

## 2-2

## Solving Two-Step Equations

## Common Core State Standards

**A-REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Also **A-CED.A.1**, **A-REI.A.1**  
**MP 2, MP 3, MP 4**

**Objective** To solve two-step equations in one variable



To get started, think about the simpler case in which you have just one chest of money.



## Getting Ready!

The diagram shows the amount of money that each player starts with in a video game. To be fair, each player should have the same amount of money. If each chest contains the same amount of money, how much money must be in each chest? How do you know?



The problem in the Solve It can be modeled by an equation. The equations in this lesson are different from the equations in Lesson 2-1 because they require two steps to solve.

**Essential Understanding** To solve two-step equations, you can use the properties of equality and inverse operations to form a series of simpler equivalent equations. You can use the properties of equality repeatedly to isolate the variable.

A two-step equation, like the one shown below, involves two operations.

Multiplication      Addition

$$2x + 3 = 15$$

To solve a two-step equation, identify the operations and undo them using inverse operations. You can undo the operations in the reverse order of the order of operations. For example, to solve  $2x + 3 = 15$ , you can use subtraction first to undo the addition, and then use division to undo the multiplication.

## Think

What operations are used in the equation?

The equation uses multiplication and addition. You can undo the addition first, and then the multiplication.



### Problem 1 Solving a Two-Step Equation

What is the solution of  $2x + 3 = 15$ ?

$$2x + 3 = 15$$

$$2x + 3 - 3 = 15 - 3 \quad \text{Subtract 3 from each side.}$$

$$2x = 12 \quad \text{Simplify.}$$

$$\frac{2x}{2} = \frac{12}{2} \quad \text{Divide each side by 2.}$$

$$x = 6 \quad \text{Simplify.}$$

**Check**  $2x + 3 = 15$

$$2(6) + 3 \stackrel{?}{=} 15 \quad \text{Substitute 6 for } x.$$

$$15 = 15 \quad \text{Simplify. The solution checks.}$$



**Got It?** 1. What is the solution of  $5 = \frac{t}{2} - 3$ ?



### Problem 2 Using an Equation as a Model

**Community Service** You are making a bulletin board to advertise community service opportunities in your town. You plan to use half a sheet of construction paper for each ad. You need 5 sheets of construction paper for a title banner. You have 18 sheets of construction paper. How many ads can you make?

#### Know

- Paper per ad:  $\frac{1}{2}$  sheet
- Paper for banner: 5 sheets
- Total paper: 18 sheets

#### Need

The number of ads you can make

#### Plan

Write and solve an equation. Let the variable represent the unknown.

## Think

How can a model help you write the equation?

The model shows that a half of a sheet per ad plus 5 sheets for the title banner is equal to 18 sheets.

----- 18 -----	
$\frac{1}{2}a$	5

Let  $a$  = the number of ads you can make.

$$\frac{1}{2}a + 5 = 18$$

$$\frac{1}{2}a + 5 - 5 = 18 - 5 \quad \text{Subtract 5 from each side.}$$

$$\frac{1}{2}a = 13 \quad \text{Simplify.}$$

$$2\left(\frac{1}{2}a\right) = 2(13) \quad \text{Multiply each side by 2.}$$

$$a = 26 \quad \text{Simplify.}$$

You can make 26 community service advertisements for the bulletin board.



**Got It?** 2. Suppose you used one quarter of a sheet of paper for each ad and four full sheets for the title banner in Problem 2. How many ads could you make?

When one side of an equation is a fraction with more than one term in the numerator, you can still undo division by multiplying each side by the denominator.

## Plan

**What operation should you perform first?**

Multiplication. When you multiply by the denominator of the fraction in the equation, you get a one-step equation. So, multiplying first gets rid of the fraction.



### Problem 3 Solving With Two Terms in the Numerator

What is the solution of  $\frac{x-7}{3} = -12$ ?

$$\frac{x-7}{3} = -12$$

$$3\left(\frac{x-7}{3}\right) = 3(-12) \quad \text{Multiply each side by 3.}$$

$$x-7 = -36 \quad \text{Simplify.}$$

$$x-7+7 = -36+7 \quad \text{Add 7 to each side.}$$

$$x = -29 \quad \text{Simplify.}$$



**Got It?** 3. a. What is the solution of  $6 = \frac{x-2}{4}$ ?

**b. Reasoning** Write the right side of the equation in part (a) as the difference of two fractions. Solve the equation. Did you find the equation in part (a) or the rewritten equation easier to solve? Why?

When you use deductive reasoning, you must state your steps and your reason for each step using properties, definitions, or rules. In Problem 4, you are asked to provide the reasons for each step of the problem using deductive reasoning.



