

Connecting **BIG** ideas and Answering the Essential Questions**1 Equivalence**

You can represent algebraic expressions in many ways. When you add, subtract, multiply, divide, and factor polynomials, you replace one expression with an equivalent expression.

**2 Properties**

The properties of real numbers are the basis of the laws of algebra. You can apply properties of real numbers, such as the Distributive Property, to polynomials.

**Adding and Subtracting Polynomials (Lesson 8-1)**

$$\begin{aligned}(3x^2 + 4x + 1) + (2x^2 + 5x + 8) \\ &= (3x^2 + 2x^2) + (4x + 5x) + (1 + 8) \\ &= 5x^2 + 9x + 9\end{aligned}$$

**Multiplying Binomials (Lesson 8-3)**

$$\begin{aligned}(m + 4)(2m - 5) &= 2m^2 - 5m + 8m - 20 \\ &= 2m^2 + 3m - 20\end{aligned}$$

**Factoring Trinomials (Lessons 8-5 and 8-6)**

$$x^2 - 6x + 8 = (x - 2)(x - 4)$$

**Factoring by Grouping (Lesson 8-8)**

$$\begin{aligned}3x^2 - 10x - 8 &= 3x^2 - 12x + 2x - 8 \\ &= (3x^2 - 12x) + (2x - 8) \\ &= 3x(x - 4) + 2(x - 4) \\ &= (3x + 2)(x - 4)\end{aligned}$$

**Multiplying Special Cases (Lesson 8-4)**

$$(2x + 3)(2x - 3) = 4x^2 - 9$$

**Factoring Special Cases (Lesson 8-7)**

$$49p^2 - 16 = (7p + 4)(7p - 4)$$

**Chapter Vocabulary**

- binomial (p. 487)
- degree of a monomial (p. 486)
- degree of a polynomial (p. 487)
- difference of two squares (p. 525)
- factoring by grouping (p. 529)
- monomial (p. 486)
- perfect-square trinomial (p. 523)
- polynomial (p. 487)
- standard form of a polynomial (p. 487)
- trinomial (p. 487)

Choose the correct term to complete each sentence.

1. A polynomial that has two terms is a(n)   ?  .
2. A monomial or the sum of two or more monomials is a(n)   ?  .
3. A(n)   ?   is an expression that is a number, a variable, or a product of a number and one or more variables.
4. A polynomial that is the product of two identical binomial factors is a(n)   ?  .
5. The sum of the exponents of the variables in a monomial is the   ?  .

## 8-1 Adding and Subtracting Polynomials

### Quick Review

A **monomial** is a number, a variable, or a product of a number and one or more variables. A **polynomial** is a monomial or the sum of two or more monomials. The **degree of a polynomial** in one variable is the same as the degree of the monomial with the greatest exponent. To add two polynomials, add the like terms of the polynomials. To subtract a polynomial, add the opposite of the polynomial.

### Example

What is the difference of  $3x^3 - 7x^2 + 5$  and  $2x^2 - 9x - 1$ ?

$$\begin{aligned}(3x^3 - 7x^2 + 5) - (2x^2 - 9x - 1) \\ &= 3x^3 - 7x^2 + 5 - 2x^2 + 9x + 1 \\ &= 3x^3 + (-7x^2 - 2x^2) + 9x + (1 + 5) \\ &= 3x^3 - 9x^2 + 9x + 6\end{aligned}$$

### Exercises

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.

6.  $4r + 3 - 9r^2 + 7r$       7.  $3 + b^3 + b^2$   
8.  $3 + 8t^2$       9.  $n^3 + 4n^5 + n - n^3$   
10.  $7x^2 + 8 + 6x - 7x^2$       11.  $p^3q^3$

Simplify. Write each answer in standard form.

12.  $(2v^3 - v + 8) + (-v^3 + v - 3)$   
13.  $(6s^4 + 7s^2 + 7) + (8s^4 - 11s^2 + 9s)$   
14.  $(4h^3 + 3h + 1) - (-5h^3 + 6h - 2)$   
15.  $(8z^3 - 3z^2 - 7) - (z^3 - z^2 + 9)$

## 8-2 Multiplying and Factoring

### Quick Review

You can multiply a monomial and a polynomial using the Distributive Property. You can factor a polynomial by finding the greatest common factor (GCF) of the terms of the polynomial.

### Example

What is the factored form of  $10y^4 - 12y^3 + 4y^2$ ?

First find the GCF of the terms of the polynomial.

$$10y^4 = 2 \cdot 5 \cdot y \cdot y \cdot y \cdot y$$

$$12y^3 = 2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot y$$

$$4y^2 = 2 \cdot 2 \cdot y \cdot y$$

The GCF is  $2 \cdot y \cdot y$  or  $2y^2$ .

Then factor out the GCF.

$$\begin{aligned}10y^4 - 12y^3 + 4y^2 &= 2y^2(5y^2) + 2y^2(-6y) + 2y^2(2) \\ &= 2y^2(5y^2 - 6y + 2)\end{aligned}$$

### Exercises

Simplify each product. Write in standard form.

16.  $5k(3 - 4k)$       17.  $4m(2m + 9m^2 - 6)$   
18.  $6g^2(g - 8)$       19.  $3d(6d + d^2)$   
20.  $-2n^2(5n - 9 + 4n^2)$       21.  $q(11 + 8q - 2q^2)$

Find the GCF of the terms of each polynomial. Then factor the polynomial.

