8-5 Factoring $x^2 + bx + c$

Common Core State Standards

A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 3, MP 4, MP 7, MP 8

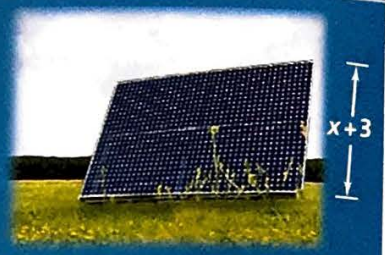
Objective To factor trinomials of the form $x^2 + bx + c$

Here's a hint. What two numbers have a sum of 7 and a product of 12?

SOLVE IT!

Getting Ready!

The area of the rectangular solar panel is given by the trinomial $x^2 + 7x + 12$. The height of the solar panel is $x + 3$. What is an expression for the length of the panel? Explain your reasoning.



MATHEMATICAL PRACTICES

Essential Understanding You can write some trinomials of the form $x^2 + bx + c$ as the product of two binomials.

To understand how, consider the product of binomials below.

$$(x + 3)(x + 7) = x^2 + (7 + 3)x + 3 \cdot 7 = x^2 + 10x + 21$$

The coefficient of the trinomial's x^2-term is 1. The coefficient of the trinomial's x-term, 10, is the *sum* of the numbers 3 and 7 in the binomials. The trinomial's constant term, 21, is the *product* of the same numbers, 3 and 7. To factor a trinomial of the form $x^2 + bx + c$ as the product of binomials, you must find two numbers that have a sum of b and a product of c.

Problem 1 Factoring $x^2 + bx + c$ Where $b > 0$, $c > 0$

What is the factored form of $x^2 + 8x + 15$?

Plan

What is an easy way to organize your factoring?

Use a table to list the pairs of factors of the constant term c and the sums of those pairs of factors.

List the pairs of factors of 15. Identify the pair that has a sum of 8.

Factors of 15	Sum of Factors
1 and 15	16
3 and 5	8 ✔

$x^2 + 8x + 15 = (x + 3)(x + 5)$

Check $(x + 3)(x + 5) = x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$ ✔

Got It? **1.** What is the factored form of $r^2 + 11r + 24$?

Some factorable trinomials have a negative coefficient of x and a positive constant term. In this case, you need to inspect the negative factors of c to find the factors of the trinomial.

Problem 2 Factoring $x^2 + bx + c$ Where $b < 0$, $c > 0$

What is the factored form of $x^2 - 11x + 24$?

Think

Why look at pairs of *negative* factors of 24?
You want the factors of 24 with a sum of -11. Only two negative numbers have a positive product *and* a negative sum.

List the pairs of negative factors of 24. Identify the pair that has a sum of -11.

Factors of 24	Sum of Factors
-1 and -24	-25
-2 and -12	-14
-3 and -8	-11 ✔
-4 and -6	-10

$x^2 - 11x + 24 = (x - 3)(x - 8)$

Check $(x - 3)(x - 8) = x^2 - 8x - 3x + 24$
$= x^2 - 11x + 24$ ✔

Got It? **2. a.** What is the factored form of $y^2 - 6y + 8$?
b. Reasoning Can you factor $x^2 - x + 2$? Explain.

When you factor trinomials with a negative constant term, you need to inspect pairs of positive and negative factors of c.

Problem 3 Factoring $x^2 + bx + c$ Where $c < 0$

What is the factored form of $x^2 + 2x - 15$?

Think

What's another way to do this problem?
Find two positive factors of 15 that *differ* by 2. The factors are 3 and 5. Then attach a negative sign to one of the factors so that their sum is positive. You get -3 and 5.

Identify the pair of factors of -15 that has a sum of 2.

Factors of -15	Sum of Factors
1 and -15	-14
-1 and 15	14
3 and -5	-2
-3 and 5	2 ✔

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Got It? **3.** What is the factored form of each polynomial?
a. $n^2 + 9n - 36$ **b.** $c^2 - 4c - 21$

Problem 4 Applying Factoring Trinomials

Geometry The area of a rectangle is given by the trinomial $x^2 - 2x - 35$. What are the possible dimensions of the rectangle? Use factoring.

Know The area of the rectangle

Need Possible dimensions of the rectangle

Plan Area = length × width, so factor the trinomial for area as the product of binomials that represent the length and width.

To factor $x^2 - 2x - 35$, identify the pair of factors of -35 that has a sum of -2.

Factors of -35	Sum of Factors
1 and -35	-34
-1 and 35	34
5 and -7	-2 ✔
-5 and 7	2

$x^2 - 2x - 35 = (x + 5)(x - 7)$

So the possible dimensions of the rectangle are $x + 5$ and $x - 7$.

