3-4 Solving Multi-Step Inequalities

Common Core State Standards

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Also A-CED.A.1**

MP 1, MP 2, MP 3, MP 4

Objectives To solve multi-step inequalities

Make sense of the problem. What information do you need before finding the minimum number of T-shirts?

SOLVE IT!

Getting Ready!

Math Club members are selling Pi Day T-shirts for $7.50 each. The goal is to raise $500 by Friday. The figure at the right shows how much they have raised by Wednesday. What is the minimum number of T-shirts they must still sell in order to reach their goal? Explain your reasoning.

MATHEMATICAL PRACTICES

You can model the situation in the Solve It with the inequality $337.50 + 7.50x \geq 500$. In this lesson, you will learn how to write and solve multi-step inequalities like this one.

Essential Understanding You solve a multi-step inequality in the same way you solve a one-step inequality. You use the properties of inequality to transform the original inequality into a series of simpler, equivalent inequalities.

Problem 1 Using More Than One Step

What are the solutions of $9 + 4t > 21$? Check the solutions.

Plan

How can you check the solutions?
Check the endpoint, 3. Then choose a value greater than 3 and check the inequality symbol.

$$9 + 4t > 21$$

$9 + 4t - 9 > 21 - 9$ Subtract 9 from each side.

$4t > 12$ Simplify.

$\frac{4t}{4} > \frac{12}{4}$ Divide each side by 4.

$t > 3$ Simplify.

Check $9 + 4(3) \stackrel{?}{=} 21$ Check the endpoint of $t > 3$ by substituting 3 for t in the related equation.

$21 = 21$ ✔ Simplify.

$9 + 4(4) \stackrel{?}{=} 21$ Check the inequality symbol of $t > 3$ by substituting 4 for t in the original inequality.

$25 > 21$ ✔ Simplify.

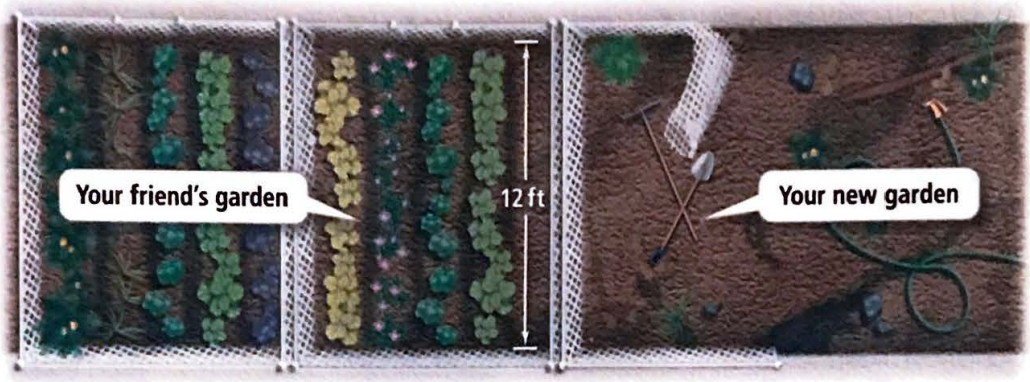
Got It? 1. What are the solutions of the inequality? Check your solutions.

a. $-6a - 7 \leq 17$ b. $-4 < 5 - 3n$ c. $50 > 0.8x + 30$

You can adapt familiar formulas to write inequalities. You use the real-world situation to determine which inequality symbol to use.

Problem 2 Writing and Solving a Multi-Step Inequality

Geometry **In a community garden, you want to fence in a vegetable garden that is adjacent to your friend's garden. You have at most 42 ft of fence. What are the possible lengths of your garden?**



Relate Since the fence will surround the garden, you can use the perimeter formula $P = 2\ell + 2w$.

twice the length	plus	twice the width	is at most	the amount of fence

Define Let ℓ = the length of the garden.