### Problem 4 Using Deductive Reasoning

What is the solution of  $-t + 8 = 3$ ? Justify each step.

Steps	Reasons
$-t + 8 = 3$	Original equation
$-t + 8 - 8 = 3 - 8$	Subtraction Property of Equality
$-t = -5$	Use subtraction to simplify.
$-1t = -5$	Multiplicative Property of $-1$
$\frac{-1t}{-1} = \frac{-5}{-1}$	Division Property of Equality
$t = 5$	Use division to simplify.

## Think

**Why isn't  $-t = -5$  the solution?**

When you solve for a variable, the coefficient must be 1, not  $-1$ .



**Got It?** 4. What is the solution of  $\frac{x}{3} - 5 = 4$ ? Justify each step.



## Lesson Check

Do you know **HOW?**

Solve each equation. Check your answer.

1.  $5x + 12 = -13$

2.  $6 = \frac{m}{7} - 3$

3.  $\frac{y-1}{4} = -2$

4.  $-x - 4 = 9$

5. **Fundraising** The junior class is selling granola bars to raise money. They purchased 1250 granola bars and paid a delivery fee of \$25. The total cost, including the delivery fee, was \$800. What was the cost of each granola bar?

Do you **UNDERSTAND?**



What properties of equality would you use to solve each equation? What operation would you perform first? Explain.

6.  $-8 = \frac{s}{4} + 3$

7.  $2x - 9 = 7$

8.  $\frac{x}{3} - 8 = 4$

9.  $-4x + 3 = -5$

10. **Reasoning** Can you solve the equation  $\frac{d-3}{5} = 6$  by adding 3 before multiplying by 5? Explain.



## Practice and Problem-Solving Exercises



**A** Practice

Solve each equation. Check your answer.

11.  $2 + \frac{a}{4} = -1$

12.  $3n - 4 = 11$

13.  $-1 = 7 + 8x$

14.  $\frac{y}{5} + 2 = -8$

15.  $4b + 6 = -2$

16.  $10 = \frac{x}{4} - 8$

17.  $10 + \frac{h}{3} = 1$

18.  $-14 = -5 + 3c$

19.  $26 = \frac{m}{6} + 5$

20.  $\frac{a}{5} - 18 = 2$

21.  $-5x - 2 = 13$

22.  $14 = -2k + 3$

Define a variable and write an equation for each situation. Then solve.

23. **Maximum Capacity** A delivery person uses a service elevator to bring boxes of books up to an office. The delivery person weighs 160 lb and each box of books weighs 50 lb. The maximum capacity of the elevator is 1000 lb. How many boxes of books can the delivery person bring up at one time?
24. **Shopping** You have \$16 and a coupon for a \$5 discount at a local supermarket. A bottle of olive oil costs \$7. How many bottles of olive oil can you buy?
25. **Rentals** Two college friends rent an apartment. They have to pay the landlord two months' rent and a \$500 security deposit when they sign the lease. The total amount they pay the landlord is \$2800. What is the rent for one month?

Solve each equation. Check your answer.

26.  $\frac{y-4}{2} = 10$

27.  $7 = \frac{x-8}{3}$

28.  $\frac{z+10}{9} = 2$

29.  $4 = \frac{a+10}{2}$

30.  $7\frac{1}{2} = \frac{x+3}{2}$

31.  $\frac{b+3}{5} = -1$

32.  $-2 = \frac{d-7}{7}$

33.  $\frac{g-3}{3} = \frac{5}{3}$

Solve each equation. Justify each step.

34.  $14 - b = 19$

35.  $20 - 3h = 2$

36.  $3 - \frac{x}{2} = 6$

37.  $-1 = 4 + \frac{x}{3}$

← See Problem 1.

← See Problem 2.

← See Problem 3.

← See Problem 4.

Solve each equation. Check your answer.

38.  $\frac{2+y}{3} = -1$

39.  $-24 = -10t + 3$

40.  $10 = 0.3x - 9.1$

41.  $\frac{1}{2} = \frac{1}{2}c - 2$

42.  $\frac{x-3}{3} = -4\frac{1}{2}$

43.  $9.4 = -d + 5.6$

44.  $\frac{d+17}{2} = 5\frac{1}{3}$

45.  $2.4 + 10m = 6.89$

46.  $\frac{1}{5}t - 3 = -17$

Solve each equation. Justify each step.