Got It? **4.** A rectangle's area is $x^2 - x - 72$. What are possible dimensions of the rectangle? Use factoring.

You can also factor some trinomials that have more than one variable. Consider the product $(p + 9q)(p + 7q)$.

$$(p + 9q)(p + 7q) = p^2 + 7pq + 9pq + 9q(7q)$$
$$= p^2 + 16pq + 63q^2$$

This suggests that a trinomial with two variables may be factorable if the first term includes the square of one variable, the middle term includes both variables, and the last term includes the square of the other variable.

Problem 5 Factoring a Trinomial With Two Variables

What is the factored form of $x^2 + 6xy - 55y^2$?

Plan

Is this problem similar to one you've seen before?
Yes. This problem is similar to factoring a trinomial in one variable of the form $x^2 + bx + c$, where $c < 0$.

List the pairs of factors of -55. Identify the pair that has a sum of 6.

Factors of -55	Sum of Factors
1 and -55	-54
-1 and 55	54
5 and -11	-6
-5 and 11	6 ✔

$x^2 + 6xy - 55y^2 = (x - 5y)(x + 11y)$

Got It? **5.** What is the factored form of $m^2 + 6mn - 27n^2$?

Lesson Check

Do you know HOW?

Factor each expression. Check your answer.

1. $x^2 + 7x + 12$
2. $r^2 - 13r + 42$
3. $p^2 + 3p - 40$
4. $a^2 + 12ab + 32b^2$
5. The area of a rectangle is given by the trinomial $n^2 - 3n - 28$. What are the possible dimensions of the rectangle? Use factoring.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Tell whether the sum of the factors of the constant term should be *positive* or *negative* when you factor the trinomial.

6. $s^2 + s - 30$
7. $w^2 + 11w + 18$
8. $x^2 - x - 20$
9. **Reasoning** Under what circumstances should you look at pairs of negative factors of the constant term when factoring a trinomial of the form $x^2 + bx + c$?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Complete.

See Problems 1 and 2.

10. $k^2 + 5k + 6 = (k + 2)(k + \blacksquare)$
11. $x^2 - 7x + 10 = (x - 5)(x - \blacksquare)$
12. $t^2 - 10t + 24 = (t - 4)(t - \blacksquare)$
13. $v^2 + 12v + 20 = (v + 10)(v + \blacksquare)$

Factor each expression. Check your answer.

14. $y^2 + 6y + 5$
15. $t^2 + 10t + 16$
16. $x^2 + 15x + 56$
17. $n^2 - 15n + 56$
18. $r^2 - 11r + 24$
19. $q^2 - 8q + 12$

Complete.

See Problem 3.

20. $q^2 + 3q - 54 = (q - 6)(q + \blacksquare)$
21. $z^2 - 2z - 48 = (z - 8)(z + \blacksquare)$
22. $n^2 - 5n - 50 = (n + 5)(n - \blacksquare)$
23. $y^2 + 8y - 9 = (y + 9)(y - \blacksquare)$

Factor each expression. Check your answer.

24. $r^2 + 6r - 27$
25. $w^2 - 7w - 8$
26. $z^2 + 2z - 8$
27. $x^2 + 5x - 6$
28. $v^2 + 5v - 36$
29. $n^2 - 3n - 10$

STEM 30. **Carpentry** The area of a rectangular desk is given by the trinomial $d^2 - 7d - 18$. What are the possible dimensions of the desk? Use factoring. See Problem 4.

31. **Design** The area of a rectangular rug is given by the trinomial $r^2 - 3r - 4$. What are the possible dimensions of the rug? Use factoring.

Choose the correct factored form for each expression.

See Problem 5.

32. $k^2 + 5kn - 84n^2$ **A.** $(k - 7n)(k - 12n)$ **B.** $(k - 7n)(k + 12n)$

33. $p^2 - 8pq - 33q^2$ **A.** $(p + 3q)(p - 11q)$ **B.** $(p - 3q)(p + 11q)$

34. $x^2 - 16xy + 48y^2$ **A.** $(x - 4y)(x + 12y)$ **B.** $(x - 4y)(x - 12y)$

Factor each expression.

35. $r^2 + 19rs + 90s^2$ **36.** $g^2 - 12gh + 35h^2$ **37.** $m^2 - 3mn - 28n^2$

38. $x^2 + 3xy - 18y^2$ **39.** $w^2 - 14wz + 40z^2$ **40.** $p^2 + 11pq + 24q^2$

B Apply

41. Writing Suppose you can factor $x^2 + bx + c$ as $(x + p)(x + q)$.

a. Explain what you know about p and q when $c > 0$.

b. Explain what you know about p and q when $c < 0$.

