2-1 Solving One-Step Equations

Common Core State Standards

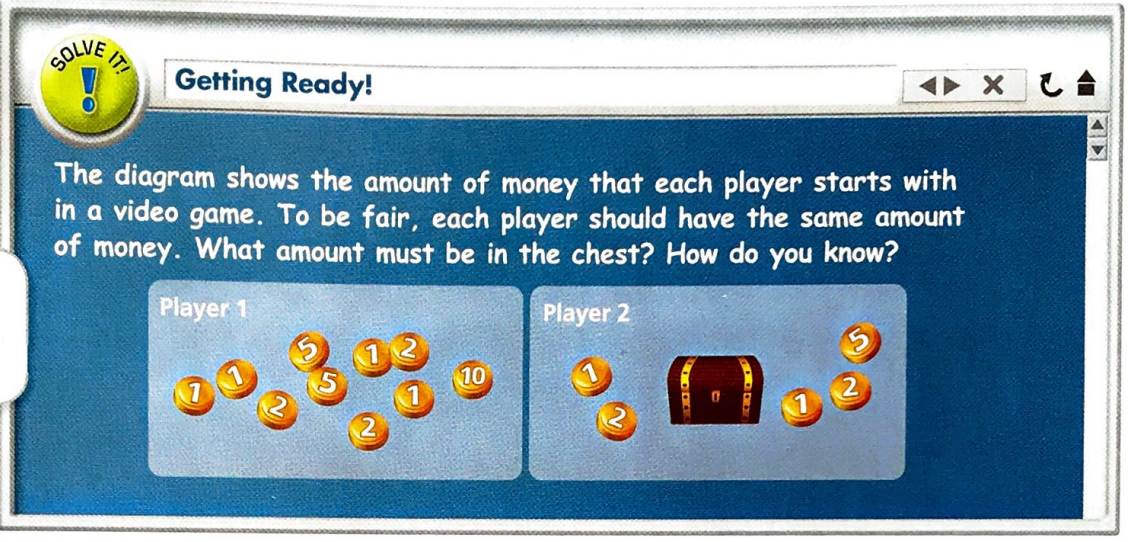
A-CED.A.1 Create equations . . . in one variable and use them to solve problems. *Include equations arising from linear . . . functions.* **Also A-REI.B.3**

MP 1, MP 3, MP 4, MP 7

Objective To solve one-step equations in one variable

First make a plan for how you are going to solve this problem.

MATHEMATICAL PRACTICES



In the Solve It, you may have used reasoning to find the amount of money in the chest. In this lesson, you will learn to solve problems like the one above by using equations.

Lesson Vocabulary

- equivalent equations
- Addition Property of Equality
- Subtraction Property of Equality
- isolate
- inverse operations
- Multiplication Property of Equality
- Division Property of Equality

Essential Understanding **Equivalent equations** are equations that have the same solution(s). You can find the solution of a one-step equation using the properties of equality and inverse operations to write a simpler equivalent equation.

take note

Property Addition and Subtraction Properties of Equality

Addition Property of Equality Adding the same number to each side of an equation produces an equivalent equation.

Algebra	Example
For any real numbers a, b, and c,	$x - 3 = 2$
if $a = b$, then $a + c = b + c$.	$x - 3 + 3 = 2 + 3$

Subtraction Property of Equality Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra	Example
For any real numbers a, b, and c,	$x + 3 = 2$
if $a = b$, then $a - c = b - c$.	$x + 3 - 3 = 2 - 3$

To solve an equation, you must **isolate** the variable. You do this by getting the variable with a coefficient of 1 alone on one side of the equation.

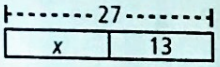
You can isolate a variable using the properties of equality and inverse operations. An **inverse operation** undoes another operation. For example, subtraction is the inverse of addition. When you solve an equation, each inverse operation you perform should produce a simpler equivalent equation.

Problem 1 Solving an Equation Using Subtraction

What is the solution of $x + 13 = 27$?

Plan

How can you visualize the equation?
You can *draw a diagram.* Use a model like the one below to help you visualize an equation. A model for the equation $x + 13 = 27$ is



Think | **Write**

You need to isolate x. Start by writing the equation.

$$x + 13 = 27$$

Undo addition by subtracting the same number from each side.

$$x + 13 - 13 = 27 - 13$$

Simplify each side of the equation.

$$x = 14$$

Substitute your answer into the original equation to check it.

$$x + 13 = 27$$
$$14 + 13 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark$$

Got It? **1. a.** What is the solution of $y + 2 = -6$? Check your answer.

b. Reasoning In Problem 1, why does subtracting 13 from both sides of the original equation result in an equivalent equation?

