10-2 Simplifying Radicals

Common Core State Standards

Prepares for A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

MP 1, MP 2, MP 3, MP 4, MP 7

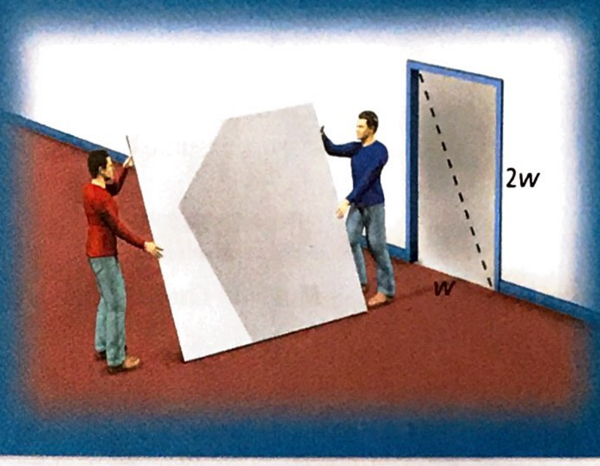
Objective To simplify radicals involving products and quotients

Use what you know about triangles to solve this problem.

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

Suppose you are bringing a mirror into your living room. What is the maximum height of a square mirror that will fit through the doorway shown? Justify your reasoning.



Lesson Vocabulary
- radical expression
- rationalize the denominator

In the Solve It, the maximum height of the mirror is a *radical expression*. A **radical expression**, such as $2\sqrt{3}$ or $\sqrt{x+3}$, is an expression that contains a radical. A radical expression is simplified if the following statements are true.

- The radicand has no perfect-square factors other than 1.
- The radicand contains no fractions.
- No radicals appear in the denominator of a fraction.

Simplified			Not Simplified		
$3\sqrt{5}$	$9\sqrt{x}$	$\frac{\sqrt{2}}{4}$	$3\sqrt{12}$	$\sqrt{\frac{x}{2}}$	$\frac{5}{\sqrt{7}}$

Essential Understanding You can simplify radical expressions using multiplication and division properties of square roots.

take note **Property** **Multiplication Property of Square Roots**

Algebra	Example
For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.	$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

You can use the Multiplication Property of Square Roots to simplify radicals by removing perfect-square factors from the radicand.

Problem 1 Removing Perfect-Square Factors

Plan

What strategy can you use to find the factor to remove? You can *solve a simpler problem* by first just listing the factors of the radicand. Then choose the greatest perfect square on the list.

What is the simplified form of $\sqrt{160}$?

$\sqrt{160} = \sqrt{16 \cdot 10}$	16 is the greatest perfect-square factor of 160.
$= \sqrt{16} \cdot \sqrt{10}$	Use the Multiplication Property of Square Roots.
$= 4\sqrt{10}$	Simplify $\sqrt{16}$.

Got It? **1.** What is the simplified form of $\sqrt{72}$?

Sometimes you can simplify radical expressions that contain variables. A variable with an even exponent is a perfect square. A variable with an odd exponent is the product of a perfect square and the variable. For example, $n^3 = n^2 \cdot n$, so $\sqrt{n^3} = \sqrt{n^2 \cdot n}$. In this lesson, assume that all variables in radicands represent nonnegative numbers.

Problem 2 Removing Variable Factors

Think

How is this problem similar to Problem 1? In both problems, you need to remove a perfect-square factor from the radicand. In this problem, however, the factor you remove contains a variable.

Multiple Choice **What is the simplified form of $\sqrt{54n^7}$?**

Ⓐ $n^3\sqrt{54n}$ Ⓑ $9n^6\sqrt{6n}$ Ⓒ $3n^3\sqrt{6n}$ Ⓓ $3n\sqrt{27n}$

$\sqrt{54n^7} = \sqrt{9n^6 \cdot 6n}$	$9n^6$, or $(3n^3)^2$, is a perfect-square factor of $54n^7$.
$= \sqrt{9n^6} \cdot \sqrt{6n}$	Use the Multiplication Property of Square Roots.
$= 3n^3\sqrt{6n}$	Simplify $\sqrt{9n^6}$.

