

3-6

Compound Inequalities

Objectives To solve and graph inequalities containing the word *and*
To solve and graph inequalities containing the word *or*



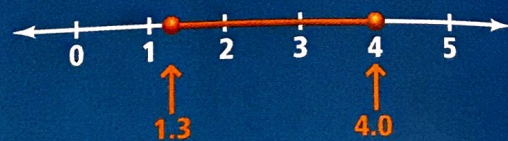
What does it mean that the number line shows a range of values?



Getting Ready!

The diagram shows the number of boxes of oranges that an orange tree can produce in 1 year. An orange grower earns \$9.50 for each box of oranges that he sells. How much could the grower expect to earn in 1 year from 1 tree? Explain your reasoning.

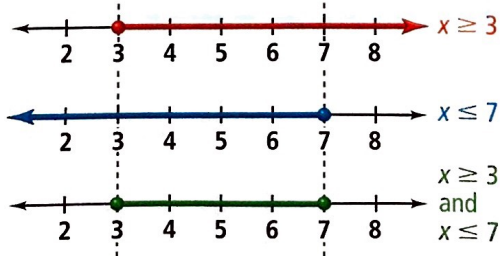
Average Annual Orange Tree Production (number of boxes per year)



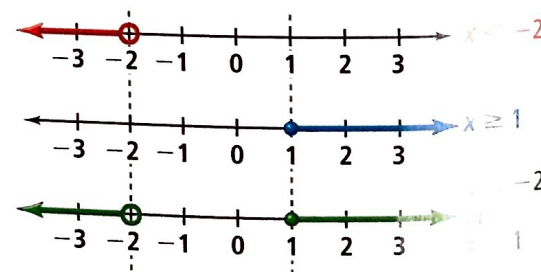
The Solve It involves a value that is between two numbers. You can use a compound inequality to represent this relationship. A **compound inequality** consists of two distinct inequalities joined by the word *and* or the word *or*.

Essential Understanding You find the solutions of a compound inequality either by identifying where the solution sets of the distinct inequalities overlap or by combining the solution sets to form a larger solution set.

The graph of a compound inequality with the word *and* contains the *overlap* of the graphs of the two inequalities that form the compound inequality.



The graph of a compound inequality with the word *or* contains *each* graph of the two inequalities that form the compound inequality.



You can rewrite a compound inequality involving *and* as a single inequality. For instance, in the inequality above, you can write $x \geq 3$ and $x \leq 7$ as $3 \leq x \leq 7$. You read this as “ x is greater than or equal to 3 and less than or equal to 7.” Another way to read it is “ x is between 3 and 7, inclusive.” In this example, *inclusive* means the solutions of the inequality include both 3 and 7.



Lesson Vocabulary

- compound inequality
- interval notation

Problem 1 Writing a Compound Inequality

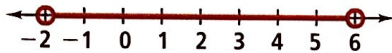
What compound inequality represents the phrase? Graph the solutions.

A all real numbers that are greater than -2 and less than 6

$$n > -2 \text{ and } n < 6$$

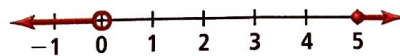
$$-2 < n \text{ and } n < 6$$

$$-2 < n < 6$$



B all real numbers that are less than 0 or greater than or equal to 5

$$t < 0 \text{ or } t \geq 5$$



Think

Why can you write an and inequality without the word and?

The compound inequality $-2 < n$ and $n < 6$ means n is greater than -2 and n is less than 6 . This means n is between -2 and 6 . You write this as $-2 < n < 6$.

Got It? 1. For parts (a) and (b) below, write a compound inequality that represents each phrase. Graph the solutions.

a. all real numbers that are greater than or equal to -4 and less than 6

b. all real numbers that are less than or equal to $2\frac{1}{2}$ or greater than 6

c. Reasoning What is the difference between “ x is between -5 and 7 ” and “ x is between -5 and 7 , inclusive”?

A solution of a compound inequality involving *and* is any number that makes *both* inequalities true. One way you can solve a compound inequality is by separating it into two inequalities.

Problem 2 Solving a Compound Inequality Involving And

What are the solutions of $-3 \leq m - 4 < -1$? Graph the solutions.

$$-3 \leq m - 4 < -1$$

$$-3 \leq m - 4 \quad \text{and} \quad m - 4 < -1$$

Write the compound inequality as two inequalities joined by the word *and*.

$$-3 + 4 \leq m - 4 + 4 \quad \text{and} \quad m - 4 + 4 < -1 + 4$$

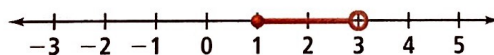
Add 4 to each side of each inequality.

$$1 \leq m \quad \text{and} \quad m < 3$$

Simplify.

$$1 \leq m < 3$$

Write the solutions as a single inequality.