Problem 2 Solving an Equation Using Addition

What is the solution of $-7 = b - 3$?

Plan

How can you get started?
Undo operations. Add 3 to each side to undo subtraction.

$$-7 = b - 3$$
$$-7 + 3 = b - 3 + 3 \quad \text{Add 3 to each side.}$$
$$-4 = b \quad \text{Simplify.}$$

Got It? **2.** What is the solution of each equation? Check your answer.

a. $m - 8 = -14$ **b.** $\frac{1}{2} = y - \frac{3}{2}$

You can use the Multiplication and Division Properties of Equality to solve equations. Division is the inverse of multiplication.

take note

Property Multiplication and Division Properties of Equality

Multiplication Property of Equality Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Algebra	Example
For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$.	$\frac{x}{3} = 2$ $\frac{x}{3} \cdot 3 = 2 \cdot 3$

Division Property of Equality Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Algebra	Example
For any real numbers a, b, and c, such that $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.	$5x = 20$ $\frac{5x}{5} = \frac{20}{5}$

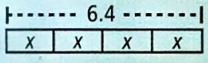
Problem 3 Solving an Equation Using Division

GRIDDED RESPONSE

What is the solution of $4x = 6.4$?

Plan

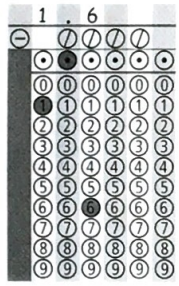
How can a model help you solve the equation?
The model tells you that you must divide 6.4 by 4 in order to solve the equation $4x = 6.4$.



$4x = 6.4$

$\frac{4x}{4} = \frac{6.4}{4}$ Divide each side by 4.

$x = 1.6$ Simplify.



Got It? **3.** What is the solution of each equation? Check your answer.

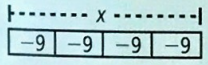
a. $10 = 15x$ **b.** $-3.2z = 14$

Problem 4 Solving an Equation Using Multiplication

What is the solution of $\frac{x}{4} = -9$?

Plan

How can a model help you solve the equation?
The model tells you that you must multiply -9 by 4 in order to solve the equation $\frac{x}{4} = -9$.



$\frac{x}{4} = -9$

$\frac{x}{4} \cdot 4 = -9 \cdot 4$ Multiply each side by 4.

$x = -36$ Simplify.

Got It? **4.** What is the solution of each equation? Check your answer.

a. $19 = \frac{r}{3}$ **b.** $\frac{x}{-9} = 8$

When the coefficient of the variable in an equation is a fraction, you can use the reciprocal of the fraction to solve the equation.

Problem 5 Solving Equations Using Reciprocals

What is the solution of $\frac{4}{5}m = 28$?

Think

Why multiply by the reciprocal?
You want the coefficient of m to be 1. The product of a number and its reciprocal is 1, so multiply by the reciprocal.

$\frac{4}{5}m = 28$

$\frac{5}{4}\left(\frac{4}{5}m\right) = \frac{5}{4}(28)$ Multiply each side by $\frac{5}{4}$, the reciprocal of $\frac{4}{5}$.

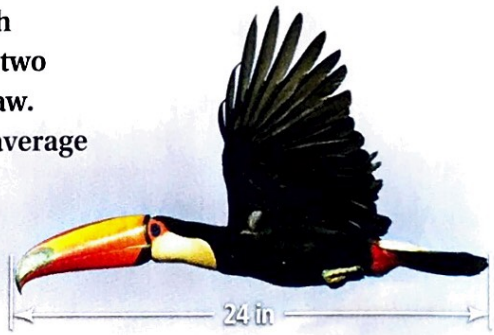
$m = 35$ Simplify.

Got It? **5. a.** What is the solution of $12 = \frac{3}{4}x$? Check your answer.

b. Reasoning Are the equations $m = 18$ and $\frac{2}{3}m = 12$ equivalent? How do you know?

Problem 6 Using a One-Step Equation as a Model STEM

Biology **Toucans and blue-and-yellow macaws are both tropical birds. The length of an average toucan is about two thirds of the length of an average blue-and-yellow macaw. Toucans are about 24 in. long. What is the length of an average blue-and-yellow macaw?**



Relate length of toucan is $\frac{2}{3}$ of length of blue-and-yellow macaw

Define Let ℓ = the length of an average blue-and-yellow macaw.

