

4-1

Using Graphs to Relate Two Quantities

Common Core State Standards

F-IF.B.4 ... interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features ... of the relationship ...

MP 1, MP 2, MP 3, MP 4

Objective To represent mathematical relationships using graphs

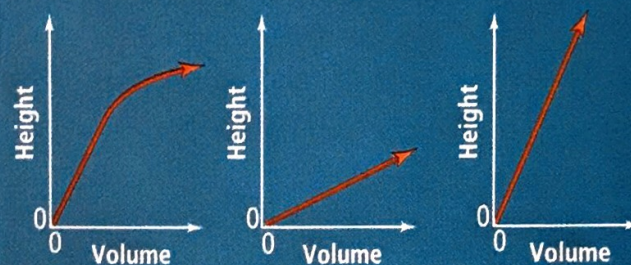


Graphs can help you see relationships.



Getting Ready!

The graphs below relate the height of the water to the volume of the water in each container.
Which graph goes with which container? Justify your reasoning.



As you may have noticed in the Solve It, the change in the height of the water as the volume increases is related to the shape of the container.

Essential Understanding You can use graphs to visually represent the relationship between two variable quantities as they both change.

Think

How can you analyze the relationship in a graph?

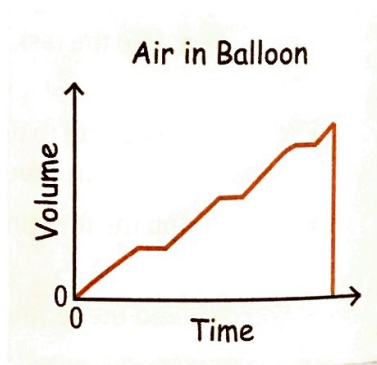
Read the titles. The axis titles tell you what variables are related. The graph itself represents the relationship as the variables change.



Problem 1 Analyzing a Graph

The graph shows the volume of air in a balloon as you blow it up, until it pops. What are the variables? Describe how the variables are related at various points on the graph.

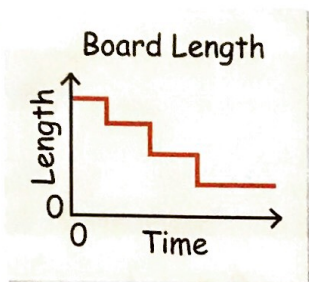
The variables are volume and time. The volume increases each time you blow, and it stays constant each time you pause to breathe. When the balloon pops in the middle of the fourth blow, the volume decreases to 0.



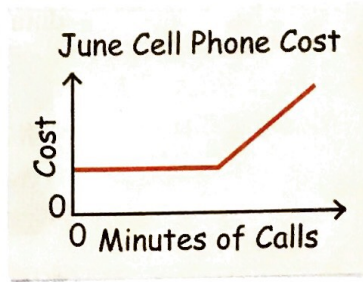


Got It? 1. What are the variables in each graph? Describe how the variables are related at various points on the graph.

a.



b.



Tables and graphs can both show relationships between variables. Data from a table are often displayed using a graph to visually represent the relationship.

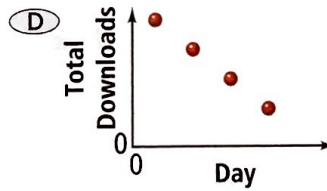
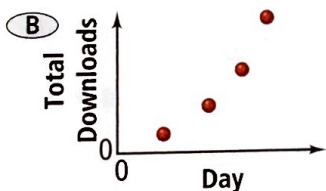
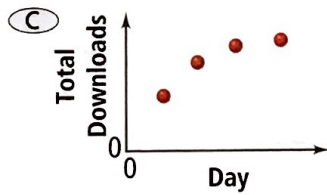
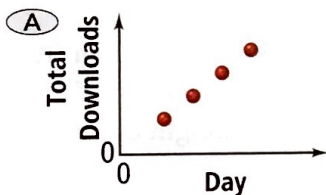


Problem 2 Matching a Table and a Graph

Multiple Choice A band allowed fans to download its new video from its Web site. The table shows the total number of downloads after 1, 2, 3, and 4 days. Which graph could represent the data shown in the table?

