3-8 Unions and Intersections of Sets

Common Core State Standards

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems . . .

MP 1, MP 2, MP 3, MP 4

Objective To find the unions and intersections of sets

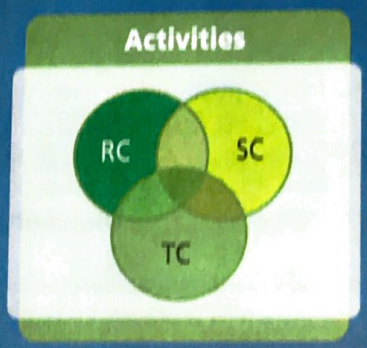
What is the meaning of the middle area where the three circles overlap?

MATHEMATICAL PRACTICES

Getting Ready!

Fifty students were polled about their after-school activities. All said that they participate in one or more of three clubs: the Robotics Club (*RC*), the Student Council (*SC*), and the Theater Club (*TC*). How many students participate in the theater club only? Use the information in the table. Explain your reasoning.

Activity	Number of Students
RC	22
SC	22
RC and *TC* only	5
RC and *SC* only	8
SC and *TC* only	6
RC, *SC*, and *TC*	3



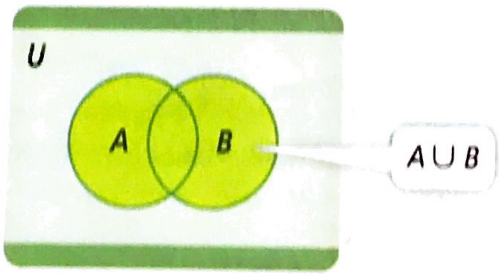
Lesson Vocabulary

- union
- intersection
- disjoint sets

Certain regions of the Venn diagram in the Solve It show *unions* and *intersections* of sets.

Essential Understanding Given two or more sets, you can describe which elements belong to *at least one* set. You can also describe which elements belong to *all* of the sets. You use symbols to represent these relationships.

The **union** of two or more sets is the set that contains all elements of the sets. The symbol for union is $\cup$. To find the union of two sets, list the elements that are in either set, or in both sets. An element is in the union if it belongs to *at least one* of the sets. In the Venn diagram below, $A \cup B$ is shaded.



Problem 1 Union of Sets

In your left pocket, you have a quarter, a paper clip, and a key. In your right pocket, you have a penny, a quarter, a pencil, and a marble. What is a set that represents the different items in your pockets?

Step 1 Write sets that represent the contents of each pocket.

Left pocket: $L = \{\text{quarter, paper clip, key}\}$

Right pocket: $R = \{\text{penny, quarter, pencil, marble}\}$

Step 2 Write the union of the sets, which represents the different items that are in your pockets.

$L \cup R = \{\text{quarter, paper clip, key, penny, pencil, marble}\}$

Think

What if an item is in both sets?

Sets L and R each contain a quarter, so a quarter is in the union of L and R. You should list it only once, though.

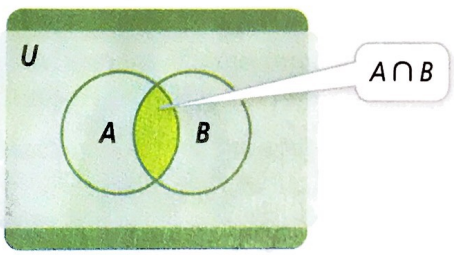
Got It? **1. a.** Write sets P and Q below in roster form. What is $P \cup Q$?

$P = \{x \mid x \text{ is a whole number less than } 5\}$

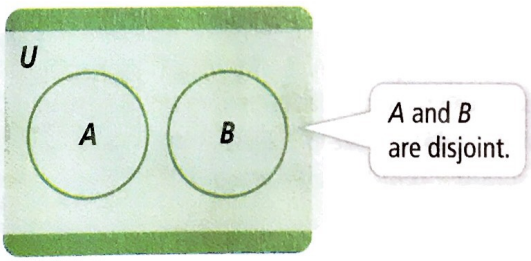
$Q = \{y \mid y \text{ is an even natural number less than } 5\}$

b. Reasoning What is true about the union of two distinct sets if one set is a subset of the other?

The **intersection** of two or more sets is the set of elements that are common to every set. An element is in the intersection if it belongs to *all* of the sets. The symbol for intersection is $\cap$. When you find the intersection of two sets, list only the elements that are in both sets. In the Venn diagram below, $A \cap B$ is shaded.