42. Error Analysis Describe and correct the error made in factoring the trinomial.

~~$x^2 - 10x - 24 = (x - 6)(x - 4)$~~

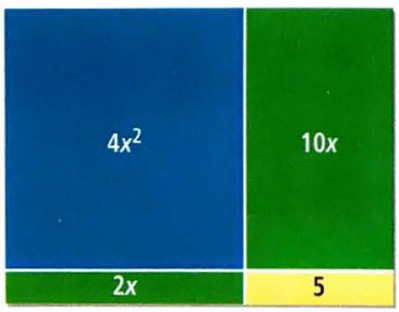
43. Think About a Plan The area of a parallelogram is given by the trinomial $x^2 - 14x + 24$. The base of the parallelogram is $x - 2$. What is an expression for the height of the parallelogram?

- What is the formula for the area of a parallelogram?
- How can you tell whether the binomial that represents the height has a positive or negative constant term?

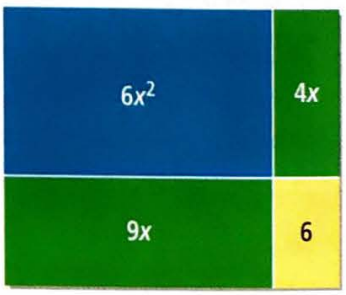
44. Recreation A rectangular skateboard park has an area of $x^2 + 15x + 54$. What are the possible dimensions of the park? Use factoring.

Write the standard form of each polynomial modeled below. Then factor each expression.

45.



46.



47. Reasoning Let $x^2 - 13x - 30 = (x + p)(x + q)$.

a. What do you know about the signs of p and q?

b. Suppose $|p| > |q|$. Which number, p or q, is a negative integer? Explain.

48. Reasoning Let $x^2 + 13x - 30 = (x + p)(x + q)$.

a. What do you know about the signs of p and q?

b. Suppose $|p| > |q|$. Which number, p or q, is a negative integer? Explain.

Factor each expression.

49. $x^2 + 27x + 50$

50. $g^2 - 18g + 45$

51. $k^2 - 18k - 63$

52. $d^2 + 30d - 64$

53. $s^2 - 10st - 75t^2$

54. $h^2 + 9hj - 90j^2$

Challenge

Factor each trinomial.

Sample $n^6 + n^3 - 42 = (n^3)^2 + n^3 - 42$
$= (n^3 - 6)(n^3 + 7)$

55. $x^{12} + 12x^6 + 35$

56. $t^8 + 5t^4 - 24$

57. $r^6 - 21r^3 + 80$

58. $m^{10} + 18m^5 + 17$

59. $x^{12} - 19x^6 - 120$

60. $p^6 + 14p^3 - 72$

Standardized Test Prep

SAT/ACT

61. What is the factored form of $x^2 + x - 42$?

Ⓐ $(x - 7)(x - 6)$ Ⓑ $(x - 7)(x + 6)$ Ⓒ $(x + 7)(x - 6)$ Ⓓ $(x + 7)(x + 6)$

62. What is the solution of the equation $6x + 7 = 25$?

Ⓕ 2 Ⓖ 3 Ⓗ $5\frac{1}{3}$ Ⓘ 8

63. A museum charges an admission price of \$12 per person when you buy tickets online. There is also a \$5 charge per order. You spend \$65 purchasing p tickets online. Which equation best represents this situation?

Ⓐ $12p + 5 = 65$ Ⓑ $5p + 12 = 65$ Ⓒ $12p - 5 = 65$ Ⓓ $65p + 12 = 5$

Short Response

64. You and your friend bike to school at the rates shown. Who is faster? Show your work.

You: 7 mi/h

Your friend: 11 ft/s

Mixed Review

Simplify each product. See Lesson 8-4.

65. $(c + 4)^2$

66. $(2v - 9)^2$

67. $(3w + 7)(3w - 7)$

Solve each equation for x. See Lesson 2-5.

68. $\frac{a}{b} = \frac{x}{d}$

69. $8(x - d) = x$

70. $m = \frac{(c + x)}{n}$

Get Ready! **To prepare for Lesson 8-6, do Exercises 71–73.**

Find the GCF of the terms of each polynomial. See Lesson 8-2.

71. $14x^2 + 7x$

72. $24x^2 - 30x + 12$

73. $6x^3 + 45x^2 + 15$