Write

2ℓ	+	$2(12)$	$\leq$	42

$$2\ell + 2(12) \leq 42$$

$$2\ell + 24 \leq 42 \quad \text{Simplify.}$$

$$2\ell + 24 - 24 \leq 42 - 24 \quad \text{Subtract 24 from each side.}$$

$$2\ell \leq 18 \quad \text{Simplify.}$$

$$\frac{2\ell}{2} \leq \frac{18}{2} \quad \text{Divide each side by 2.}$$

$$\ell \leq 9 \quad \text{Simplify.}$$

The length of the garden must be 9 ft or less.

Got It? 2. You want to make a rectangular banner that is 18 ft long. You have no more than 48 ft of trim for the banner. What are the possible widths of the banner?

Problem 3 Using the Distributive Property

Think

You can use reasoning and *guess-and-check* to solve the problem. If either 9 or 10 is a solution, at least one other answer choice would also be a solution. So, eliminate 9 and 10 as possible answers. Guess that either 8 or 11 is correct and check your guess.

Multiple Choice Which is a solution of $3(t + 1) - 4t \geq -5$?

(A) 8 (B) 9 (C) 10 (D) 11

$3(t + 1) - 4t \geq -5$	
$3t + 3 - 4t \geq -5$	Distributive Property
$-t + 3 \geq -5$	Combine like terms.
$-t + 3 - 3 \geq -5 - 3$	Subtract 3 from each side.
$-t \geq -8$	Simplify.
$\frac{-t}{-1} \leq \frac{-8}{-1}$	Divide each side by -1. Reverse the inequality symbol.
$t \leq 8$	Simplify.

8 is a solution of the inequality $t \leq 8$. The correct answer is A.

Got It? **3.** What are the solutions of $15 \leq 5 - 2(4m + 7)$? Check your solutions.

Some inequalities have variables on both sides of the inequality symbol. You need to gather the variable terms on one side of the inequality and the constant terms on the other side.

Problem 4 Solving an Inequality With Variables on Both Sides

Think

Why subtract $3n$ instead of $6n$ from each side of the inequality?
You can subtract either $3n$ or $6n$ from each side. However, subtracting $3n$ gives you a variable term with a positive coefficient.

What are the solutions of $6n - 1 > 3n + 8$?

$6n - 1 > 3n + 8$	
$6n - 1 - 3n > 3n + 8 - 3n$	To gather variables on the left, subtract $3n$ from each side.
$3n - 1 > 8$	Simplify.
$3n - 1 + 1 > 8 + 1$	To gather the constants on the right, add 1 to each side.
$3n > 9$	Simplify.
$\frac{3n}{3} > \frac{9}{3}$	Divide each side by 3.
$n > 3$	Simplify.

Got It? **4. a.** What are the solutions of $3b + 12 > 27 - 2b$? Check your solutions.

b. Reasoning The first step in solving Problem 4 was to subtract $3n$ from each side of the inequality. What else could have been the first step in solving the inequality? Explain.

Sometimes solving an inequality gives a statement that is *always* true, such as $4 > 1$. In that case, the solutions are all real numbers. If the statement is *never* true, as is $9 \leq -5$, then the inequality has no solution.

Problem 5 Inequalities With Special Solutions

Think

Is there another way to solve this inequality?
Yes. Instead of using the Distributive Property, you can first divide each side by 2.

A **What are the solutions of $10 - 8a \geq 2(5 - 4a)$?**

$$10 - 8a \geq 2(5 - 4a)$$

$10 - 8a \geq 10 - 8a$ Distributive Property

$10 - 8a + 8a \geq 10 - 8a + 8a$ Add $8a$ to each side.

$10 \geq 10$ Simplify.

Since the inequality $10 \geq 10$ is always true, the solutions of $10 - 8a \geq 2(5 - 4a)$ are all real numbers.

Think

Without solving, how can you tell that this inequality has no solution?
The variable terms on each side of the inequality are equal, but -5 is *not* greater than 7.

B **What are the solutions of $6m - 5 > 7m + 7 - m$?**

$$6m - 5 > 7m + 7 - m$$

$6m - 5 > 6m + 7$ Simplify.

$6m - 5 - 6m > 6m + 7 - 6m$ Subtract $6m$ from each side.

$-5 > 7$ Simplify.

Since the inequality $-5 > 7$ is never true, the inequality $6m - 5 > 7m + 7 - m$ has no solution.

