3-3 Solving Inequalities Using Multiplication or Division

Common Core State Standards

A-CED.A.1 Create . . . inequalities in one variable and use them to solve problems . . . **Also N-Q.A.2, A-REI.B.3**

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To use multiplication or division to solve inequalities

This pattern can help you remember the properties you'll learn in this lesson.

SOLVE IT!

Getting Ready!

Consider the inequality $4 > 1$. Copy and complete each statement at the right by replacing each ■ with $<$ or $>$. What happens to the inequality symbol when you multiply each side by a positive number? What happens to the inequality symbol when you multiply each side by a negative number? Justify your reasoning.

$4 \cdot 3 \blacksquare 1 \cdot 3$

$4 \cdot 2 \blacksquare 1 \cdot 2$

$4 \cdot 1 \blacksquare 1 \cdot 1$

$4 \cdot -1 \blacksquare 1 \cdot -1$

$4 \cdot -2 \blacksquare 1 \cdot -2$

$4 \cdot -3 \blacksquare 1 \cdot -3$

MATHEMATICAL PRACTICES In the Solve It, you may have noticed that multiplying both sides of an inequality by a negative number affects the inequality symbol.

Essential Understanding Just as you used multiplication and division to solve equations in Chapter 2, you can use multiplication and division to solve inequalities.

take note

Key Concept Multiplication Property of Inequality

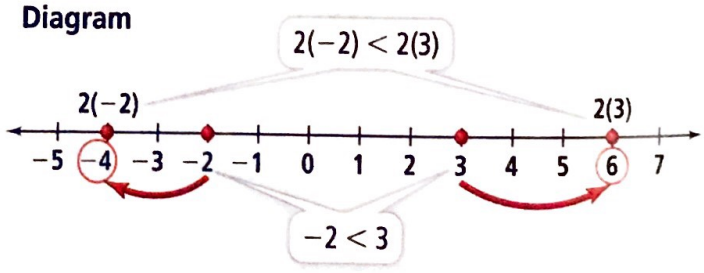
Words

Let a, b, and c be real numbers with $c > 0$.

If $a > b$, then $ac > bc$.

If $a < b$, then $ac < bc$.

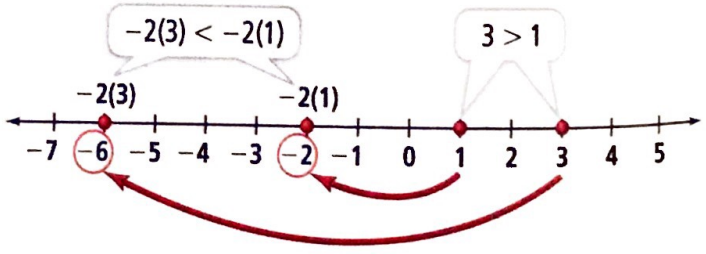
Diagram



Let a, b, and c be real numbers with $c < 0$.

If $a > b$, then $ac < bc$.

If $a < b$, then $ac > bc$.



These properties are also true for inequalities using $\geq$ and $\leq$.

Here's Why It Works Multiplying or dividing each side of an inequality by a negative number changes the meaning of the inequality. You need to reverse the inequality symbol to make the inequality true. Here is an example:

$3 > 1$

$-2(3) \blacksquare -2(1)$ Multiply by -2.

$-6 \blacksquare -2$ Simplify.

$-6 < -2$ Reverse the inequality symbol to make the inequality true.

Problem 1 Multiplying by a Positive Number

What are the solutions of $\frac{x}{3} < -2$? Graph the solutions.

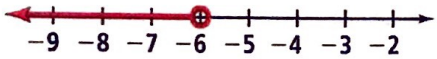
Think

Why multiply by 3? You can multiply by any multiple of 3. But multiplying by 3 isolates the variable.

$\frac{x}{3} < -2$

$3\left(\frac{x}{3}\right) < 3(-2)$ Multiply each side by 3.

$x < -6$ Simplify.



Got It? 1. What are the solutions of $\frac{c}{8} > \frac{1}{4}$? Graph the solutions.

Problem 2 Multiplying by a Negative Number

What are the solutions of $-\frac{3}{4}w \geq 3$? Graph and check the solutions.

Write

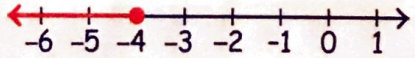
Think

Multiplying by a negative number changes the inequality. Reverse the inequality symbol to make it a true statement.

$-\frac{3}{4}w \geq 3$

$-\frac{4}{3}\left(-\frac{3}{4}w\right) \leq -\frac{4}{3}(3)$

$w \leq -4$



To check the endpoint of $w \leq -4$, make sure that -4 is the solution of the equation $-\frac{3}{4}w = 3$.

$-\frac{3}{4}(-4) \stackrel{?}{=} 3$

$3 = 3$ ✔

To check the inequality symbol of $w \leq -4$, make sure that a number less than -4 is a solution of the original inequality.