The correct answer is C.

Got It? **2.** What is the simplified form of $-m\sqrt{80m^9}$?

You can use the Multiplication Property of Square Roots to write $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

Problem 3 Multiplying Two Radical Expressions

Think

What property allows you to multiply the whole numbers first? The Commutative Property of Multiplication allows you to change the order of the factors.

What is the simplified form of $2\sqrt{7t} \cdot 3\sqrt{14t^2}$?

$2\sqrt{7t} \cdot 3\sqrt{14t^2} = 6\sqrt{7t \cdot 14t^2}$	Multiply the whole numbers and use the Multiplication Property of Square Roots.
$= 6\sqrt{98t^3}$	Simplify under the radical symbol.
$= 6\sqrt{49t^2 \cdot 2t}$	$49t^2$, or $(7t)^2$, is a perfect-square factor of $98t^3$.
$= 6\sqrt{49t^2} \cdot \sqrt{2t}$	Use the Multiplication Property of Square Roots.
$= 6 \cdot 7t\sqrt{2t}$	Simplify $\sqrt{49t^2}$.
$= 42t\sqrt{2t}$	Simplify.

Got It? 3. What is the simplified form of each expression in parts (a)–(c)?

a. $3\sqrt{6} \cdot \sqrt{18}$ b. $\sqrt{2a} \cdot \sqrt{9a^3}$ c. $7\sqrt{5x} \cdot 3\sqrt{20x^5}$

d. **Reasoning** In Problem 3, can you simplify the given product by first simplifying $\sqrt{14t^2}$? Explain.

Problem 4 Writing a Radical Expression

Art **A rectangular door in a museum is three times as tall as it is wide. What is a simplified expression for the maximum length of a painting that fits through the door?**

Know
The door is w units wide and $3w$ units high.

Need
The diagonal length d of the doorway

Plan
Use the Pythagorean Theorem.

Think
How is this like problems you have done before?
The width and height of the door are two legs of a right triangle. This is like finding the hypotenuse of a right triangle using the Pythagorean Theorem.

$d^2 = w^2 + (3w)^2$	Pythagorean Theorem
$d^2 = w^2 + 9w^2$	Simplify $(3w)^2$.
$d^2 = 10w^2$	Combine like terms.
$d = \sqrt{10w^2}$	Find the principal square root of each side.
$d = \sqrt{w^2} \cdot \sqrt{10}$	Multiplication Property of Square Roots
$d = w\sqrt{10}$	Simplify $\sqrt{w^2}$.

An expression for the maximum length of the painting is $w\sqrt{10}$, or about $3.16w$.

Got It? 4. A door's height is four times its width w. What is the maximum length of a painting that fits through the door?

You can simplify some radical expressions using the following property.

take note

Property Division Property of Square Roots

Algebra	Example
For $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.	$\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$

When a radicand has a denominator that is a perfect square, it is easier to apply the Division Property of Square Roots first and then simplify the numerator and denominator of the result. When the denominator of a radicand is not a perfect square, it may be easier to simplify the fraction first.

Problem 5 Simplifying Fractions Within Radicals

Think
Which method should you use?
If the denominator is a perfect square, apply the Division Property of Square Roots first. If not, simplify the fraction first.

What is the simplified form of each radical expression?

A $\sqrt{\frac{64}{49}}$

$\sqrt{\frac{64}{49}} = \frac{\sqrt{64}}{\sqrt{49}}$ Use the Division Property of Square Roots.

$= \frac{8}{7}$ Simplify $\sqrt{64}$ and $\sqrt{49}$.

B $\sqrt{\frac{8x^3}{50x}}$

$\sqrt{\frac{8x^3}{50x}} = \sqrt{\frac{4x^2}{25}}$ Divide the numerator and denominator by $2x$.