Video Downloads

Day	Total Downloads
1	346
2	1011
3	3455
4	10,426



Know

The relationship represented by a table

Need

A graph that could represent the relationship

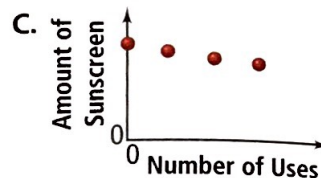
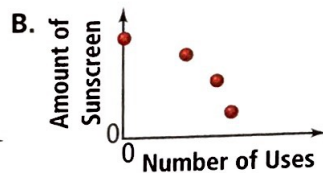
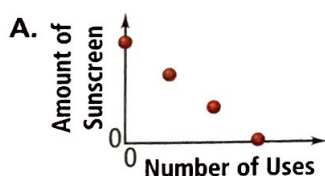
Plan

Compare the pattern of changes in the table to each graph.

In the table, the total number of downloads increases each day, and each increase is noticeably greater than the previous increase. So the graph should rise from left to right, and each rise should be steeper than the previous rise. The correct answer is B.

- Got It?** 2. The table shows the amount of sunscreen left in a can based on the number of times the sunscreen has been used. Which graph could represent the data shown in the table?

Sunscreen				
Number of Uses	0	1	2	3
Amount of Sunscreen (oz)	5	4.8	4.6	4.4



In Problem 2, the number of downloads, which is on the vertical axis of each graph, depends on the day, which is on the horizontal axis. When one quantity depends on another, show the independent quantity on the horizontal axis and the dependent quantity on the vertical axis.

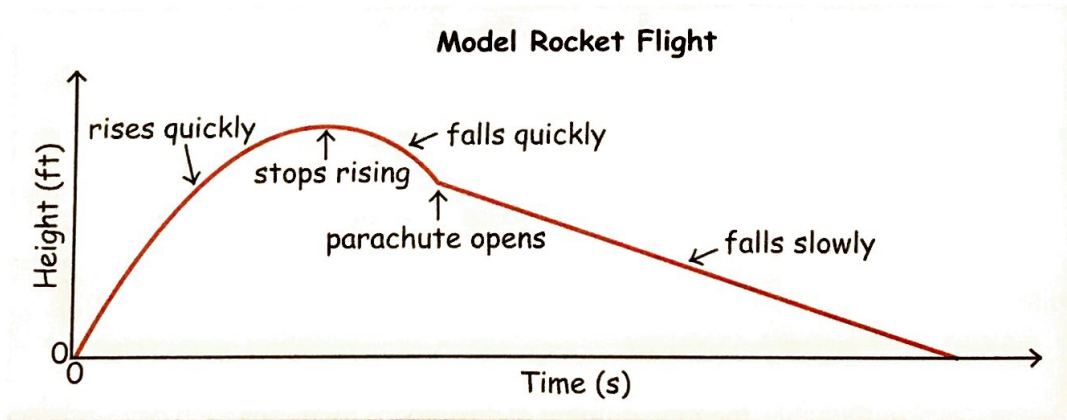
Problem 3 Sketching a Graph STEM

Rocketry A model rocket rises quickly and then slows to a stop as its fuel burns out. It begins to fall quickly until the parachute opens, after which it falls slowly back to Earth. What sketch of a graph could represent the height of the rocket during its flight? Label each section.

Think

How can you get started?

Identify the two variables that are being related, such as *height* and *time*. Then look for key words that describe the relationship, such as *rises quickly* or *falls slowly*.

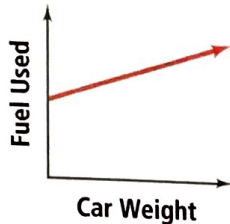


- Got It?** 3. a. Suppose you start to swing yourself on a playground swing. You move back and forth and swing higher in the air. Then you slowly swing to a stop. What sketch of a graph could represent how your height from the ground might change over time? Label each section.
- b. **Reasoning** If you jumped from the swing instead of slowly swinging to a stop, how would the graph in part (a) be different? Explain.

Lesson Check

Do you know HOW?

1. What are the variables in the graph at the right? Use the graph to describe how the variables are related.
2. Describe the relationship between time and temperature in the table below.

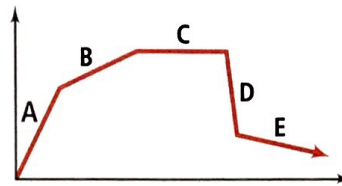


Time (number of hours after noon)	1	3	5	7
Temperature (°F)	61	62	58	51

Do you UNDERSTAND?