Got It? 2. What are the solutions of $-2 < 3y - 4 < 14$? Graph the solutions.

Plan

How do you know to join the two inequalities with *and*?

The compound inequality $-3 \leq m - 4 < -1$ means that the quantity $m - 4$ is between -3 and -1 , including -3 . So use the word *and*.

You can also solve an inequality like $-3 \leq m - 4 < -1$ by working on all three parts of the inequality at the same time. You work to isolate the variable between the inequality symbols. This method is used in Problem 3.

Problem 3 Writing and Solving a Compound Inequality

Test Average To earn a B in your algebra course, you must achieve an unrounded test average between 84 and 86, inclusive. You scored 86, 85, and 80 on the first three tests of the grading period. What possible scores can you earn on the fourth and final test to earn a B in the course?

Think

What is another way to solve this problem?

You can *work backward* to solve this problem. You can start with the inequality $84 \leq x \leq 86$ where x represents the average of your test scores. Then rewrite the inequality in terms of the sum of your 4 test scores.

Know

- Test average must be between 84 and 86, inclusive
- First 3 test scores

Need

Possible scores you can earn on the last test to get a B in the course

Plan

Write an expression for your test average. Then write and solve a compound inequality.

$84 \leq \frac{86 + 85 + 80 + x}{4} \leq 86$	Write a compound inequality.
$4(84) \leq 4\left(\frac{251 + x}{4}\right) \leq 4(86)$	Multiply each part by 4.
$336 \leq 251 + x \leq 344$	Simplify.
$336 - 251 \leq 251 + x - 251 \leq 344 - 251$	Subtract 251 from each part.
$85 \leq x \leq 93$	Simplify.

Your score on the fourth test must be between 85 and 93, inclusive.



Got It? 3. Reasoning Suppose you scored 78, 78, and 79 on the first three tests. Is it possible for you to earn a B in the course? Assume that 100 is the maximum grade you can earn in the course and on the test. Explain.

A solution of a compound inequality involving *or* is any number that makes *either* inequality true. To solve a compound inequality involving *or*, you must solve separately the two inequalities that form the compound inequality.

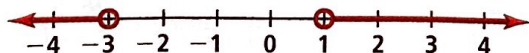
Problem 4 Solving a Compound Inequality Involving Or

What are the solutions of $3t + 2 < -7$ or $-4t + 5 < 1$? Graph the solutions.

$3t + 2 < -7$	or	$-4t + 5 < 1$
$3t + 2 - 2 < -7 - 2$	or	$-4t + 5 - 5 < 1 - 5$
$3t < -9$	or	$-4t < -4$
$\frac{3t}{3} < \frac{-9}{3}$	or	$\frac{-4t}{-4} > \frac{-4}{-4}$
$t < -3$	or	$t > 1$

Reverse the inequality symbol when you divide by a negative number.

The solutions are given by $t < -3$ or $t > 1$.



Got It? 4. What are the solutions of $-2y + 7 < 1$ or $4y + 3 \leq -5$? Graph the solutions.

Plan

How is this inequality different from others you've solved?

It contains the word *or*. Unlike an *and* inequality, it's formed by two inequalities with solutions that do not overlap.

You can use an inequality such as $x \leq -3$ to describe a portion of the number line called an *interval*. You can also use *interval notation* to describe an interval on the number line. **Interval notation** includes the use of three special symbols. These symbols include

parentheses: Use (or) when a $<$ or $>$ symbol indicates that the interval's endpoints are *not* included.

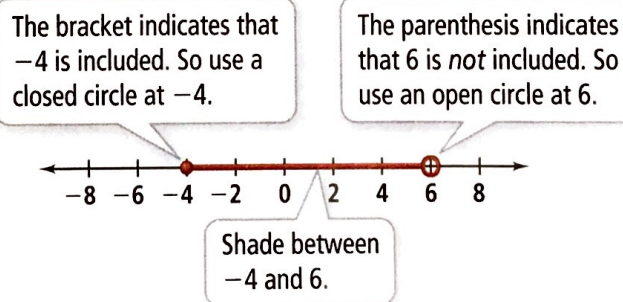
brackets: Use [or] when a \leq or \geq symbol indicates that the interval's endpoints are included.

infinity: Use ∞ when the interval continues forever in a *positive* direction. Use $-\infty$ when the interval continues forever in a *negative* direction.

