10-1 The Pythagorean Theorem

Common Core State Standards

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

MP 1, MP 2, MP 3, MP 4

Objectives To solve problems using the Pythagorean Theorem
To identify right triangles

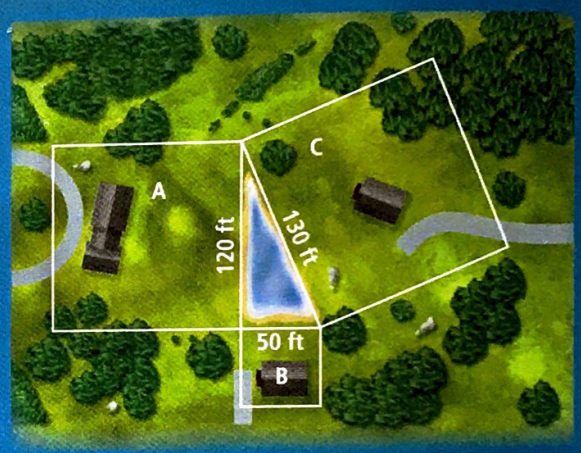
This is almost like an optical illusion. What do you see, three squares or three sides of a triangle?

MATHEMATICAL PRACTICES

SOLVE IT!

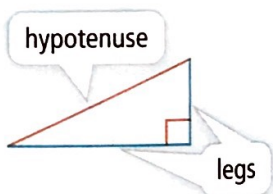
Getting Ready!

The diagram shows three square house lots that border a pond shaped like a right triangle. What is the area of each house lot? Can you write an equation to relate all three areas? Explain.



Lesson Vocabulary
- hypotenuse
- leg
- Pythagorean Theorem
- conditional
- hypothesis
- conclusion
- converse

There are special names for the sides of a right triangle like the one in the Solve It. The side opposite the right angle is the **hypotenuse.** It is the longest side. Each of the sides forming the right angle is a **leg.** The **Pythagorean Theorem,** named after the Greek mathematician Pythagoras, relates the lengths of the legs and the length of the hypotenuse.



Essential Understanding The lengths of the sides of a right triangle have a special relationship. If you know the lengths of any two of the sides, you can find the length of the third side.

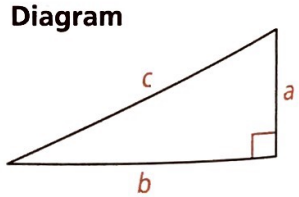
take note

Theorem The Pythagorean Theorem

Words
In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Algebra
$a^2 + b^2 = c^2$

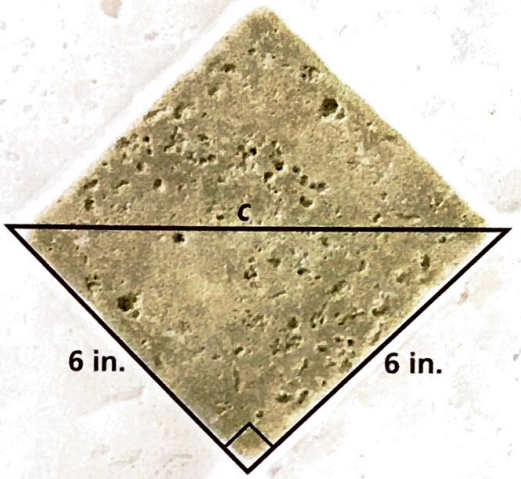
Diagram



You can use the Pythagorean Theorem to find the length of a right triangle's hypotenuse given the lengths of its legs. Using the Pythagorean Theorem to solve for a side length involves finding a principal square root, because side lengths are always positive.

Problem 1 Finding the Length of a Hypotenuse

The tiles at the right are squares with 6-in. sides. What is the length of the hypotenuse of the right triangle shown?



Plan

What do you know? What do you need? You know the lengths a and b of the two legs. You need to find the length c of the hypotenuse. Substitute for a and b in $a^2 + b^2 = c^2$, and then solve for c.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 6^2 = c^2$	Substitute 6 for a and b.
$72 = c^2$	Simplify.
$\sqrt{72} = c$	Find the principal square root.
$8.5 \approx c$	Use a calculator.

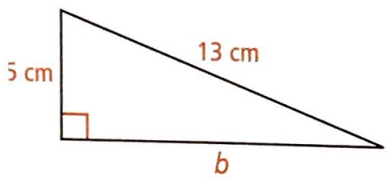
The length of the hypotenuse is about 8.5 in.

Got It? **1.** What is the length of the hypotenuse of a right triangle with legs of lengths 9 cm and 12 cm?

You can also use the Pythagorean Theorem to find the length of a leg of a right triangle.

Problem 2 Finding the Length of a Leg

What is the side length b in the triangle at the right?