Got It? **5.** What are the solutions of each inequality?

a. $9 + 5n \leq 5n - 1$ **b.** $8 + 6x \geq 7x + 2 - x$

Lesson Check

Do you know HOW?

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

1. $7 + 6a > 19$
2. $2(t + 2) - 3t \geq -1$
3. $6z - 15 < 4z + 11$
4. $18x - 5 \leq 3(6x - 2)$
5. The perimeter of a rectangle is at most 24 cm. Two opposite sides are both 4 cm long. What are the possible lengths of the other two sides?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. Reasoning How can you tell that the inequality $3t + 1 > 3t + 2$ has no solution just by looking at the terms in the inequality?

7. Reasoning Can you solve the inequality $2(x - 3) \leq 10$ *without* using the Distributive Property? Explain.

8. Error Analysis Your friend says that the solutions of the inequality $-2(3 - x) > 2x - 6$ are all real numbers. Do you agree with your friend? Explain. What if the inequality symbol were $\geq$?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Solve each inequality. Check your solutions.

See Problem 1.

9. $5f + 7 \leq 22$

10. $6n - 3 > -18$

11. $-5y - 2 < 8$

12. $6 - 3p \geq -9$

13. $9 \leq -12 + 6r$

14. $6 \leq 12 + 4j$

Write and solve an inequality.

See Problem 2.

15. Family Trip On a trip from Buffalo, New York, to St. Augustine, Florida, a family wants to travel at least 250 mi in the first 5 h of driving. What should their average speed be in order to meet this goal?

16. Geometry An isosceles triangle has at least two congruent sides. The perimeter of a certain isosceles triangle is at most 12 in. The length of each of the two congruent sides is 5 in. What are the possible lengths of the remaining side?

Solve each inequality.

See Problems 3 and 4.

17. $3(k - 5) + 9k \geq -3$

18. $-(7c - 18) - 2c > 0$

19. $-3(j + 3) + 9j < -15$

20. $-4 \leq 4(6y - 12) - 2y$

21. $30 > -(5z + 15) + 10z$

22. $-4(d + 5) - 3d > 8$

23. $4x + 3 < 3x + 6$

24. $4v + 8 \geq 6v + 10$

25. $5f + 8 \geq 2 + 6f$

26. $6 - 3p \leq 4 - p$

27. $3m - 4 \leq 6m + 11$

28. $4t + 17 > 7 + 5t$

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

See Problem 5.

29. $-3(w - 3) \geq 9 - 3w$

30. $-5r + 6 \leq -5(r + 2)$

31. $-2(6 + s) \geq -15 - 2s$

32. $9 + 2x < 7 + 2(x - 3)$

33. $2(n - 8) < 16 + 2n$

34. $6w - 4 \leq 2(3w + 6)$

B Apply

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

35. $-3(x - 3) \geq 5 - 4x$

36. $3s + 6 \leq -5(s + 2)$

37. $3(2 + t) \geq 15 - 2t$

38. $\frac{4}{3}s - 3 < s + \frac{2}{3} - \frac{1}{3}s$

39. $4 - 2n \leq 5 - n + 1$

40. $-2(0.5 - 4t) \geq -3(4 - 3.4)$

41. $4(a - 2) - 6a \leq -9$

42. $4(3n - 1) \geq 2(n + 3)$

43. $17 - (4k - 2) \geq 2(k + 3)$

44. Think About a Plan Your cell phone plan costs \$39.99 per month plus \$.15 for each text message you send or receive. You have at most \$45 to spend on your cell phone bill. What is the maximum number of text messages that you can send or receive next month?

- What information do you know? What information do you need?
- What inequality can you use to find the maximum number of text messages that you can send or receive?
- What are the solutions of the inequality? Are they reasonable?

45. Rental Rates The student council wants to rent a ballroom for the junior prom. The ballroom's rental rate is \$1500 for 3 h and \$125 for each additional half hour. Suppose the student council raises \$2125. What is the maximum number of hours for which they can rent the ballroom?