Inequality	Graph	Interval Notation
$x \geq 2$		$[2, \infty)$
$x < 2$		$(-\infty, 2)$
$1 < x \leq 5$		$(1, 5]$
$x < -3$ or $x \geq 4$		$(-\infty, -3)$ or $[4, \infty)$

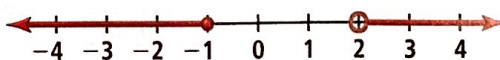
Problem 5 Using Interval Notation

A What is the graph of $[-4, 6)$? How do you write $[-4, 6)$ as an inequality?



The inequality $-4 \leq x < 6$ represents the interval notation $[-4, 6)$.

B What is the graph of $x \leq -1$ or $x > 2$? How do you write $x \leq -1$ or $x > 2$ in interval notation?



The interval notation $(-\infty, -1]$ or $(2, \infty)$ represents the inequality $x \leq -1$ or $x > 2$.

- Got It?** 5. a. What is the graph of $(-2, 7]$? How do you write $(-2, 7]$ as an inequality?
 b. What is the graph of $y > 7$? How do you write $y > 7$ in interval notation?

Plan

How do you write interval notation as an inequality?

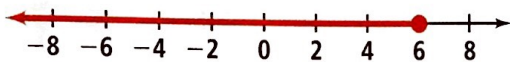
It may help to read its meaning aloud first. $[-4, 6)$ means all real numbers greater than or equal to -4 and less than 6 .



Lesson Check

Do you know HOW?

1. What compound inequality represents the phrase “all real numbers that are greater than or equal to 0 and less than 8”? Graph the solutions.
2. What are the solutions of $-4 \leq r - 5 < -1$? Graph the solutions.
3. Your test scores in science are 83 and 87. What possible scores can you earn on your next test to have a test average between 85 and 90, inclusive?
4. Write the interval represented on the number line below as an inequality and in interval notation.



Do you UNDERSTAND?



5. **Vocabulary** Which of the following are compound inequalities?
 - A. $x > 4$ or $x < -4$
 - B. $x \geq 6$
 - C. $8 \leq 5x < 30$
 - D. $7x > 42$ or $-5x \leq 10$
6. **Error Analysis** A student writes the inequality $x \geq 17$ in interval notation as $[17, \infty]$. Explain why this is incorrect.
7. **Reasoning** What are the solutions of $3x - 7 \leq 14$ or $4x - 8 > 20$? Write your solutions as a compound inequality and in interval notation.
8. **Writing** Compare the graph of a compound inequality involving *and* with the graph of a compound inequality involving *or*.



Practice and Problem-Solving Exercises



A Practice

Write a compound inequality that represents each phrase. Graph the solutions.

See Problem 1

9. all real numbers that are between -5 and 7

10. The circumference of a women's basketball must be between 28.5 in. and 29 in., inclusive.

Solve each compound inequality. Graph your solutions.

See Problems 2 and 3

11. $-4 < k + 3 < 8$

12. $5 \leq y + 2 \leq 11$

13. $3 < 4p - 5 \leq 15$

14. $15 \leq \frac{20 + 11 + k}{3} \leq 19$

15. $\frac{1}{4} < \frac{2x - 7}{2} < 5$

16. $-3 \leq \frac{6 - q}{9} \leq 3$

Solve each compound inequality. Graph your solutions.

See Problem 4

17. $6b - 1 < -7$ or $2b + 1 > 5$

18. $5 + m > 4$ or $7m < -35$

19. $4d + 5 \geq 13$ or $7d - 2 < 12$

20. $7 - c < 1$ or $4c \leq 12$

21. $5y + 7 \leq -3$ or $3y - 2 \geq 13$

22. $5z - 3 > 7$ or $4z - 6 < -10$

Write each interval as an inequality. Then graph the solutions.

See Problem 5

23. $(-\infty, 2]$

24. $[-4, 5]$

25. $(-\infty, -1]$ or $(3, \infty)$

26. $[6, \infty)$

Write each inequality in interval notation. Then graph the interval.

27. $x > -2$

28. $x \leq 0$

29. $x < -2$ or $x \geq 1$

30. $-3 \leq x < 4$

B Apply

Solve each inequality. Write each set in interval notation.

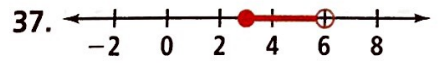
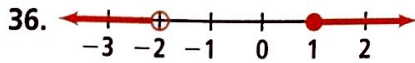
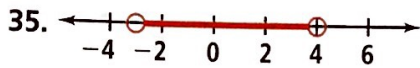
31. $7 < x + 6 \leq 12$

32. $-9 < 3m + 6 \leq 18$

33. $f + 14 < 9$ or $-9f \leq -45$

34. $12h - 3 \geq 15h$ or $5 > -0.2h + 10$

Write a compound inequality that each graph could represent.



Solve each compound inequality. Justify each step.