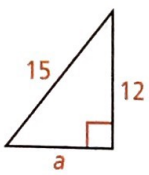
Think

How is this problem different from Problem 1? In Problem 1, the length of the hypotenuse was unknown. In this problem, the length of a leg is unknown.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + b^2 = 13^2$	Substitute 5 for a and 13 for c.
$25 + b^2 = 169$	Simplify.
$b^2 = 144$	Subtract 25 from each side.
$b = 12$	Find the principal square root of each side.

The side length b is 12 cm.

Got It? **2.** What is the side length a in the triangle at the right?



An *if-then* statement such as "If an animal is a horse, then it has four legs" is called a **conditional.** Conditionals have two parts. The part following *if* is the **hypothesis.** The part following *then* is the **conclusion.**

The **converse** of a conditional switches the hypothesis and the conclusion. Sometimes the converse of a true conditional is not true.

You can write the Pythagorean Theorem as a conditional: "If a triangle is a right triangle with legs of lengths a and b and hypotenuse of length c, then $a^2 + b^2 = c^2$." The converse of the Pythagorean Theorem is always true.

take note

Property The Converse of the Pythagorean Theorem

If a triangle has sides of lengths a, b, and c, and $a^2 + b^2 = c^2$, then the triangle is a right triangle with hypotenuse of length c.

You can use the Pythagorean Theorem and its converse to determine whether a triangle is a right triangle. If the side lengths satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle. If they do not, then it is not a right triangle.

Problem 3 Identifying Right Triangles

Multiple Choice **Which set of lengths could be the side lengths of a right triangle?**

(A) 6 in., 24 in., 25 in. (B) 4 m, 8 m, 10 m (C) 10 in., 24 in., 26 in. (D) 8 ft, 15 ft, 16 ft

Plan

Why should you check each answer choice?
If you find two answer choices that appear to be correct, then you know you have made a mistake.

Determine whether the lengths satisfy $a^2 + b^2 = c^2$. The greatest length is c.

$6^2 + 24^2 \stackrel{?}{=} 25^2$	$4^2 + 8^2 \stackrel{?}{=} 10^2$	$10^2 + 24^2 \stackrel{?}{=} 26^2$	$8^2 + 15^2 \stackrel{?}{=} 16^2$
$36 + 576 \stackrel{?}{=} 625$	$16 + 64 \stackrel{?}{=} 100$	$100 + 576 \stackrel{?}{=} 676$	$64 + 225 \stackrel{?}{=} 256$
$612 \neq 625$	$80 \neq 100$	$676 = 676$ ✔	$289 \neq 256$

By the Converse of the Pythagorean Theorem, the lengths 10 in., 24 in., and 26 in. could be the side lengths of a right triangle. The correct answer is C.

Got It? **3. a.** Could the lengths 20 mm, 47 mm, and 52 mm be the side lengths of a right triangle? Explain.

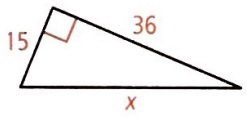
b. Reasoning If a, b, and c satisfy the equation $a^2 + b^2 = c^2$, are $2a$, $2b$, and $2c$ also possible side lengths of a right triangle? How do you know?

Lesson Check

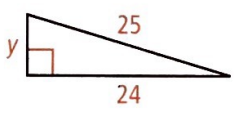
Do you know HOW?

Find each missing side length.

1.



2.

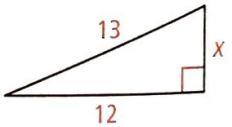


3. Could the lengths 12 cm, 35 cm, and 37 cm be the side lengths of a right triangle? Explain.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary What is the converse of the conditional, "If you study math, then you are a student"?

5. Error Analysis A student found the length x in the triangle at the right by solving the equation $12^2 + 13^2 = x^2$. Describe and correct the error.



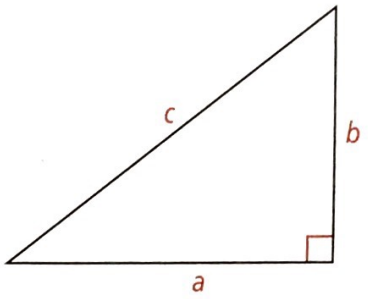
Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the triangle at the right. Find the missing side length. If necessary, round to the nearest tenth.

See Problems 1 and 2.



6. $a = 3, b = 4$
7. $a = 6, c = 10$
8. $b = 1, c = \frac{5}{4}$
9. $a = 5, c = 13$
10. $a = 0.3, b = 0.4$
11. $a = 8, b = 15$
12. $a = 1, c = \frac{5}{3}$
13. $b = 6, c = 7.5$
14. $b = 3.5, c = 3.7$
15. $a = 1.1, b = 6$
16. $a = 8, c = 17$
17. $a = 9, b = 40$
18. $b = 2.4, c = 7.4$
19. $a = 4, b = 7.5$
20. $a = 0.9, c = 4.1$

21. **Fitness** A jogger goes half a mile north and then turns west. If the jogger finishes 1.3 mi from the starting point, how far west did the jogger go?