$= \frac{\sqrt{4x^2}}{\sqrt{25}}$ Use the Division Property of Square Roots.

$= \frac{\sqrt{4} \cdot \sqrt{x^2}}{\sqrt{25}}$ Use the Multiplication Property of Square Roots.

$= \frac{2x}{5}$ Simplify $\sqrt{4}$, $\sqrt{x^2}$, and $\sqrt{25}$.

Got It? **5.** What is the simplified form of each radical expression?

a. $\sqrt{\frac{144}{9}}$ **b.** $\sqrt{\frac{36a}{4a^3}}$ **c.** $\sqrt{\frac{25y^3}{z^2}}$

When a radicand in a denominator is not a perfect square, you may need to **rationalize the denominator** to remove the radical. To do this, multiply the numerator and denominator by the same radical expression. Choose an expression that makes the radicand in the denominator a perfect square. It may be helpful to start by simplifying the original radical in the denominator.

Problem 6 Rationalizing Denominators

Think
Does multiplying an expression by $\frac{\sqrt{7}}{\sqrt{7}}$ change its value?
No. The fraction $\frac{\sqrt{7}}{\sqrt{7}}$ is equal to 1. Multiplying an expression by 1 won't change its value.

What is the simplified form of each expression?

A $\frac{\sqrt{3}}{\sqrt{7}}$

$\frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ Multiply by $\frac{\sqrt{7}}{\sqrt{7}}$.

$= \frac{\sqrt{21}}{\sqrt{49}}$

$= \frac{\sqrt{21}}{7}$

B $\frac{\sqrt{7}}{\sqrt{8n}}$

$\frac{\sqrt{7}}{\sqrt{8n}} = \frac{\sqrt{7}}{2\sqrt{2n}}$

$= \frac{\sqrt{7}}{2\sqrt{2n}} \cdot \frac{\sqrt{2n}}{\sqrt{2n}}$ Multiply by $\frac{\sqrt{2n}}{\sqrt{2n}}$.

$= \frac{\sqrt{14n}}{2\sqrt{4n^2}}$

$= \frac{\sqrt{14n}}{4n}$

Got It? **6.** What is the simplified form of each radical expression?

a. $\frac{\sqrt{2}}{\sqrt{3}}$ **b.** $\frac{\sqrt{5}}{\sqrt{18m}}$ **c.** $\sqrt{\frac{7s}{3}}$

Lesson Check

Do you know HOW?

Simplify each radical expression.

1. $\sqrt{98}$
2. $\sqrt{16b^5}$
3. $3\sqrt{5m} \cdot 4\sqrt{\frac{1}{5}m^3}$
4. $\sqrt{\frac{15x}{x^3}}$
5. $\frac{\sqrt{5}}{\sqrt{3}}$
6. $\frac{\sqrt{6}}{\sqrt{2n}}$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** Is the radical expression in simplified form? Explain.
   a. $\frac{\sqrt{31}}{3}$ b. $7\sqrt{\frac{6}{11}}$ c. $-5\sqrt{175}$
8. **Compare and Contrast** Simplify $\frac{3}{\sqrt{12}}$ two different ways. Which way do you prefer? Explain.
9. **Writing** Explain how you can tell whether a radical expression is in simplified form.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each radical expression.

See Problems 1 and 2.

10. $\sqrt{225}$
11. $\sqrt{99}$
12. $\sqrt{128}$
13. $-\sqrt{60}$
14. $-4\sqrt{117}$
15. $5\sqrt{700}$
16. $\sqrt{192s^2}$
17. $\sqrt{50t^5}$
18. $3\sqrt{18a^2}$
19. $-21\sqrt{27x^9}$
20. $3\sqrt{150b^8}$
21. $-2\sqrt{243y^3}$

Simplify each product.

See Problem 3.