Disjoint sets have no elements in common. The intersection of disjoint sets is the empty set. The diagram below shows two disjoint sets.



Problem 2 Intersection of Sets

Set $X = \{x \mid x \text{ is a natural number less than } 19\}$, set $Y = \{y \mid y \text{ is an odd integer}\}$, and set $Z = \{z \mid z \text{ is a multiple of } 6\}$.

A What is $X \cap Z$?

List the elements that are both natural numbers less than 19 and multiples of 6: $X \cap Z = \{6, 12, 18\}$.

B What is $Y \cap Z$?

List the elements that are both odd integers and multiples of 6. There are no multiples of 6 that are also odd, so Y and Z are disjoint sets. They have no elements in common. $Y \cap Z = \varnothing$, the empty set.

Think

Why are sets Y and Z disjoint?
Every element of Z is a multiple of 6, so every element of Z is *even*. Y contains only *odd* numbers. So no element of Z belongs to Y.

Got It? **2.** Let $A = \{2, 4, 6, 8\}$, $B = \{0, 2, 5, 7, 8\}$, and $C = \{n \mid n \text{ is an odd whole number}\}$.

a. What is $A \cap B$? **b.** What is $A \cap C$? **c.** What is $C \cap B$?

You can draw Venn diagrams to solve problems involving relationships between sets.

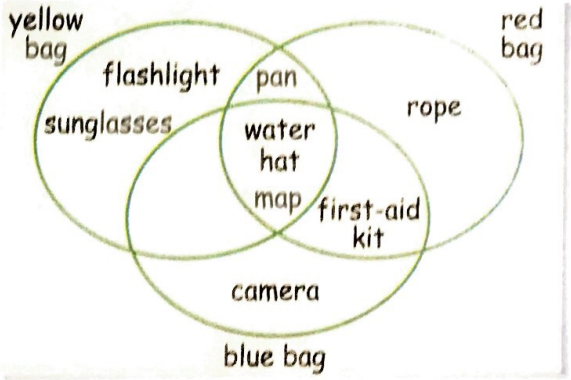
Problem 3 Making a Venn Diagram

Camping **Three friends are going camping. The items in each of their backpacks form a set. Which items do all three friends have in common?**

Think

How do you know where in the diagram to place each item?
Items that the friends have in common belong in an intersection. Use the Venn diagram to determine the correct intersection.

Draw a Venn diagram to represent the union and intersection of the sets.



All three friends have a hat, a map, and a bottle of water in their backpacks.

Got It? **3.** Let $A = \{x | x \text{ is one of the first five letters in the English alphabet}\}$, $B = \{x | x \text{ is a vowel}\}$, and $C = \{x | x \text{ is a letter in the word VEGETABLE}\}$. Which letters are in all three sets?

You can also use Venn diagrams to show the *number* of elements in the union or intersection of sets.

Problem 4 Using a Venn Diagram to Show Numbers of Elements

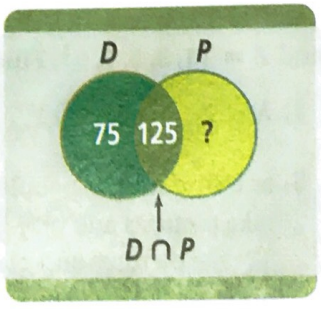
Polling **Of 500 commuters polled, some drive to work, some take public transportation, and some do both. Two hundred commuters drive to work and 125 use both types of transportation. How many commuters take public transportation?**

Know
- Number who commute: 500
- Number who drive: 200
- Number who drive *and* use public transportation: 125

Need
Number who use public transportation

Plan
- Draw a Venn diagram
- Calculate the number of commuters who only drive
- Calculate the number of commuters who only use public transportation

Step 1 Draw a Venn diagram. Let D = commuters who drive and P = commuters who take public transportation.



Step 2 The intersection of D and P represents the commuters who use both methods of transportation: $D \cap P$ has 125 commuters.

Step 3 Find the number of commuters who only drive: $200 - 125 = 75$. Enter 75 into the Venn diagram.

Step 4 The total number of commuters is 500. Subtract to find the number of commuters who use only public transportation: $500 - 200 = 300$.

The number of commuters using public transportation is $300 + 125 = 425$.

Got It? **4.** Of 30 students in student government, 20 are honor students and 9 are officers and honor students. All of the students are officers, honor students, or both. How many are officers but not honor students?