46. Writing Suppose a friend is having difficulty solving $3.75(q - 5) > 4(q + 3)$. Explain how to solve the inequality, showing all the necessary steps and identifying the properties you would use.

47. Biology The average normal body temperature for humans is 98.6°F. An abnormal increase in body temperature is classified as hyperthermia, or fever. Which inequality represents the body temperature in degrees Celsius of a person with hyperthermia? (*Hint*: To convert from degrees Celsius C to degrees Fahrenheit F, use the formula $F = \frac{9}{5}C + 32$.)

(A) $\frac{9}{5}C + 32 \geq 98.6$ (B) $\frac{9}{5}C + 32 \leq 98.6$ (C) $\frac{9}{5}C + 32 < 98.6$ (D) $\frac{9}{5}C + 32 > 98.6$

48. Open-Ended Write two different inequalities that you can solve by subtracting 3 from each side and then dividing each side by -5. Solve each inequality.

49. **a.** Solve $6v + 5 \leq 9v - 7$ by gathering the variable terms on the left side and the constant terms on the right side of the inequality.

b. Solve $6v + 5 \leq 9v - 7$ by gathering the constant terms on the left side and the variable terms on the right side of the inequality.

c. Compare the results of parts (a) and (b).

d. Which method do you prefer? Explain.

50. Mental Math Determine whether each inequality is *always true* or *never true*.

a. $5s + 7 \geq 7 + 5s$ **b.** $4t + 6 > 4t - 3$ **c.** $5(m + 2) < 5m - 4$

51. Commission A sales associate in a shoe store earns \$325 per week, plus a commission equal to 4% of her sales. This week her goal is to earn at least \$475. At least how many dollars' worth of shoes must she sell in order to reach her goal?

52. A student uses the table below to help solve $7y + 2 < 6(4 - y)$.

y	$7y + 2$	$<$	$6(4 - y)$
0.5	$7(0.5) + 2 = 5.5$	True	$6(4 - 0.5) = 21$
1	$7(1) + 2 = 9$	True	$6(4 - 1) = 18$
1.5	$7(1.5) + 2 = 12.5$	True	$6(4 - 1.5) = 15$
2	$7(2) + 2 = 16$	False	$6(4 - 2) = 12$

a. Reasoning Based on the table, would you expect the solution of $7y + 2 < 6(4 - y)$ to be of the form $y < c$ or $y > c$, where c is a real number? Explain.

b. Estimate Based on the table, estimate the value of c.

c. Solve the inequality. Compare the actual solution to your estimated solution.

Error Analysis Describe and correct the error in each solution.

53.

$4y + 4 \leq -3y + 6$

$4y \leq -3y + 2$

$y \leq 2$

54.

$5(p + 3) > 4p + 2$

$5p + 3 > 4p + 2$

$5p > 4p - 1$

$p > -1$

Challenge

55. Geometry The base of a triangle is 12 in. Its height is $(x + 6)$ in. Its area is no more than 72 in.^{2}. What are the possible integer values of x?

56. Part-Time Jobs You can earn money by tutoring for \$8 per hour and by walking dogs for \$7.50 per hour. You have 15 h available to work. What is the greatest number of hours you can spend walking dogs and still make at least \$115?

57. Freight Handling The elevator of a building can safely carry no more than 4000 lb. A worker moves supplies in 50-lb boxes from the loading dock to the fourth floor of the building. The worker weighs 210 lb. The cart he uses weighs 95 lb.

a. What is the greatest number of boxes he can move in one trip?

b. The worker needs to deliver 275 boxes. How many trips must he make?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2

Look back at the information on page 163 about the athletic boosters' concession stand. In part (c) of the Apply What You've Learned section in Lesson 3-2, you listed possible combinations of packages of empty boxes the boosters can buy in order to have enough empty boxes for the next game.

a. Consider the possibility that the athletic boosters buy four packages of 75 empty boxes to augment their supply of 40 that they already have on hand. Let x be the number of boxes of popcorn that the boosters sell at the next game. Write an inequality that models the boosters making a profit (that is, where net revenue exceeds total costs).

b. Solve your inequality from part (a). Interpret your solution in terms of the situation.