STEM 22. **Construction** A construction worker is cutting along the diagonal of a rectangular board 15 ft long and 8 ft wide. What will be the length of the cut?

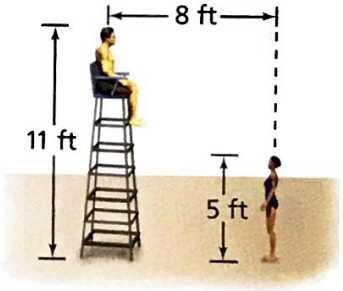
Determine whether the given lengths can be side lengths of a right triangle.

See Problem 3.

23. 15 ft, 36 ft, 39 ft
24. 12 m, 60 m, 61 m
25. 13 in., 35 in., 38 in.
26. 16 cm, 63 cm, 65 cm
27. 14 in., 48 in., 50 in.
28. 16 yd, 30 yd, 34 yd

B Apply

29. **Swimming** A swimmer asks a question to a lifeguard sitting on a tall chair, as shown in the diagram. The swimmer needs to be close to the lifeguard to hear the answer. What is the distance between the swimmer's head and the lifeguard's head?

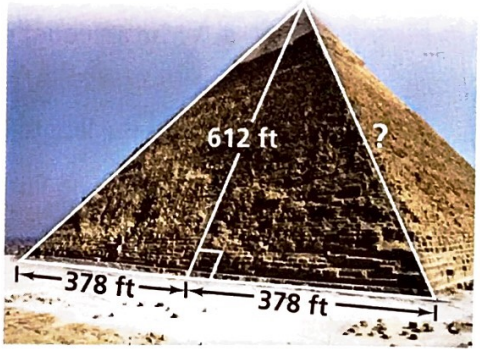


Any set of three positive integers that satisfies the equation $a^2 + b^2 = c^2$ is a *Pythagorean triple*. Determine whether each set of numbers is a Pythagorean triple.

30. 11, 60, 61
31. 13, 84, 85
32. 40, 41, 58
33. 50, 120, 130
34. 32, 126, 130
35. 28, 45, 53

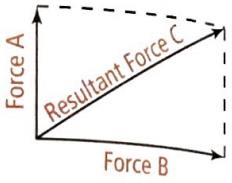
36. **Think About a Plan** A banner shaped like a right triangle has a hypotenuse of length 26 ft and a leg of length 10 ft. What is the area of the banner?
- What information do you need to find the area of a triangle?
- How can you find the length of the other leg?

37. **History** Originally, each face of the Great Pyramid of Giza was a triangle with the dimensions shown. How far was a corner of the base from the pyramid's top? Round to the nearest foot.



38. Two sides of a right triangle measure 10 in. and 8 in.
   a. **Writing** Explain why this is not enough information to be sure of the length of the third side.
   b. Give two possible values for the length of the third side.

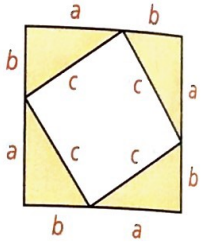
STEM **39. Physics** If two forces pull at right angles to each other, the resultant force can be represented by the diagonal of a rectangle, as shown at the right. This diagonal is a hypotenuse of a right triangle. A 50-lb force and a 120-lb force combine for a resultant force of 130 lb. Are the forces pulling at right angles to each other? Explain.



40. A rectangular box is 4 cm wide, 4 cm tall, and 10 cm long. What is the diameter of the smallest circular opening through which the box will fit? Round to the nearest tenth of a centimeter.

Challenge

41. Reasoning Use the diagram at the right.



a. Find the area of the larger square. Write your answer as a trinomial.

b. Find the area of the smaller square.

c. Find the area of each triangle in terms of a and b.

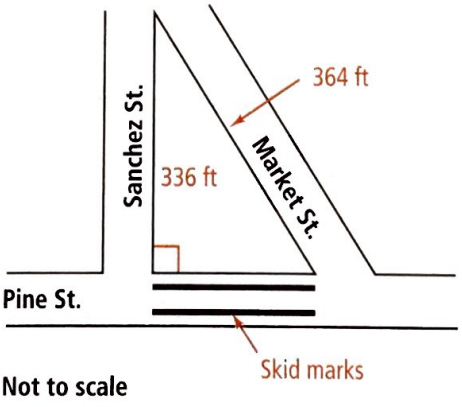
d. The area of the larger square equals the sum of the area of the smaller square and the areas of the four triangles. Write this equation and simplify. What do you notice?

42. Geometry The lengths of the sides of a right triangle are three consecutive integers. Write and solve an equation to find the three integers.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1, MP 4

Look back at the information on page 613 about the car skidding on Pine Street. The diagram showing the streets and the skid marks is shown again below.



Not to scale

a. Write an equation that you can use to find the length of the skid marks. Explain how you wrote the equation.

b. Find the length of the skid marks.

c. How do you know that your answer in part (b) is reasonable?