$-\frac{3}{4}(-5) \stackrel{?}{\geq} 3$

$3\frac{3}{4} \geq 3$ ✔

Got It? 2. What are the solutions of $-\frac{n}{3} < -1$? Graph and check.

Solving inequalities using division is similar to solving inequalities using multiplication. If you divide each side of an inequality by a negative number, you need to reverse the direction of the inequality symbol.

Take note

Key Concept Division Property of Inequality

Let a, b, and c be real numbers with $c > 0$.

If $a > b$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

Examples

$6 > 3$, so $\frac{6}{3} > \frac{3}{3}$.

$9 < 12$, so $\frac{9}{3} < \frac{12}{3}$.

Let a, b, and c be real numbers with $c < 0$.

If $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

$6 > 3$, so $\frac{6}{-3} < \frac{3}{-3}$.

$9 < 12$, so $\frac{9}{-3} > \frac{12}{-3}$.

These properties are also true for inequalities using $\geq$ and $\leq$.

Problem 3 Dividing by a Positive Number

Part-Time Job You walk dogs in your neighborhood after school. You earn \$4.50 per dog. How many dogs do you need to walk to earn at least \$75?

Relate cost per dog times number of dogs is at least amount wanted

Define Let d = the number of dogs.

Write $4.50 \cdot d \geq 75$

$4.50d \geq 75$

$\frac{4.50d}{4.50} \geq \frac{75}{4.50}$ Divide each side by 4.50.

$d \geq 16\frac{2}{3}$ Simplify.

However, since d represents the number of dogs, it must be a positive integer. So you must walk at least 17 dogs to earn at least \$75.

Think

What types of solutions make sense for this situation?
Only whole-number solutions make sense because you cannot walk part of a dog.

Got It? 3. a. A student club plans to buy food for a soup kitchen. A case of vegetables costs \$10.68. The club can spend at most \$50 for this project. What are the possible numbers of cases the club can buy?

b. **Reasoning** In Problem 3, why do you round to the greater whole number?

Problem 4 Dividing by a Negative Number

What are the solutions of $-9y \leq 63$? Graph the solutions.

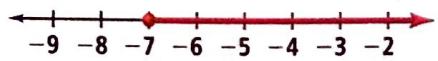
Think

How is this inequality different from the one in Problem 3?
The coefficient is negative. You can still use the properties of inequality to solve, but pay attention to the direction of the symbol.

$-9y \leq 63$

$\frac{-9y}{-9} \geq \frac{63}{-9}$ Divide each side by -9. Reverse the inequality symbol.

$y \geq -7$ Simplify each side.



Got It? **4.** What are the solutions of $-5x > -10$? Graph the solutions.

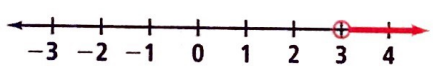
Lesson Check

Do you know HOW?

Match the inequality with its graph.

1. $x + 2 > -1$
2. $-\frac{x}{3} < -1$
3. $x - 4 \leq -1$
4. $-3x \geq 9$

A.


B.


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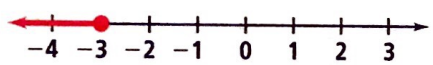

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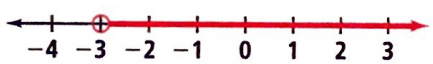


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Which operation would you use to solve the inequality? Explain.

   a. $1 \leq -\frac{x}{2}$ **b.** $y - 4 > -5$ **c.** $-6w < -36$

6. Error Analysis Describe and correct the error in the solution.

$-\frac{n}{5} > 2$

$-5\left(-\frac{n}{5}\right) > -5(2)$

$n > -10$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each inequality. Graph and check your solution.

See Problems 1 and 2.

7. $\frac{x}{5} \geq -2$
8. $\frac{w}{6} < 1$
9. $4 > \frac{p}{8}$
10. $1 \leq -\frac{5}{4}y$
11. $-\frac{v}{2} \geq 1.5$
12. $-3 < \frac{x}{3}$
13. $-7 \leq \frac{7}{3}x$
14. $8 > \frac{2}{3}k$
15. $0 \leq -\frac{3}{11}m$
16. $-\frac{3}{2}b < 6$
17. $-\frac{3}{4} < -\frac{3}{8}m$
18. $-5 \geq -\frac{5}{9}y$

Solve each inequality. Graph and check your solution.

See Problems 3 and 4.

19. $3m \geq 6$
20. $4t < -12$
21. $-30 > -5c$
22. $-4w \leq 20$
23. $11z > -33$
24. $56 < -7d$
25. $18b \leq -3$
26. $-7y \geq 17$
27. $-5h < 65$
28. $8t \leq 64$
29. $63 \geq 7q$
30. $-12x > 132$

31. Text Messages Text messages cost \$.15 each. You can spend no more than \$10. How many text messages can you send?

32. Aquarium Fish Tetras cost \$3.99 each. You can spend at most \$25. How many tetras can you buy for your aquarium?

B Apply

Write four solutions to each inequality.