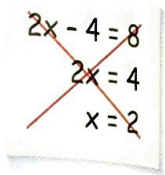
47.  $15 = 9 - 3p$

48.  $4 - 5k = -16$

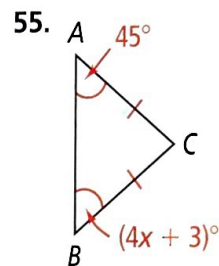
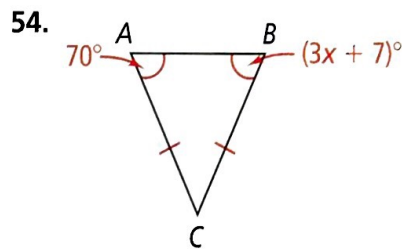
49.  $9 + \frac{c}{-5} = -5$

50.  $\frac{q}{-3} + 12 = 2$

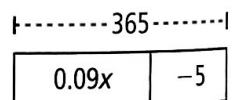
- © 51. **Error Analysis** Describe and correct the error in finding the solution of the equation at the right.
- © 52. **Writing** Without solving the equation  $-3x + 5 = 44$ , tell whether the value of  $x$  is positive or negative. How do you know?
- © 53. a. Solve the equation  $2x - 1 = 7$  by undoing subtraction first.  
 b. Solve the equation in part (a) by undoing multiplication first. Do you get the same answer you got in part (a)?  
 c. **Reasoning** Which method from parts (a) and (b) do you prefer? Explain.



**Geometry** In each triangle, the measure of  $\angle A$  equals the measure of  $\angle B$ . Find the value of  $x$ .



- © 56. **Think About a Plan** A Web site allows musicians to post their songs online. Then people using the Web site can buy any of the posted songs. Suppose each musician must pay a one-time fee of \$5 to use the Web site. Each musician earns \$.09 every time a particular song of his or hers is downloaded. If a musician earned \$365 for a particular song, how many times was the song downloaded?
- How can the model at the right help you solve the problem?
  - How does the model tell you which operations to use in the equation?
- © 57. **Open-Ended** Write a real-world problem that you can model with the two-step equation  $8b + 6 = 38$ . Then solve the problem.
58. **Home Improvement** A contractor is adding a back porch on to a house. The porch needs to hold 20 people and furniture that weighs 250 lb. The contractor calculates that the porch needs to hold 3750 lb to meet that specification. What value did the contractor use for the weight of a person?



- STEM** 59. **Earth Science** The air temperature beneath Earth's surface increases by about  $10^{\circ}\text{C}$  per kilometer. The surface temperature and the air temperature at the bottom of a mine are shown. How many kilometers below Earth's surface is the bottom of the mine?



Surface:  $18^{\circ}\text{C}$   
Bottom of mine:  $38^{\circ}\text{C}$   
Not drawn to scale

60. **Car-Sharing Program** Members of a car-sharing program pay a fee of \$50 per month plus \$7.65 for every hour they use a car. A member's bill was \$149.45 last month. How many hours did the customer use a car last month?



61. **Word Processing** You format a document in three columns of equal width. The document is 8.5 in. wide. You want left and right margins of 1 in. each. Between the columns there is a "gutter" that is one eighth as wide as each column. What is the width of each column?

Tell whether each equation has a solution. If so, find the solution. If not, explain why not.

62.  $2x - 0 = 0$

63.  $0(-2x) = 4$

64.  $\frac{x-2}{2} = 0$

65.  $\frac{x-2}{0} = 2$

## Standardized Test Prep

## GRIDDED RESPONSE



66. William's age  $w$  and Jamie's age  $j$  are related by the equation  $w = 2j - 12$ . When William is 36.5 years old, how old is Jamie?
67. Dominique paints faces at an annual carnival. Her goal this year is to earn \$100. She spends \$15 on supplies and will work for 2.5 h. How much will she need to earn in dollars per hour in order to reach her goal?
68. The cost of a gallon of milk  $m$  is \$.50 more than five times the cost of a gallon of water  $w$ . If a gallon of milk costs \$3.75, what is the cost of a gallon of water?

## Mixed Review

Solve each equation.

69.  $-5x = -25$

70.  $7 = 3.2 + y$

71.  $\frac{y}{4} = 36$

72.  $z - 2 = 4.5$

➡ See Lesson 2-1.

Tell whether each statement is *true* or *false*. If it is false, give a counterexample.

➡ See Lesson 1-4.

73. The difference of the absolute value of two numbers is the same as the difference of the two numbers themselves.
74. Adding 1 to a number always increases its absolute value.

**Get Ready!** To prepare for Lesson 2-3, do Exercises 75-78.

Simplify each expression.

➡ See Lesson 1-7.

75.  $7(5 - t)$

76.  $-2(-2x + 5)$

77.  $-3(2 - b)$

78.  $5(2 - 5n)$