 538 total electoral votes to be declared the winner. Suppose a candidate has earned 238 electoral votes in states outside the southeastern U.S.

What is the least number of states in the southeastern U.S. that the candidate could win and still become president? What are these states? Justify your reasoning.

Lesson Vocabulary

- equivalent inequalities

You can model the situation in the Solve It with the inequality $238 + x \geq 270$, where x represents the number of electoral votes needed. You can find its solutions using one of the *properties of inequality*.

Essential Understanding Just as you used properties of equality to solve equations in Chapter 2, you can use properties of inequality to solve inequalities.

The Addition Property of Inequality is shown below. Applying this property to an inequality produces an equivalent inequality. **Equivalent inequalities** are inequalities that have the same solutions.

take note

Key Concept Addition Property of Inequality

Words

Let a, b, and c be real numbers.

If $a > b$, then $a + c > b + c$.

If $a < b$, then $a + c < b + c$.

This property is also true for $\geq$ and $\leq$.

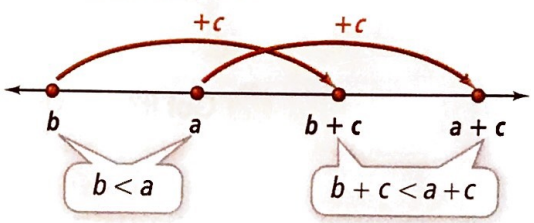
Examples

$5 > 4$, so $5 + 3 > 4 + 3$.

$-2 < 0$, so $-2 + 1 < 0 + 1$.

Diagram

The diagram below illustrates one way to think about this rule.



Problem 1 Using the Addition Property of Inequality

What are the solutions of $x - 15 > -12$? Graph the solutions.

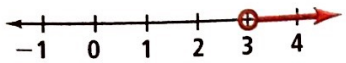
Think

Do you know how to solve a related problem?
Yes. You know how to solve the related equation $x - 15 = -12$ using the Addition Property of Equality.

$x - 15 > -12$

$x - 15 + 15 > -12 + 15$ Add 15 to each side.

$x > 3$ Simplify.



The solutions of $x > 3$ are all real numbers greater than 3.

Got It? 1. What are the solutions of $n - 5 < -3$? Graph the solutions.

In Problem 1, how can you check that the final inequality $x > 3$ describes the solutions of the original inequality $x - 15 > -12$? The original inequality has infinitely many solutions, so you cannot check them all. However, you can verify that the final inequality is correct by checking its endpoint and the direction of the inequality symbol. You will do this in Problem 2.

Problem 2 Solving an Inequality and Checking Solutions

What are the solutions of $10 \geq x - 3$? Graph and check the solutions.

Think

You need to isolate x. Undo subtraction by adding the same number to each side.

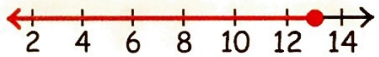
Write

$10 \geq x - 3$

$10 + 3 \geq x - 3 + 3$

$13 \geq x$

The graph of $13 \geq x$ (or $x \leq 13$) contains 13 and all real numbers to the left of 13.



To check the endpoint 13 of $13 \geq x$, make sure that 13 is the solution of the related *equation* $10 = x - 3$.

$10 = x - 3$

$10 \stackrel{?}{=} 13 - 3$

$10 = 10$ ✔

To check the inequality symbol of $13 \geq x$, make sure that a number *less than* 13 is a solution of the original inequality.

$10 \geq x - 3$

$10 \stackrel{?}{\geq} 12 - 3$

$10 \geq 9$ ✔

Got It? 2. What are the solutions of $m - 11 \geq -2$? Graph and check the solutions.

The Subtraction Property of Inequality is shown below.

take note

Key Concept Subtraction Property of Inequality

Words

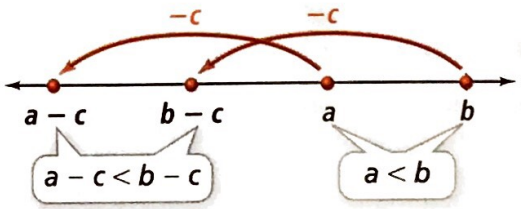
Let a, b, and c be real numbers.
If $a > b$, then $a - c > b - c$.
If $a < b$, then $a - c < b - c$.
This property is also true for $\geq$ and $\leq$.