33. $\frac{x}{2} \leq -1$ **34.** $\frac{r}{3} \geq -4$ **35.** $-1 \geq \frac{r}{3}$ **36.** $0.5 > \frac{1}{2}c$

Tell what you can do to the first inequality in order to get the second.

37. $-\frac{c}{4} > 3; c < -12$ **38.** $\frac{n}{5} \leq -2; n \leq -10$ **39.** $5z > -25; z > -5$ **40.** $\frac{3}{4}b \leq 3; b \leq 4$

Replace each ■ with the number that makes the inequalities equivalent.

41. $■s > 14; s < -7$ **42.** $■x \geq 25; x \leq -5$ **43.** $-8u \leq ■; u \geq -0.5$ **44.** $-2a > ■; a < -9$

Determine whether each statement is *always, sometimes,* or *never* true. Justify your answer.

45. If $x > 3$ and $y < 1$, then $xy > 0$.

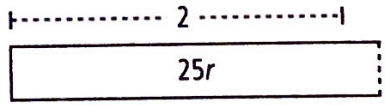
46. If $x < 0$ and $y < 0$, then $xy > 0$.

47. If $x \geq 0$ and $y > 1$, then $xy > 0$.

48. If $x > 0$ and $y \geq 0$, then $xy > 0$.

49. Think About a Plan A friend calls you and asks you to meet at the park 2 mi away in 25 min. You set off on your skateboard after the call. At what speeds (in miles per minute) can you ride your skateboard to be at the park in at most 25 min?

- How are the distance you travel, your speed, and time related?
- How can an inequality help you solve the problem?
- How can the model below help you solve the problem?



Solve each inequality. Justify each step.

50. $-4.5 > 9p$ **51.** $-1 \geq \frac{t}{3}$ **52.** $\frac{3}{4}n < 4$ **53.** $0.5 \leq \frac{1}{2}c$

54. $-8u < 4$ **55.** $\frac{n}{5} \leq -2$ **56.** $-12 > 4a$ **57.** $1 < -\frac{5}{7}s$

58. Trip A family is taking a cross-country trip by car. They drive at an average speed of 55 mi/h, and their goal is to travel at least 400 mi/day. How many hours per day do they need to drive?

59. Lunch You have \$30. You are going to buy a sandwich and a drink for yourself and two friends from the menu at the right. You will spend the remainder on snacks. What is the least number of snacks you might buy? What is the greatest number of snacks you might buy? Explain.

60. **Open-Ended** Write an inequality that can be solved by dividing by a negative number and has the solution $x < \frac{1}{3}$.

61. **Patterns** Consider the pattern of inequalities $\frac{x}{2} < 10$, $\frac{x}{3} < 10$, $\frac{x}{4} < 10$, . . . Suppose a real number a is a solution of a certain inequality in the pattern. What other inequalities in the pattern do you know have a as a solution? Explain.

62. **Reasoning** If $ax \leq ay$ and $ay \leq az$, is $x \leq z$? Explain.

Challenge

63. **Basketball** A company sells men's basketballs with a circumference of 29.5 in. They also sell youth basketballs with a circumference of 27.75 in. The company has cube-shaped packaging boxes with edges that are either 8 in., 9 in., or 10 in. long. What is the smallest box in which each ball can be packaged?

STEM

64. **Construction** A contractor is building a rectangular walkway $3\frac{1}{3}$ ft wide by 35 ft long using square cement pavers. Each paver has an area of $\frac{4}{9}$ ft^2. What is the least number of pavers he needs to make the walkway?

Standardized Test Prep

SAT/ACT

65. The mayor of Renee's town chose 160 students from her school to attend a city debate. This amount is no more than $\frac{1}{4}$ of the students in Renee's school. What is the least possible number of students that attend Renee's school?

(A) 40 (B) 160 (C) 320 (D) 640

66. An art teacher has a box of 100 markers. The teacher gives 7 markers to each student in the class and has 16 markers left over. How many students are in the class?

(F) 11 (G) 12 (H) 13 (I) 14

Short Response

67. The width of a rectangle is 3 in. shorter than the length. The perimeter of the rectangle is 18 in. What is the length of the rectangle? Show your work.

Mixed Review

Solve each inequality. See Lesson 3-2.

68. $x + 5 \leq -6$

69. $y - 4.7 \geq 8.9$

70. $q - 5 < 0$

71. $\frac{1}{2} > \frac{3}{4} + c$

72. $-\frac{2}{3} < b + \frac{1}{3}$

73. $y - 21 \leq 54$

Get Ready! **To prepare for Lesson 3-4, do Exercises 74–76.**

Solve each equation. See Lesson 2-3.

74. $-x + 8 + 4x = 14$

75. $-6(2y + 2) = 12$

76. $0.5t + 3.5 - 2.5t = 1.5t$