7 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Equivalence
One way to represent numbers is to use exponents. A number raised to the 0 power is equal to 1.

Zero and Negative Exponents (Lesson 7-1)

$10^0 = 1$

$10^{-3} = \frac{1}{10^3}$

2 Properties
Just as there are properties that describe how to rewrite expressions involving addition and multiplication, there are properties that describe how to rewrite and simplify exponential and radical expressions.

Properties of Exponents (Lessons 7-2, 7-3, and 7-4)

$5^2 \cdot 5^4 = 5^{2+4} = 5^6$

$\left(3^{\frac{3}{4}}\right)^4 = 3^{\frac{3}{4} \cdot 4} = 3^3$

$\frac{7^8}{7^5} = 7^{8-5} = 7^3$

Rational Exponents and Radicals (Lesson 7-5)

$4^{\frac{1}{2}} = \sqrt{4}$

$x^{\frac{1}{3}} = \sqrt[3]{x}$

$b^{\frac{2}{3}} = \sqrt[3]{b^2}$

3 Function
The family of exponential functions has equations of the form $y = a \cdot b^x$. They can be used to model exponential growth or decay and to model geometric sequences.

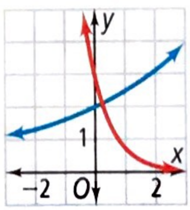
Exponential Functions (Lessons 7-6 and 7-7)

Exponential Growth

$y = 2 \cdot \left(\frac{5}{4}\right)^x$

Exponential Decay

$y = 3 \cdot \left(\frac{1}{4}\right)^x$



Geometric Sequences (Lesson 7-8)

An *explicit formula* is a function rule that relates each term of a sequence to the term number.

A *recursive formula* is a function rule that relates each term of a sequence to the ones before it.

Chapter Vocabulary

- compound interest (p. 461)
- decay factor (p. 462)
- exponential decay (p. 462)
- exponential function (p. 453)
- exponential growth (p. 460)
- geometric sequence (p. 467)
- growth factor (p. 460)
- index (p. 448)

Choose the correct term to complete each sentence.

1. A _?_ is a number sequence that has a common ratio between terms.

2. For a function $y = a \cdot b^x$, where $a > 0$ and $b > 1$, b is the _?_.

3. For a function $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$, b is the _?_.

4. The function $y = a \cdot b^x$ models _?_ for $a > 0$ and $b > 1$.

5. The function $y = a \cdot b^x$ models _?_ for $a > 0$ and $0 < b < 1$.

7-1 Zero and Negative Exponents

Quick Review

You can use zero and negative integers as exponents. For every nonzero number a, $a^0 = 1$. For every nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$. When you evaluate an exponential expression, you can simplify the expression before substituting values for the variables.

Example

What is the value of $a^2b^{-4}c^0$ for $a = 3$, $b = 2$, and $c = -5$?

$a^2b^{-4}c^0 = \frac{a^2c^0}{b^4}$ Use the definition of negative exponents.

$= \frac{a^2(1)}{b^4}$ Use the definition of zero exponent.

$= \frac{3^2}{2^4}$ Substitute.

$= \frac{9}{16}$ Simplify.

Exercises

Simplify each expression.

6. 5^0

7. 7^{-2}

8. $\frac{4x^{-2}}{y^{-8}}$

9. $\frac{1}{p^2q^{-4}r^0}$

Evaluate each expression for $x = 2$, $y = -3$, and $z = -5$.

10. x^0y^2

11. $(-x)^{-4}y^2$

12. x^0z^0

13. $\frac{5x^0}{y^{-2}}$

14. $y^{-2}z^2$

15. $\frac{2x}{y^2z^{-1}}$

16. Reasoning Is it true that $(-3b)^4 = -12b^4$? Explain why or why not.

7-2 Multiplying Powers With the Same Base

Quick Review

To multiply powers with the same base, add the exponents.

$a^m \cdot a^n = a^{m+n}$, where $a \neq 0$ and m and n are real numbers

Example

What is the simplified form of each expression?

a. $3^{10} \cdot 3^4 = 3^{10+4} = 3^{14}$

b. $(a^4)(a^3) = a^{4+3} = a^7$

c. $(x^{\frac{3}{5}})(x^{\frac{1}{5}}) = x^{\frac{3}{5}+\frac{1}{5}} = x^{\frac{4}{5}}$

d. $(b^{\frac{3}{4}})(b^{\frac{1}{2}}) = b^{\frac{3}{4}+\frac{1}{2}} = b^{\frac{3}{4}+\frac{2}{4}} = b^{\frac{5}{4}}$

Exercises

Complete each equation.