22. $\sqrt{8} \cdot \sqrt{32}$
23. $\frac{1}{3}\sqrt{6} \cdot \sqrt{24}$
24. $4\sqrt{10} \cdot 2\sqrt{90}$
25. $5\sqrt{6} \cdot \frac{1}{6}\sqrt{216}$
26. $-5\sqrt{21} \cdot (-3\sqrt{42})$
27. $\sqrt{18n} \cdot \sqrt{98n^3}$
28. $3\sqrt{5c} \cdot 7\sqrt{15c^2}$
29. $\sqrt{2y} \cdot \sqrt{128y^5}$
30. $-6\sqrt{15s^3} \cdot 2\sqrt{75}$
31. $-9\sqrt{28a^2} \cdot \frac{1}{3}\sqrt{63a}$
32. $10\sqrt{12x^3} \cdot 2\sqrt{6x^3}$
33. $-\frac{1}{3}\sqrt{18c^5} \cdot \left(-6\sqrt{8c^9}\right)$

See Problem 4.

STEM 34. **Construction** Students are building rectangular wooden frames for the set of a school play. The height of a frame is 6 times the width w. Each frame has a brace that connects two opposite corners of the frame. What is a simplified expression for the length of a brace?

35. **Park** A park is shaped like a rectangle with a length 5 times its width w. What is a simplified expression for the distance between opposite corners of the park?

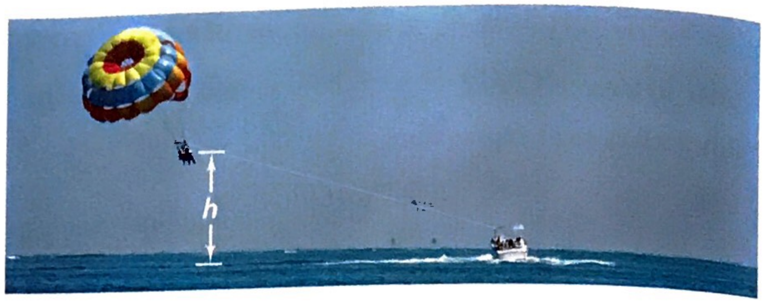
Simplify each radical expression.

See Problems 5 and 6.

36. $\sqrt{\frac{16}{25}}$
37. $7\sqrt{\frac{6}{32}}$
38. $-4\sqrt{\frac{100}{729}}$
39. $\sqrt{\frac{3x^3}{64x^2}}$
40. $-5\sqrt{\frac{162t^3}{2t}}$
41. $11\sqrt{\frac{49a^5}{4a^3}}$
42. $\frac{1}{\sqrt{11}}$
43. $\frac{\sqrt{5}}{\sqrt{8x}}$
44. $\frac{3\sqrt{6}}{\sqrt{15}}$
45. $\frac{22}{\sqrt{11}}$
46. $\frac{2\sqrt{24}}{\sqrt{48t^4}}$
47. $\frac{8\sqrt{7s}}{\sqrt{28s^3}}$

B Apply

48. Look for a Pattern From a viewing height of h feet, the approximate distance d to the horizon, in miles, is given by the equation $d = \sqrt{\frac{3h}{2}}$.



a. To the nearest mile, what is the distance to the horizon from a height of 150 ft? 225 ft? 300 ft?

b. How does the distance to the horizon increase as the height increases?

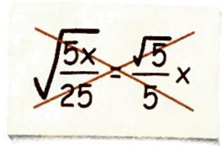
49. Think About a Plan A square picture on the front page of a newspaper occupies an area of 24 in.^{2}. What is the length of each side of the picture? Write your answer as a radical in simplified form.

- How can you find the side length of a square if you know the area?
- What property can you use to write your answer in simplified form?

Explain why each radical expression is or is not in simplified form.

50. $\frac{13x}{\sqrt{4}}$ **51.** $\frac{3}{\sqrt{3}}$ **52.** $-4\sqrt{5}$ **53.** $5\sqrt{30}$

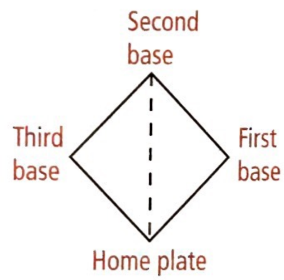
54. Error Analysis A student simplified the radical expression at the right. What mistake did the student make? What is the correct answer?