Recall from Lesson 3-6 that the graph of a compound inequality with the word *and* contains the *overlap* of the graphs of the two inequalities that form the compound inequality. You can think of the overlap as the intersection of two sets. Similarly, you can think of the solutions of an *or* inequality as the union of two sets.

Problem 5 Writing Solutions of an Inequality

What are the solutions of $|2x - 1| < 3$? Write the solutions as either the union or the intersection of two sets.

$$|2x - 1| < 3$$

$-3 < 2x - 1 < 3$ Write a compound inequality.

$-2 < 2x < 4$ Add 1 to each expression.

$-1 < x < 2$ Divide each side by 2.

Think

Is the solution of the inequality a union or an intersection?
The solution is a compound inequality joined by the word *and*. So the solution is an intersection.

The solutions of the inequality are given by $-1 < x < 2$. You can write this as $x > -1$ and $x < 2$. This compound inequality is the intersection of two sets, which you can write as follows: $\{x|x > -1\} \cap \{x|x < 2\}$.

Got It? 5. Solve each inequality. Write the solutions as either the union or the intersection of two sets.

a. $8 \leq x + 5 < 11$

b. $|4x - 6| > 14$

Lesson Check

Do you know HOW?

Let $X = \{2, 4, 6, 8, 10\}$, $Y = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and $Z = \{1, 3, 5, 7, 9\}$. Find each union or intersection.

1. $X \cup Y$
2. $X \cap Y$
3. $X \cap Z$
4. $Y \cup Z$

5. In a survey of 80 people who use their cell phones to take pictures and play games, 49 take pictures and 35 take pictures and play games. How many people only use their cell phones to play games?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** Suppose A and B are nonempty sets. Which set contains more elements: $A \cup B$ or $A \cap B$? Explain your reasoning.

7. **Compare and Contrast** How are unions and intersections of sets different?

Determine whether each statement is *true* or *false*.

8. If x is an element of set A and x is not an element of set B, then x is an element of $A \cup B$.

9. If x is not an element of set A and x is an element of set B, then x is an element of $A \cap B$.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find each union or intersection. Let $A = \{1, 3, 4\}$, $B = \{x|x$ is an even whole number less than $9\}$, $C = \{2, 5, 7, 10\}$, and $D = \{x|x$ is an odd whole number less than $10\}$.

See Problems 1 and 2.

10. $A \cup B$
11. $A \cup C$
12. $A \cup D$
13. $B \cup C$
14. $B \cup D$
15. $C \cup D$
16. $A \cap B$
17. $A \cap C$
18. $A \cap D$
19. $B \cap C$
20. $B \cap D$
21. $C \cap D$

Draw a Venn diagram to represent the union and intersection of these sets. See Problem 3.

22. The letters in the words ALGEBRA, GEOMETRY, and CALCULUS are represented by the sets $V = \{A, L, G, E, B, R\}$, $W = \{G, E, O, M, T, R, Y\}$, and $X = \{C, A, L, U, S\}$, respectively.

23. Let $E = \{x | x \text{ is a positive, composite number less than } 10\}$, $F = \{1, 2, 4, 5, 6, 8, 9\}$, and $G = \{x | x \text{ is a positive, even number less than or equal to } 10\}$.

24. Let $L = \{A, B, C, 1, 2, 3, \text{horse, cow, pig}\}$, $M = \{-1, 0, 1, B, Y, \text{pig, duck, } \Delta\}$, and $N = \{C, 3, \text{duck, } \Delta\}$.

25. Camping Twenty-eight girls went camping. There were two main activities: volleyball and swimming. Fourteen girls went swimming, 5 participated in both activities, and 4 girls did neither. How many girls only played volleyball? See Problem 4.

26. Winter Sports A ski shop owner surveys 200 people who ski or snowboard. If 196 people ski and 154 people do both activities, how many people snowboard?

Solve each inequality. Write the solutions as either the union or intersection of two sets. See Problem 5.

27. $|3x - 5| < 14$

28. $-6 < n + 7 \leq 21$

29. $|8w - 1| \geq 7$

30. $3 \leq |5d + 11|$

31. $2|x - 7| > 28$

32. $|4.5t - 1.5| \leq 12$

B Apply

Find each union or intersection. Let $W = \{5, 6, 7, 8\}$, $X = \{3, 6, 9\}$, $Y = \{2, 3, 7, 8\}$, and $Z = \{x | x \text{ is an even whole number less than } 10\}$.

33. $W \cup Y \cup Z$

34. $X \cap Y \cap Z$

35. $W \cap X \cap Z$

36. Writing Let $M = \{x | x \text{ is a multiple of } 3\}$ and $N = \{x | x \text{ is a multiple of } 5\}$. Describe the intersection of M and N.