3. Match one of the labeled segments in the graph below with each of the following verbal descriptions: *rising slowly*, *constant*, and *falling quickly*.



4. **Reasoning** Describe a real-world relationship that could be represented by the graph sketched above.

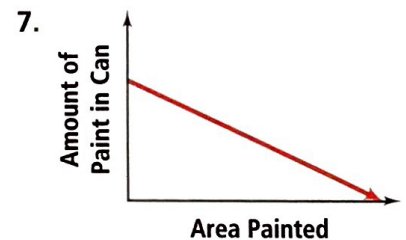
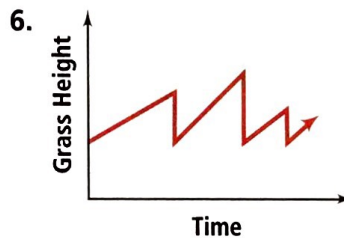
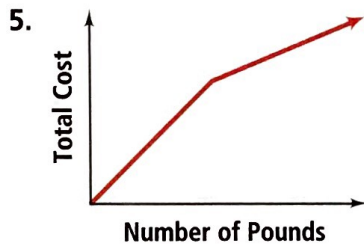
Practice and Problem-Solving Exercises



A Practice

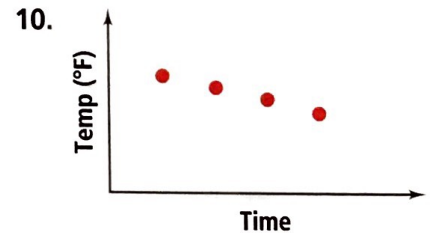
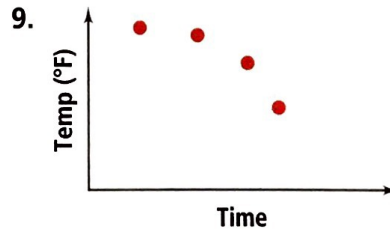
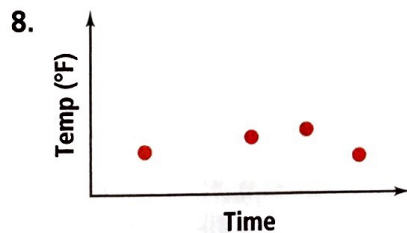
What are the variables in each graph? Describe how the variables are related at various points on the graph.

See Problem 1.



Match each graph with its related table. Explain your answers.

See Problem 2.



A.

Time	Temperature (°F)
1 P.M.	91°
3 P.M.	89°
5 P.M.	81°
7 P.M.	64°

B.

Time	Temperature (°F)
1 P.M.	61°
3 P.M.	60°
5 P.M.	59°
7 P.M.	58°

C.

Time	Temperature (°F)
1 P.M.	24°
3 P.M.	26°
5 P.M.	27°
7 P.M.	21°

Sketch a graph to represent each situation. Label each section.

11. hours of daylight each day over the course of one year
12. your distance from the ground as you ride a Ferris wheel
13. your pulse rate as you watch a scary movie

B Apply

14. **Think About a Plan** The *shishi-odoshi*, a popular Japanese garden ornament, was originally designed to frighten away deer. Using water, it makes a sharp rap each time a bamboo tube rises. Sketch a graph that could represent the volume of water in the bamboo tube as it operates.



Tube begins filling.



Full tube begins falling.



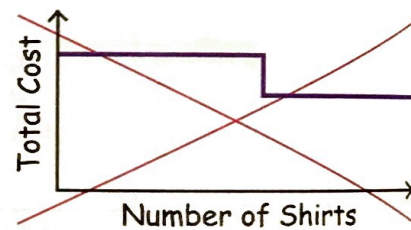
Tube falls and empties water.



Tube rises and hits rock, making noise.

- What quantities vary in this situation?
- How are these quantities related?