Examples

$-3 < 5$, so $-3 - 2 < 5 - 2$.
$3 > -4$, so $3 - 1 > -4 - 1$.

Diagram

The diagram below illustrates one way to think about this rule.



Problem 3 Using the Subtraction Property of Inequality

What are the solutions of $t + 6 > -4$? Graph the solutions.

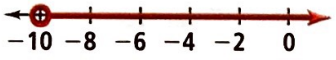
Think

How is this inequality different from others you have seen before?
The expression $t + 6$ involves addition, so you have to use subtraction to undo the addition and isolate the variable.

$$t + 6 > -4$$

$t + 6 - 6 > -4 - 6$ Subtract 6 from each side.

$t > -10$ Simplify.



The solutions of $t > -10$ are all real numbers to the right of -10.

Got It? **3.** What are the solutions of $-1 \geq y + 12$? Graph the solutions.

Problem 4 Writing and Solving an Inequality STEM

Computers **The hard drive on your computer has a capacity of 120 gigabytes (GB). You have used 85 GB. You want to save some home videos to your hard drive. What are the possible sizes of the home video collection you can save?**

Think

How do you know which inequality symbol to use?
Words and phrases like *at most, no more than,* and *maximum* may indicate that you should use $\leq$.

Relate current hard drive space used plus size of videos is at most hard drive capacity

Define Let v = the size of the video collection.

Write $85 + v \leq 120$

$85 + v - 85 \leq 120 - 85$ Subtract 85 from each side.

$v \leq 35$ Simplify.

The home video collection can be any size less than or equal to 35 GB.

Got It? **4. a.** A club has a goal to sell at least 25 plants for a fundraiser. Club members sell 8 plants on Wednesday and 9 plants on Thursday. What are the possible numbers of plants the club can sell on Friday to meet their goal?

b. Reasoning Can you use the same inequality symbol to represent phrases like *at least*, *no less than*, and *greater than or equal to*? Explain.

Lesson Check

Do you know HOW?

Solve each inequality. Graph and check your solutions.

1. $p - 4 < 1$
2. $8 \geq d - 2$
3. $y + 5 < -7$
4. $4 + c > 7$
5. A cyclist takes her bicycle on a chairlift to the top of a slope. The chairlift can safely carry 680 lb. The cyclist weighs 124 lb, and the bicycle weighs 32 lb. What are the possible additional weights the chairlift can safely carry?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Writing** How can you use the addition and subtraction properties of inequality to produce equivalent inequalities?
7. **Reasoning** What can you do to the first inequality in each pair in order to get the second inequality?
   - **a.** $x + 4 \leq 10; x \leq 6$
   - **b.** $m - 1 > 3; m > 4$
   - **c.** $5 \geq 3 + n; 2 \geq n$
   - **d.** $-6 < y - 2; -4 < y$
8. **Compare and Contrast** Suppose you solve the two inequalities $y + 4 \leq 6$ and $y - 4 \leq 6$. How are your methods of solving the inequalities similar? How are they different?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Tell what number you would add to each side of the inequality to solve the inequality. See Problems 1 and 2.

9. $f - 6 \geq -3$
10. $1 < d - 7$
11. $a - 3.3 \geq 2.6$
12. $5 > -18 + m$

Solve each inequality. Graph and check your solutions.

13. $y - 2 > 11$
14. $v - 4 < -3$
15. $-6 > c - 2$
16. $8 \leq f - 4$
17. $t - 4 \geq -7$
18. $s - 10 \leq 1$
19. $9 < p - 3$
20. $-3 \geq x - 1$
21. $0 < -\frac{1}{3} + f$
22. $z - 12 \leq -4$
23. $-\frac{3}{4} > r - \frac{3}{4}$
24. $y - 1 \geq 1.5$
25. $4.3 > -0.4 + s$
26. $-2.5 > n - 0.9$
27. $c - \frac{4}{7} < \frac{6}{7}$
28. $p - 1\frac{1}{2} > 1\frac{1}{2}$

Tell what number you would subtract from each side of the inequality to solve the inequality. See Problem 3.