17. $3^2 \cdot 3^{\square} = 3^{10}$

18. $a^6 \cdot a^{\square} = a^8$

19. $x^2y^5 \cdot x^{\square}y^{\square} = x^5y^{11}$

20. $a^{\frac{1}{2}} \cdot a^{\square} = a$

21. $x^{\frac{2}{3}} \cdot x^{\square} = x^{\frac{11}{12}}$

22. $m^{\frac{3}{4}}n^{\frac{1}{2}} \cdot m^{\square}n^{\square} = m^{\frac{5}{4}}n$

Simplify each expression.

23. $2d^2 \cdot d^3$

24. $(x^3)(x^4)$

25. $(x^3y^5)(-y^7x)$

26. $(s^{\frac{3}{5}})(s^{\frac{2}{3}})$

27. $(p^{\frac{1}{3}}q)(q^{\frac{1}{2}}p)$

28. $2m^{\frac{3}{4}}n^2 \cdot 3m^{\frac{1}{4}}n$

29. Estimation Each square inch of your body has about 6.5×10^2 pores. Suppose the back of your hand has an area of about 0.12×10^2 in.^{2}. About how many pores are on the back of your hand? Write your answer in scientific notation.

7-3 More Multiplication Properties of Exponents

Quick Review

To raise a power to a power, multiply the exponents.

$(a^m)^n = a^{mn}$, where $a \neq 0$ and m and n are real numbers

To raise a product to a power, raise each factor in the product to the power.

$(ab)^n = a^n b^n$, where $a \neq 0$, $b \neq 0$, and n are real numbers

Example

What is the simplified form of each expression?

a. $(x^5)^7 = x^{5 \cdot 7} = x^{35}$

b. $(pq)^8 = p^8 q^8$

c. $\left(x^{\frac{1}{3}}\right)^3 = x^{\frac{1}{3} \cdot 3} = x^{\frac{3}{3}} = x$

d. $\left(ab\right)^{\frac{2}{3}} = a^{\frac{2}{3}} b^{\frac{2}{3}}$

Exercises

Complete each equation.

30. $(5^5)^{\square} = 5^{15}$

31. $(b^{-4})^{\square} = b^{20}$

32. $(4x^3y^5)^{\square} = 16x^6y^{10}$

33. $\left(x^{\frac{2}{3}}\right)^{\square} = x^2$

34. $\left(a^{\frac{1}{2}}\right)^{\square} = a^{\frac{1}{4}}$

35. $\left(2x^2y^{\frac{1}{4}}\right)^{\square} = 4x^4y^{\frac{1}{2}}$

Simplify each expression.

36. $(q^3r)^4$

37. $(1.34^2)^5(1.34)^{-8}$

38. $(12x^2y^{-2})^5(4xy^{-3})^{-7}$

39. $(-2r^{-4})^2(-3r^2z^8)^{-1}$

40. $\left(x^{\frac{4}{7}}\right)^7$

41. $\left(a^{\frac{3}{4}}b^{\frac{7}{8}}\right)^4$

7-4 Division Properties of Exponents

Quick Review

To divide powers with the same base, subtract the exponents.

$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m and n are integers

To raise a quotient to a power, raise the numerator and the denominator to the power.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $a \neq 0$, $b \neq 0$, and n is an integer

Example

What is the simplified form of $\left(\frac{5x^4}{z^2}\right)^3$?

$$\left(\frac{5x^4}{z^2}\right)^3 = \frac{(5x^4)^3}{(z^2)^3} = \frac{5^3x^{4 \cdot 3}}{z^{2 \cdot 3}} = \frac{125x^{12}}{z^6}$$

Exercises

Simplify each expression.

42. $\frac{w^2}{w^5}$

43. $\frac{21x^3}{3x^{-1}}$

44. $\left(\frac{n^5}{v^3}\right)^7$

45. $\left(\frac{3c^3}{e^5}\right)^{-4}$

Simplify each quotient. Write your answer in scientific notation.

46. $\frac{4.2 \times 10^8}{2.1 \times 10^{11}}$

47. $\frac{3.1 \times 10^4}{1.24 \times 10^2}$

48. $\frac{4.5 \times 10^3}{9 \times 10^7}$

49. $\frac{5.1 \times 10^5}{1.7 \times 10^2}$

50. Writing List the steps that you would use to simplify $\left(\frac{5a^8}{10a^6}\right)^{-3}$.

7-5 Rational Exponents and Radicals

Quick Review

If the nth root of a is a real number, m is an integer, and $\frac{m}{n}$ is in lowest terms, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

Example

Write the expression $(8x)^{\frac{1}{3}}$ in radical form.

$(8x)^{\frac{1}{3}} = 8^{\frac{1}{3}}x^{\frac{1}{3}} = 2\sqrt[3]{x}$

Write the expression $\sqrt[3]{b^2}$ as a power with a rational exponent.