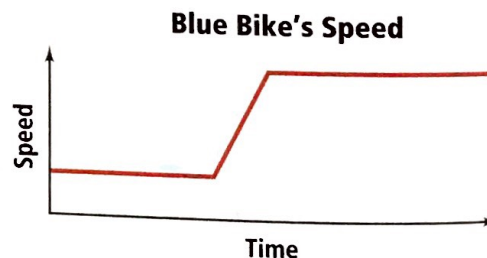
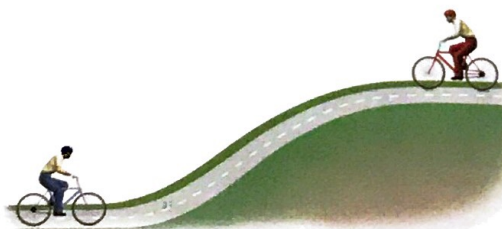
15. **Error Analysis** T-shirts cost \$12.99 each for the first 5 shirts purchased. Each additional T-shirt costs \$4.99 each. Describe and correct the error in the graph at the right that represents the relationship between total cost and number of shirts purchased.



16. **Open-Ended** Describe a real-world relationship between the area of a rectangle and its width, as the width varies and the length stays the same. Sketch a graph to show this relationship.

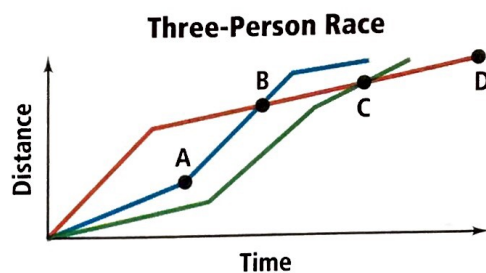
17. **Skiing** Sketch a graph of each situation. Are the graphs the same? Explain.
- a. your speed as you travel on a ski lift from the bottom of a ski slope to the top
 - b. your speed as you ski from the top of a ski slope to the bottom

18. **Reasoning** The diagram at the left below shows a portion of a bike trail.
- a. Explain whether the graph below is a reasonable representation of how the speed might change for the rider of the blue bike.

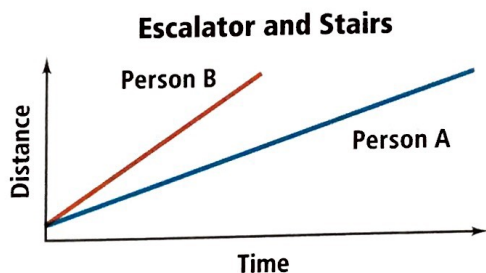


- b. Sketch two graphs that could represent a bike's speed over time. Sketch one graph for the blue bike, and the other for the red bike.

19. **Track** The sketch at the right shows the distance three runners travel during a race. Describe what occurs at times A, B, C, and D. In what order do the runners finish? Explain.



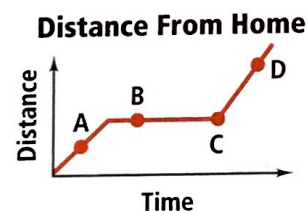
20. **Reasoning** The graph at the right shows the vertical distance traveled as Person A walks up a set of stairs and Person B walks up an escalator next to the stairs. Copy the graph. Then draw a line that could represent the vertical distance traveled as Person C rides the escalator standing still. Explain your reasoning.



Standardized Test Prep

SAT/ACT

21. The graph at the right shows your distance from home as you walk to the bus stop, wait for the bus, and then ride the bus to school. Which point represents a time that you are waiting for the bus?



- (A) A (C) C
(B) B (D) D

22. What is the solution of $-2x < 4$?

- (F) $x < 2$ (G) $x > 2$ (H) $x < -2$ (I) $x > -2$

Short Response

23. You earn \$8.50 per hour. Then you receive a raise to \$9.35 per hour. Find the percent increase. Then find your pay per hour if you receive the same percent increase two more times. Show your work.

Mixed Review

Let $A = \{-3, 1, 4\}$, $B = \{x \mid x \text{ is an odd number greater than } -2 \text{ and less than } 10\}$, and $C = \{1, 4, 7, 12\}$. Find each union or intersection.

24. $A \cup B$

25. $A \cap B$

26. $B \cup C$

27. $A \cap C$

Get Ready! To prepare for Lesson 4-2, do Exercises 28 and 29.

Use a table, an equation, and a graph to represent each relationship.

28. Donald is 4 years older than Connie.

29. You make 3 cards per hour.

◀ See Lesson 3-8.

◀ See Lesson 1-9.