29. $x + 3 > 0$
30. $9 < \frac{7}{5} + s$
31. $6.8 \geq m + 4.2$
32. $\ell + \frac{1}{3} \geq \frac{7}{3}$

Solve each inequality. Graph and check your solutions.

33. $x + 5 \leq 10$

34. $n + 6 > -2$

35. $2 < 9 + c$

36. $-1 \geq 5 + b$

37. $\frac{1}{4} + a \geq -\frac{3}{4}$

38. $8.6 + z < 14$

39. $\frac{1}{3} < n + 3$

40. $3.8 \geq b + 4$

41. $\frac{3}{5} + d \geq -\frac{2}{5}$

42. Fitness Your goal is to take at least 10,000 steps per day. According to your pedometer, you have walked 5274 steps. Write and solve an inequality to find the possible numbers of steps you can take to reach your goal.

See Problem 4.

43. Fundraising The environmental club is selling indoor herb gardens for Earth Day. Each member is encouraged to sell at least 10 gardens. You sell 3 gardens on Monday and 4 gardens on Tuesday. Write and solve an inequality to find the possible numbers of gardens you can sell to reach your goal.

44. Monthly Budget You earn $250 per month from your part-time job. You are in a kayaking club that costs $20 per month, and you save at least $100 each month. Write and solve an inequality to find the possible amounts you have left to spend each month.

B Apply

Tell what you can do to the first inequality in order to get the second.

45. $36 \leq -4 + y$; $40 \leq y$

46. $9 + b > 24$; $b > 15$

47. $m - \frac{1}{2} < \frac{3}{8}$; $m < \frac{7}{8}$

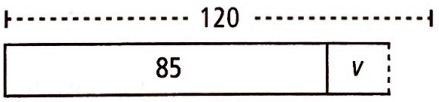
Tell whether the two inequalities in each pair are equivalent.

48. $45 \leq -5 + z$; $40 \leq z$

49. $7 + c > 33$; $c > 26$

50. $n - \frac{1}{4} < \frac{5}{4}$; $n < 1$

You can draw a model to represent an inequality. For example, the model below represents the inequality $85 + v < 120$. Draw a model to represent each inequality below.



51. $17 + x < 51$

52. $12 + y > 18$

53. $-3 + m \leq 13$

Solve each inequality. Justify each step.

54. $y - 4 + 2 \geq 10$

55. $\frac{3}{5} + d \leq 2\frac{3}{5}$

56. $z - 1.4 < 3.9$

57. $-5 > p - \frac{1}{5}$

58. $a + 5.2 < -4.6$

59. $-3.1 > z - 1.9$

60. $\frac{5}{8} + v - \frac{7}{16} > 0$

61. $-4p - 2 + 5p > 10$

62. $5y + 5 - 4y < 8$

63. $h - \frac{1}{8} \geq -1$

64. $8v - 7v - 3 \geq -6$

65. $5 \geq m - \frac{7}{16}$

66. Government The U.S. Senate is composed of 2 senators from each of the 50 states. In order for a treaty to be ratified, at least two thirds of the senators present must approve the treaty. Suppose all senators are present and 48 of them have voted in favor of a treaty. What are the possible numbers of additional senators who must vote in favor of the treaty in order to ratify it?

67. **a.** If $56 + 58 = t$, does $t = 56 + 58$?

b. If $56 + 58 \leq r$, is $r \leq 56 + 58$? Justify your answer.

c. Explain the differences between these two examples.

68. Think About a Plan You want to qualify for a regional diving competition. At today's competition, you must score at least 53 points. Out of a possible 10 points, your scores on each of the first 5 dives are shown at the right. What scores can you earn on the armstand dive that will qualify you for the regional diving competition?

- What information do you know? What information do you need?
- How might writing and solving an inequality help you?
- What does the solution of the inequality mean in terms of the original situation?