37. Think About a Plan Blood type is determined partly by which *antigens* a red blood cell has. An antigen is a protein on the surface of a red blood cell. Type A contains the A antigen. Type B contains the B antigen. Type AB contains both A and B antigens. Type O does not have any antigens. A hospital has 25 patients with the A antigen, 17 with the B antigen, 10 with the A and B antigens, and 30 without A or B antigens. How many patients are represented by the data?

- How can a Venn diagram help you solve the problem?
- What strategies can you use to complete the Venn diagram?

38. Sports In a survey of students about favorite sports, the results include 22 who like tennis, 25 who like football, 9 who like tennis and football, 17 who like tennis and baseball, 20 who like football and baseball, 6 who like all three sports, and 4 who like none of the sports. How many students like only tennis and football? How many students like only tennis and baseball? How many students like only baseball and football?

39. Reasoning Suppose A and B are sets such that $A \subseteq B$. What is true about $A \cap B$?

Draw a Venn diagram to represent the union and intersection of these sets. See Problem 3.

22. The letters in the words ALGEBRA, GEOMETRY, and CALCULUS are represented by the sets $V = \{A, L, G, E, B, R\}$, $W = \{G, E, O, M, T, R, Y\}$, and $X = \{C, A, L, U, S\}$, respectively.

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34. $X \cap Y \cap Z$

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39. Reasoning Suppose A and B are sets such that $A \subseteq B$. What is true about $A \cap B$?

The *cross product* of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs with the first element in A and with the second element in B. In set-builder notation, you write:

$$A \times B = \{(a, b) | a \text{ is an element of } A, b \text{ is an element of } B\}$$

For example, suppose $A = \{1, 2\}$ and $B = \{7, 10, 12\}$. Then:

$$A \times B = \{(1, 7), (1, 10), (1, 12), (2, 7), (2, 10), (2, 12)\}$$

Given sets A and B, find $A \times B$.

40. $A = \{1, 2, 3\}$, $B = \{-3, -2, -1, 0\}$

41. $A = \{\pi, 2\pi, 3\pi, 4\pi\}$, $B = \{2, 4\}$

42. $A = \{\text{grape, apple, orange}\}$, $B = \{\text{jam, juice}\}$

43. $A = \{\text{reduce, reuse, recycle}\}$, $B = \{\text{plastic}\}$

Challenge

44. Use a Venn diagram to determine whether the statement $(A \cap B)' = A' \cap B'$ is *true* or *false*.

45. Reasoning Is the statement $(A \cup B) \cap C = A \cup (B \cap C)$ *always, sometimes,* or *never* true? Justify your answer.

Standardized Test Prep

SAT/ACT

46. Set $X = \{x | x \text{ is a factor of } 12\}$ and set $Y = \{y | y \text{ is a factor of } 16\}$. Which set represents $X \cap Y$?

(A) $\varnothing$ (B) $\{1, 2, 4\}$ (C) $\{0, 1, 2, 4\}$ (D) $\{1, 2, 3, 4, 6, 8, 12, 16\}$

47. Which compound inequality is equivalent to $|x + 4| < 8$?

(F) $-12 < x < 4$ (H) $-12 > x > 4$

(G) $x < -12$ or $x > 4$ (I) $x > -12$ or $x < 4$

Short Response

48. Suppose you earn \$80 per week at your summer job. Your employer offers you a \$20 raise or a 20% raise. Which should you take? Explain.

Mixed Review

Solve each equation or inequality. See Lesson 3-7.

49. $|x| = 4$ **50.** $|n| + 7 = 9$ **51.** $4|f - 5| = 12$ **52.** $3|3y + 2| = 18$

53. $|4d| \leq 20$ **54.** $|x - 3| \geq 7$ **55.** $|2w + 6| > 24$ **56.** $2|3x| + 1 = 9$

Tell whether the ordered pair is a solution to the given equation. See Lesson 1-9.

57. $x + 3 = y$; $(1, 4)$ **58.** $2x - 5 = y$; $(-1, 8)$ **59.** $\frac{1}{2}x + 7 = y$; $(8, 11)$

Get Ready! **To prepare for Lesson 4-1, do Exercises 60–63.**

Graph each point on the same coordinate grid. See Review p. 60.

60. $(1, 4)$ **61.** $(-1, -5)$ **62.** $(3, -6)$ **63.** $(-2, 1)$