Think

How else can you solve this problem?
You can work backward. The toucan's length is $\frac{2}{3}$ the macaw's length, so the macaw's length is $\frac{3}{2}$ the toucan's length. You can multiply the length of the toucan by $\frac{3}{2}$.

Write $24 = \frac{2}{3} \cdot \ell$

$24 = \frac{2}{3}\ell$

$\frac{3}{2}(24) = \frac{3}{2}\left(\frac{2}{3}\ell\right)$ Multiply each side by $\frac{3}{2}$.

$36 = \ell$ Simplify.

An average blue-and-yellow macaw is 36 in. long.

Check $24 = \frac{2}{3}\ell$

$24 \stackrel{?}{=} \frac{2}{3}(36)$ Substitute 36 for ℓ.

$24 = 24$ Simplify. The solution checks.

Got It? **6.** An online DVD rental company offers gift certificates that you can use to purchase rental plans. You have a gift certificate for \$30. The plan you select costs \$5 per month. How many months can you purchase with the gift certificate?

Lesson Check

Do you know HOW?

Solve each equation. Check your answer.

1. $x + 7 = 3$
2. $9 = m - 4$
3. $5y = 24$
4. **Books** You have already read 117 pages of a book. You are one third of the way through the book. Write and solve an equation to find the number of pages in the book.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Vocabulary Which property of equality would you use to solve each equation? Why?

5. $3 + x = -34$
6. $2x = 5$
7. $x - 4 = 9$
8. $\frac{x}{7} = 9$
9. **Reasoning** Write a one-step equation. Then write two equations that are equivalent to your equation. How can you prove that all three equations are equivalent?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each equation using addition or subtraction. Check your answer. See Problems 1 and 2.

10. $6 = x + 2$
11. $27 + n = 46$
12. $23 = v + 5$
13. $4 = q + 13$
14. $f + 9 = 20$
15. $-5 + a = 21$
16. $-17 = 3 + k$
17. $5.5 = -2 + d$
18. $c + 4 = -9$
19. $67 = w - 65$
20. $23 = b - 19$
21. $g - 3.5 = 10$
22. $y - 19 = 37$
23. $q - 11 = -9$
24. $-2.5 = p + 7.1$
25. $j - 3 = -7$

Solve each equation using multiplication or division. Check your answer. See Problems 3 and 4.

26. $-8n = -64$
27. $-7y = 28$
28. $5b = 145$
29. $6a = 0.96$
30. $-96 = 4c$
31. $11 = 2.2t$
32. $17.5 = 5s$
33. $7r = -\frac{7}{2}$
34. $\frac{m}{7} = 12$
35. $35 = \frac{j}{5}$
36. $\frac{k}{7} = 13$
37. $-39 = \frac{q}{3}$
38. $14 = \frac{z}{2}$
39. $\frac{q}{-9} = -9$
40. $-13 = \frac{m}{-5}$
41. $\frac{k}{4} = -\frac{17}{2}$

Solve each equation. Check your answer. See Problem 5.

42. $\frac{2}{3}q = 18$
43. $\frac{3}{4}x = 9$
44. $\frac{5}{8}y = -1$
45. $\frac{3}{5}m = -15$
46. $\frac{1}{5}x = \frac{2}{7}$
47. $36 = \frac{4}{9}d$
48. $-6 = \frac{3}{7}n$
49. $\frac{3}{8}p = 9$

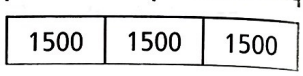
Define a variable and write an equation for each situation. Then solve. See Problem 6.

50. **Music** You have a rack that can hold 30 CDs. You can fit 7 more CDs on the rack before the rack is full. How many CDs are in the rack?
51. **Population** In a 3-year period, a city's population decreased by 7525 to about 581,600. What was the city's population at the beginning of the 3-year period?

B Apply

52. Writing If a one-step equation includes addition, should you expect to solve it by using addition? Why or why not?

53. Think About a Plan Costumes for a play at a community theater cost \$1500, which is one third of the total budget. What is the total budget for the play?



- How can the model at the right help you solve the problem?
- How does the model tell you which operation to use in the equation?

54. Entertainment On a quiz show, a contestant was penalized 250 points for an incorrect answer, leaving the contestant with 1050 points. How many points did the contestant have before the penalty?

Solve each equation. Check your answer.