OFFICIAL SCORE CARD	
DIVE	SCORE
Front Dive	*9.8*
Back Dive	*8.9*
Reverse Dive	*8.4*
Inward Dive	*8.2*
Twisting Dive	*9.4*
Armstand Dive	*?*

69. Qualifying Scores To enter a competition, students must score a total of at least 450 points on five qualifying tests. Each test is worth 100 points. On the first four tests, your scores were 94, 88, 79, and 95. What are three possible scores you can earn on the last test to enter the competition?

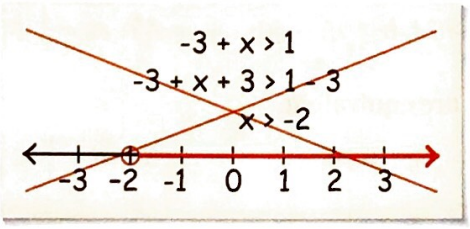
Error Analysis **Describe and correct the error in solving each inequality or in graphing the solution.**

70.

$-3 + x > 1$

$-3 + x + 3 > 1 - 3$

$x > -2$

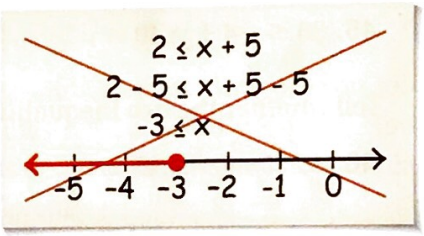


71.

$2 \leq x + 5$

$2 - 5 \leq x + 5 - 5$

$-3 \leq x$



72. **a.** **Open-Ended** Use each of the inequality symbols $<$, $\leq$, $>$ and $\geq$ to write four inequalities involving addition or subtraction.

b. Solve each inequality from part (a) and graph your solutions.

73. **a.** Mallory says that she can solve the inequality $a + 3.2 \geq 8.6$ by replacing a with 5, 6, and 7. When $a = 5$, the inequality is false. When $a = 6$ and when $a = 7$, the inequality is true. So Mallory says that the solution is $a \geq 6$. Is her reasoning correct? Justify your answer.

b. **Reasoning** Explain why substituting values into the inequality does not guarantee that Mallory's solution is correct.

74. Geometry Suppose a triangle has side lengths a, b, and c, where c is the length of the longest side. You can use the following equation and inequalities to determine whether the triangle is right, acute, or obtuse.

- If $a^2 + b^2 = c^2$, then the triangle is right.
- If $a^2 + b^2 > c^2$, then the triangle is acute.
- If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Classify each triangle with the following side lengths as *right*, *acute*, or *obtuse*.

a. 4 in., 5 in., 6 in. **b.** 3 cm, 4 cm, 5 cm **c.** 10 m, 15 m, 20 m

75. Banking To avoid a service fee, your checking account balance must be at least $500 at the end of each month. Your current balance is $536.45. You use your debit card to spend $125.19. What possible amounts can you deposit into your account by the end of the month to avoid paying the service fee?

Challenge

Reasoning Decide whether each inequality is true for all real numbers. If the inequality is not true, give a counterexample.

76. $x + y > x - y$

77. If $x \leq y$, then $x + w \leq y + w$.

78. If $w < z$, then $x - w > x - z$.

79. If $x > y$, then $x > y + w$.

80. Reasoning Find real numbers a, b, c, and d for which it is true that $a < b$ and $c < d$, but it is not true that $a - c < b - d$.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 4

Look back at the information on page 163 about the athletic boosters' concession stand.

a. Write an inequality that relates the number of empty boxes the boosters need to buy for the next game, the number of empty boxes on hand, and the maximum number of boxes of popcorn the boosters expect to sell.

b. Solve your inequality from part (a). How can you interpret your result in the context of the situation?

c. The table from page 163 is shown again below. Which packages of empty boxes can the athletic boosters buy in order to have enough for the next game? (Consider that they might buy more than one package of empty boxes. Also note that once the boosters have enough empty boxes, they do not need to buy any additional packages.)

Empty Popcorn Boxes

Number of Boxes per Package	Price per Box
75	$.25
200	$.21
300	$.19
400	$.17