38. $4r - 3 > 11$ or $4r - 3 \leq -11$

39. $2 \leq 0.75v \leq 4.5$

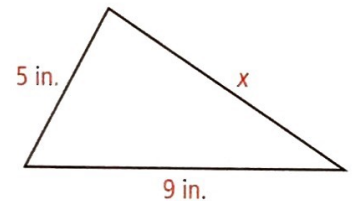
40. $\frac{4y + 2}{5} - 5 > 3$ or $\frac{4 - 3y}{6} > 4$

41. $-\frac{4}{3} \leq \frac{1}{7}w - \frac{3}{4} < 1$

- STEM** 42. **Chemistry** The acidity of the water in a swimming pool is considered normal if the average of three pH readings is between 7.2 and 7.8, inclusive. The first two readings for a swimming pool are 7.4 and 7.9. What possible values for the third reading p will make the average pH normal?

- ©** 43. **Think About a Plan** The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The lengths of two sides of a triangle are given. What are the possible lengths x of the third side of the triangle?

- Is there an upper limit on the value of x ? Is there a lower limit?
- How can you use your answers to the previous question to write one or more inequalities involving x ?



Use your answers to Exercise 43 to answer Exercises 44–47. The lengths of two sides of a triangle are given. Find the possible lengths of the third side.

44. 3.75 in., 7 in.

45. 15 ft, 21 ft

46. 14 mm, 35 mm

47. 6 m, 17 m

- STEM** 48. **Physics** The force exerted on a spring is proportional to the distance the spring is stretched from its relaxed position. Suppose you stretch a spring a distance of d inches by applying a force of F pounds. For your spring, $\frac{d}{F} = 0.8$. You apply forces between 25 lb and 40 lb, inclusive. What inequality describes the distances the spring is stretched?

- ©** 49. **Reasoning** Describe the solutions of $4x - 9 < 7$ or $3x - 10 > 2$.

50. **Nutrition** A sedentary 15-year-old male should consume no more than 2200 Calories per day. A moderately active 15-year-old male should consume between 2400 and 2800 Calories per day. An active 15-year-old male should consume between 2800 and 3200 Calories per day. Model these ranges on a number line. Represent each range of calories using interval notation.

**Challenge**

51. Heart Rates Recommended heart rates during exercise vary with age and physical condition. For a healthy person doing moderate to intense exercise, such as hiking, the inequality $0.5(220 - a) \leq R \leq 0.9(220 - a)$ gives a target range for the heart rate R (in beats per minute), based on age a (in years).

- What is the target range for heart rates for a person 15 years old?
- How old is a person whose target range is between 99 and 178.2 beats per minute?



52. Chemistry Matter is in a liquid state when its temperature is between its melting point and its boiling point. The melting point of the element mercury is -38.87°C , and its boiling point is 356.58°C . What is the range of temperatures in degrees Fahrenheit for which mercury is *not* in a liquid state? (Hint: $C = \frac{5}{9}(F - 32)$) Express the range as an inequality and in interval notation.

Standardized Test Prep**SAT/ACT**

53. A taxi traveled 5 mi to John's home and then drove him to the airport 10 mi away. Which inequality represents the possible distances d of the taxi from the airport when it started traveling toward John's home?

- (A) $5 \leq d \leq 10$ (B) $5 \leq d \leq 15$ (C) $0 \leq d \leq 5$ (D) $0 \leq d \leq 10$

54. A student must earn at least 24 credits in high school in order to graduate. Which inequality or graph does NOT describe this situation?

- (F) $c \leq 24$ (H) $24 \leq c$
 (G) $c \geq 24$ (I) 

Short Response

55. The County Water Department charges a monthly administration fee of \$10.40 plus \$.0059 for each gallon of water used, up to, but not including, 7500 gal. What are the minimum and maximum numbers of gallons of water used by customers whose monthly charge is at least \$35 but no more than \$50? Express amounts to the nearest gallon.

Mixed Review

Let $A = \{1, 3, 5, 7\}$, let $B = \{4, 8, 12\}$, and let the universal set be $U = \{1, 2, 3, 4, 5, 7, 8, 12, 15\}$.

See Lesson 3-5.

56. What are the subsets of A ?

57. What is B' ?

58. Is B' a subset of A ?

Solve each inequality.

See Lesson 3-4.

59. $5 < 6b + 3$

60. $12n \leq 3n + 27$

61. $2 + 4r \geq 5(r - 1)$

Get Ready! To prepare for Lesson 3-7, do Exercises 62–64.

Complete each statement with $<$, $=$, or $>$.

See Lesson 1-5.

62. $|3 - 7| \square 4$

63. $|-5| + 2 \square 6$

64. $|6 - 2\frac{1}{4}| \square 3\frac{5}{8}$