$\sqrt[3]{b^2} = b^{\frac{2}{3}}$

Exercises

Write each expression in radical form.

51. $m^{\frac{1}{2}}$

52. $p^{\frac{2}{3}}r^{\frac{4}{5}}$

53. $(36x^4)^{\frac{1}{2}}$

54. $(125x)^{\frac{1}{3}}$

55. $(64)^{\frac{1}{2}}x^{\frac{3}{4}}$

56. $25^{\frac{1}{3}}(x^2y)^{\frac{1}{2}}$

Write each expression as a power with a rational exponent.

57. $\sqrt{xy}$

58. $\sqrt[4]{a}$

59. $\sqrt[3]{b^2}$

60. $\sqrt[3]{x^6y^9}$

61. $\sqrt[4]{81x^2}$

62. $\sqrt[5]{x^2y^3}$

7-6 Exponential Functions

Quick Review

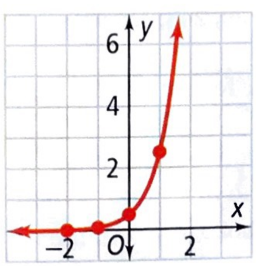
An **exponential function** involves repeated multiplication of an initial amount a by the same positive number b. The general form of an exponential function is $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Example

What is the graph of $y = \frac{1}{2} \cdot 5^x$?

Make a table of values. Graph the ordered pairs.

x	y
−2	$\frac{1}{50}$
−1	$\frac{1}{10}$
0	$\frac{1}{2}$
1	$\frac{5}{2}$
2	$\frac{25}{2}$



Exercises

Evaluate each function for the domain $\{1, 2, 3\}$.

63. $f(x) = 4^x$

64. $y = 0.01^x$

65. $y = 40\left(\frac{1}{2}\right)^x$

66. $f(x) = 3 \cdot 2^x$

Graph each function.

67. $f(x) = 2.5^x$

68. $y = 0.5(0.5)^x$

69. $f(x) = \frac{1}{2} \cdot 3^x$

70. $y = 0.1^x$

71. **Biology** A population of 50 bacteria in a laboratory culture doubles every 30 min. The function $p(x) = 50 \cdot 2^x$ models the population, where x is the number of 30-min periods.

a. How many bacteria will there be after 2 h?

b. How many bacteria will there be after 1 day?

7-7 Exponential Growth and Decay

Quick Review

When $a > 0$ and $b > 1$, the function $y = a \cdot b^x$ models **exponential growth.** The base b is called the **growth factor.** When $a > 0$ and $0 < b < 1$, the function $y = a \cdot b^x$ models **exponential decay.** In this case the base b is called the **decay factor.**

Example

The population of a city is 25,000 and decreases 1% each year. Predict the population after 6 yr.

$y = 25{,}000 \cdot 0.99^x$	Exponential decay function
$= 25{,}000 \cdot 0.99^6$	Substitute 6 for x.
$\approx 23{,}537$	Simplify.

The population will be about 23,537 after 6 yr.

Exercises

Tell whether the function represents *exponential growth* or *exponential decay.* Identify the growth or decay factor.

72. $y = 5.2 \cdot 3^x$

73. $f(x) = 7 \cdot 0.32^x$

74. $y = 0.15\left(\frac{3}{2}\right)^x$

75. $g(x) = 1.3\left(\frac{1}{4}\right)^x$

76. Finance Suppose \$2000 is deposited in an account paying 2.5% interest compounded quarterly. What will the account balance be after 12 yr?

77. Music A band performs a free concert in a local park. There are 200 people in the crowd at the start of the concert. The number of people in the crowd grows 15% every half hour. How many people are in the crowd after 3 h? Round to the nearest person.

7-8 Geometric Sequences

Quick Review

In a geometric sequence the ratio of any term to its preceding term is a constant value.

Example

Find the common ratio of the geometric sequence.

2, 6, 18, 54, . . .

$\times 3$ $\quad \times 3$ $\quad \times 3$

The common ratio of the geometric sequence is 3.

Write a recursive formula to represent the geometric sequence.

256, 64, 16, 4, . . .

$\times \frac{1}{4}$ $\quad \times \frac{1}{4}$ $\quad \times \frac{1}{4}$

$a_1 = 265;\ a_n = a_{n-1} \cdot \frac{1}{4}$

Exercises

Find the common ratio of each geometric sequence.

78. 10, 20, 40, 80, . . .

79. 1, 10, 100, 1000, . . .

80. 100, 20, 4, 0.8, . . .

81. 6561, 2187, 729, 243, . . .

Write a recursive formula to represent the geometric sequence.

82. 20, 60, 180, 540, . . .

83. 5, 2.5, 1.25, 0.625, . . .

84. 3, 12, 48, 192, . . .

85. 10, 1, 0.1, 0.01, . . .