55. Reasoning You can simplify radical expressions with negative exponents by first rewriting the expressions using positive exponents. What are the simplified forms of the following radical expressions?

a. $\frac{\sqrt{3}}{\sqrt{f^{-3}}}$ **b.** $\frac{\sqrt{x^{-3}}}{\sqrt{x}}$ **c.** $\frac{\sqrt{5a^{-2}}}{\sqrt{10a^{-1}}}$ **d.** $\frac{\sqrt{(2m)^{-3}}}{m^{-1}}$

56. Sports The bases in a softball diamond are located at the corners of a 3600-ft^2 square. How far is a throw from second base to home plate?



57. Suppose a and b are positive integers.

a. Verify that if $a = 18$ and $b = 10$, then $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

b. Open-Ended Find two other pairs of positive integers a and b such that $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

Simplify each radical expression.

58. $\sqrt{12} \cdot \sqrt{75}$ **59.** $\sqrt{26 \cdot 2}$ **60.** $\frac{\sqrt{72}}{\sqrt{64}}$ **61.** $\frac{-2}{\sqrt{a^3}}$

62. $\frac{\sqrt{180}}{\sqrt{3}}$ **63.** $\frac{\sqrt{x^2}}{\sqrt{y^3}}$ **64.** $\frac{-3\sqrt{2}}{\sqrt{6}}$ **65.** $\sqrt{8} \cdot \sqrt{10}$

66. $\sqrt{20a^2b^3}$ **67.** $\sqrt{a^3b^5c^3}$ **68.** $\sqrt{\frac{3m}{16m^2}}$ **69.** $\frac{16a}{\sqrt{6a^3}}$

Solve each equation. Leave your answer in simplified radical form.

70. $x^2 + 6x - 9 = 0$ **71.** $n^2 - 2n + 1 = 5$ **72.** $3y^2 - 4y - 2 = 0$

73. Open-Ended What are three numbers whose square roots can be written in the form $a\sqrt{3}$ for some integer value of a?

Challenge

Simplify each radical expression.

74. $\sqrt{24} \cdot \sqrt{2x} \cdot \sqrt{3x}$

75. $2b(\sqrt{5b})^2$

76. $\sqrt{45a^7} \cdot \sqrt{20a}$

77. Geometry The equation $r = \sqrt{\frac{A}{\pi}}$ gives the radius r of a circle with area A. What is the radius of a circle with the given area? Write your answer as a simplified radical and as a decimal rounded to the nearest hundredth.

a. $50\ \text{ft}^2$

b. $32\ \text{in.}^2$

c. $10\ \text{m}^2$

78. For a linear equation in standard form $Ax + By = C$, where $A \neq 0$ and $B \neq 0$, the distance d between the x- and y-intercepts is given by $d = \sqrt{\left(\frac{C}{A}\right)^2 + \left(\frac{C}{B}\right)^2}$. What is the distance between the x- and y-intercepts of the graph of $4x - 3y = 2$?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at the information about the formula $s = \sqrt{30fd}$ on page 613. The table giving several coefficients of friction is shown again below.

Typical Coefficients of Friction

Surface Material	Coefficient of Friction
Gravel	0.6
Asphalt	0.7
Cement	0.9

a. Based on the surface material of Pine Street, write and simplify a formula that gives the speed s of a car in terms of the length d of the skid marks, where s is in miles per hour and d is in feet.

b. In the Apply What You've Learned in Lesson 10-1, you found the length of the skid marks left by the car described on page 613. Use this value and the formula you found in part (a) to write a radical expression that gives the speed, in miles per hour, of the car. Give your answer in simplest radical from.

c. Suppose Pine Street was paved with gravel instead of asphalt. How would this change your answers to parts (a) and (b)? Does a decrease in the coefficient of friction lead to an increase or a decrease in the calculated speed of the car? Explain.