22.  $12p^4 + 16p^3 + 8p$       23.  $3b^4 - 9b^2 + 6b$   
24.  $45c^5 - 63c^3 + 27c$       25.  $4g^2 + 8g$   
26.  $3t^4 - 6t^3 - 9t + 12$       27.  $30h^5 - 6h^4 - 15h^3$

28. **Reasoning** The GCF of two numbers  $p$  and  $q$  is 5. Can you find the GCF of  $6p$  and  $6q$ ? Explain your answer.



## 8-3 and 8-4 Multiplying Binomials

### Quick Review

You can use algebra tiles, tables, or the Distributive Property to multiply polynomials. The FOIL method (First, Outer, Inner, Last) can be used to multiply two binomials. You can also use rules to multiply special case binomials.

### Example

What is the simplified form of  $(4x + 3)(3x + 2)$ ?

Use FOIL to multiply the binomials. Find the product of the first terms, the outer terms, the inner terms, and the last terms. Then add.

$$\begin{aligned}(4x + 3)(3x + 2) &= (4x)(3x) + (4x)(2) + (3)(3x) + (3)(2) \\ &= 12x^2 + 8x + 9x + 6 \\ &= 12x^2 + 17x + 6\end{aligned}$$

### Exercises

Simplify each product. Write in standard form.

29.  $(w + 1)(w + 12)$       30.  $(2s - 3)(5s + 4)$   
 31.  $(3r - 2)^2$       32.  $(6g + 7)(g - 8)$   
 33.  $(7q + 2)(3q + 8)$       34.  $(4n^3 + 5)(3n + 5)$   
 35.  $(t + 9)(t - 3)$       36.  $(6c + 5)^2$   
 37.  $(7h - 3)(7h + 3)$       38.  $(y - 6)(3y + 7)$   
 39.  $(4a - 7)(8a + 3)$       40.  $(4b - 3)(4b + 3)$
41. **Geometry** A rectangle has dimensions  $3x + 5$  and  $x + 7$ . Write an expression for the area of the rectangle as a product and as a polynomial in standard form.

## 8-5 and 8-6 Factoring Quadratic Trinomials

### Quick Review

You can write some quadratic trinomials as the product of two binomial factors. When you factor a polynomial, be sure to factor out the GCF first.

### Example

What is the factored form of  $x^2 + 7x + 12$ ?

List the pairs of factors of 12. Identify the pair with a sum of 7.

Factors of 12	Sum of Factors
1, 12	13
2, 6	8
3, 4	7 ✓

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

### Exercises

Factor each expression.

42.  $g^2 - 5g - 14$       43.  $2n^2 + 3n - 2$   
 44.  $6k^2 - 10kl + 4l^2$       45.  $p^2 + 8p + 12$   
 46.  $r^2 + 6r - 40$       47.  $6m^2 + 25mn + 11n^2$   
 48.  $t^2 - 13t - 30$       49.  $2g^2 - 35g + 17$   
 50.  $3x^2 + 3x - 6$       51.  $d^2 - 18d + 45$   
 52.  $w^2 - 15w - 54$       53.  $21z^2 - 70z + 49$   
 54.  $-2h^2 + 4h + 70$       55.  $x^2 + 21x + 38$   
 56.  $10v^2 + 11v - 8$       57.  $5g^2 + 15g + 10$
58. **Reasoning** Can you factor the expression  $2x^2 + 15x + 9$ ? Explain why or why not.

## 8-7 Factoring Special Cases

### Quick Review

When you factor a perfect-square trinomial, the two binomial factors are the same.

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

When you factor a difference of squares of two terms, the two binomial factors are the sum and the difference of the two terms.

$$a^2 - b^2 = (a + b)(a - b)$$

### Example

What is the factored form of  $81t^2 - 90t + 25$ ?

First rewrite the first and last terms as squares. Then determine if the middle term equals  $-2ab$ .

$$\begin{aligned} 81t^2 - 90t + 25 &= (9t)^2 - 90t + 5^2 \\ &= (9t)^2 - 2(9t)(5) + 5^2 \\ &= (9t - 5)^2 \end{aligned}$$

### Exercises

Factor each expression.

59.  $s^2 - 20s + 100$

60.  $16q^2 + 56q + 49$

61.  $r^2 - 64$

62.  $9z^2 - 16$

63.  $25m^2 + 80m + 64$

64.  $49n^2 - 4$

65.  $g^2 - 225$

66.  $9p^2 - 42p + 49$

67.  $36h^2 - 12h + 1$

68.  $w^2 + 24w + 144$

69.  $32v^2 - 8$

70.  $25x^2 - 36$

71. **Geometry** Find an expression for the length of a side of a square with an area of  $9n^2 + 54n + 81$ .

72. **Reasoning** Suppose you are using algebra tiles to factor a quadratic trinomial. What do you know about the factors of the trinomial when the tiles form a square?

## 8-8 Factoring by Grouping

### Quick Review

When a polynomial has four or more terms, you may be able to group the terms and find a common binomial factor. Then you can use the Distributive Property to factor the polynomial.

### Example

What is the factored form of  $2r^3 - 12r^2 + 5r - 30$ ?

First factor out the GCF from each group of two terms. Then factor out a common binomial factor.

$$\begin{aligned} 2r^3 - 12r^2 + 5r - 30 &= 2r^2(r - 6) + 5(r - 6) \\ &= (2r^2 + 5)(r - 6) \end{aligned}$$

### Exercises

Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

73.  $6y^3 - 3y^2 + 2y - 1$

74.  $8m^3 + 40m^2 + 6m + 15$

Factor completely.

75.  $6d^4 + 4d^3 - 6d^2 - 4d$

76.  $11b^3 - 6b^2 + 11b - 6$

77.  $45z^3 + 20z^2 + 9z + 4$

78.  $9a^3 - 12a^2 + 18a - 24$