55. $\frac{2}{7} = \frac{1}{3} + a$

56. $23 = 7x$

57. $z - 4\frac{2}{3} = 2\frac{2}{3}$

58. $\frac{2}{3}g = -4\frac{1}{2}$

59. $6\frac{1}{4} = \frac{r}{5}$

60. $h + 2.8 = -3.7$

61. $\frac{3}{2}f = \frac{1}{2}$

62. $-4 = \frac{2}{9}d$

63. $1.6m = 1.28$

64. $4d = -2.4$

65. $4\frac{1}{4} = 1\frac{3}{4} + p$

66. $-5.3 + z = 8.9$

67. $-2\frac{1}{2} = \frac{t}{10}$

68. $5b = 8.5$

69. $\frac{3}{5}n = -\frac{3}{10}$

70. Picnics At a party of 102 people, 17 lb of potato salad is served.

a. Write and solve an equation to find how many people each pound of potato salad serves.

b. Write and solve an equation to find the average number of pounds of potato salad that each person is served. Round your answer to the nearest hundredth.

71. Error Analysis Describe and correct the error in solving the equation at the right.

$$-36 = \frac{x}{9}$$
$$\left(\frac{1}{9}\right)(-36) = \left(\frac{1}{9}\right)\left(\frac{x}{9}\right)$$
$$-4 = x$$

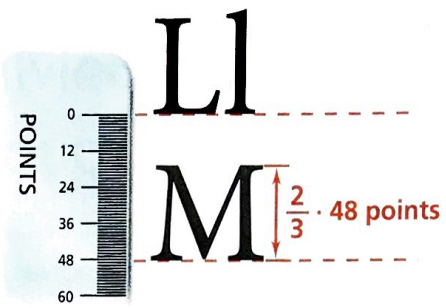
72. U.S. History Between 1788 and 2008, the U.S. Constitution was amended 27 times. How many years have passed on average between one amendment and the next, to the nearest tenth of a year?

73. Volleyball In volleyball, players serve the ball to the opposing team. If the opposing team fails to hit the ball, the service is called an ace. A player's ace average is the number of aces served divided by the number of games played. A certain player has an ace average of 0.3 and has played in 70 games this season. How many aces has the player served?

74. Open-Ended Write a problem that you can model with a one-step equation. Write the equation and solve the problem.

75. Language According to one count, the letter *e* makes up one eighth of a typical document written in English. A document contains 2800 letters. About how many letters in the document are *not e*?

76. Typography A point is a unit of length that can be used to measure the distance between two lines of text. Font sizes are often stated in points. Capital letters measure two thirds of the stated point size, as shown in the diagram for a font size of 48 points. There are 72 points in 1 inch. What point size produces capital letters that are $\frac{1}{2}$ in. tall?



Challenge

77. Reasoning In a school's musical, a choir member sang in the backup chorus for half the songs in the show, which was 12 songs. A student concludes that one half of 12 is 6, so there were 6 songs in the show. Write an equation that would help the student understand the correct number of songs in the musical.

78. Cooking Uncooked rice has about $\frac{4}{13}$ the weight of cooked rice. You want to make 6.5 lb of rice for a recipe. How many pounds of uncooked rice do you need?

Standardized Test Prep

SAT/ACT

79. Luis helped raise money for his school by jogging in the school jog-a-thon. The total amount of money he raised can be represented by the expression $1.75m$, where m is the number of miles he jogged. If Luis raised a total of \$21, how many miles did he jog?

Ⓐ 12 Ⓑ 19.25 Ⓒ 22.75 Ⓓ 36.75

80. What operation should you use to solve $14 + c = 39$?

Ⓕ squaring Ⓖ subtraction Ⓗ multiplication Ⓘ division

81. Sonya is checking orders at the fabric store where she works. Some of the orders are in decimals and some are in fractions. Which of the following statements is *not* true?

Ⓐ $\frac{10}{4} = 2.5$ Ⓑ $1.3 = 1\frac{1}{3}$ Ⓒ $0.03 = \frac{3}{100}$ Ⓓ $\frac{6}{5} = 1.2$

Mixed Review

82. If the pattern shown in the table continues, what amount will have been raised by Week 5?

Scholarship Funds				
Week	0	1	2	3
Amount (thousands)	0	2	4	6

See Lesson 1-9.

Simplify each expression. Justify each step.

See Lesson 1-4.

83. $4(13x)$ **84.** $2.2 + (3.8 - x)$ **85.** $(m + 4.5) - 0.5$

Get Ready! **To prepare for Lesson 2-2, do Exercises 86–88.**

Simplify each expression.

See Lesson 1-2.

86. $2[2 - (2 - 3) - 2]$ **87.** $\left(\frac{1}{2} + \frac{1}{3}\right)^2$ **88.** $-1 + 2 \cdot 3 - 4$