

4-2

Patterns and Linear Functions

Common Core State Standards

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **Also F-IF.B.4**

MP 1, MP 2, MP 3, MP 4

Objective To identify and represent patterns that describe linear functions



One relationship is between the length of a shadow and the time of day.



SOLVE IT! Getting Ready!

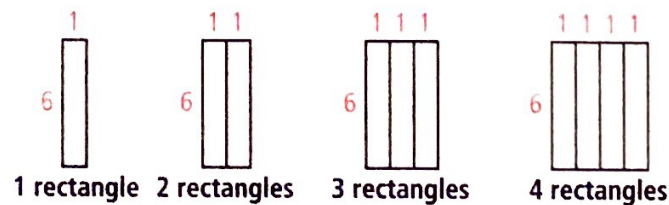
Identify quantities in the picture that vary in response to other quantities. Describe each relationship.

In the Solve It, you identified variables whose value *depends* on the value of another variable. In a relationship between variables, the **dependent variable** changes in response to another variable, the **independent variable**. Values of the independent variable are called **inputs**. Values of the dependent variable are called **outputs**.

Essential Understanding The value of one variable may be uniquely determined by the value of another variable. Such relationships may be represented using tables, words, equations, sets of ordered pairs, and graphs.

Problem 1 Representing a Geometric Relationship

In the diagram below, what is the relationship between the number of rectangles and the perimeter of the figure they form? Represent this relationship using a table, words, an equation, and a graph.



Lesson Vocabulary

- dependent variable
- independent variable
- input
- output
- function
- linear function

Think

Which variable is the dependent variable? The perimeter depends on the number of rectangles, so perimeter is the dependent variable.

- Step 1** Make a table. Use x as the independent variable and y as the dependent variable.
Let x = the number of rectangles.
Let y = the perimeter of the figure.

Number of Rectangles, x	Perimeter, y	Ordered Pair (x, y)
1	$2(1) + 2(6) = 14$	(1, 14)
2	$2(2) + 2(6) = 16$	(2, 16)
3	$2(3) + 2(6) = 18$	(3, 18)
4	$2(4) + 2(6) = 20$	(4, 20)

Write each pair of input and output values x and y as an ordered pair (x, y) .

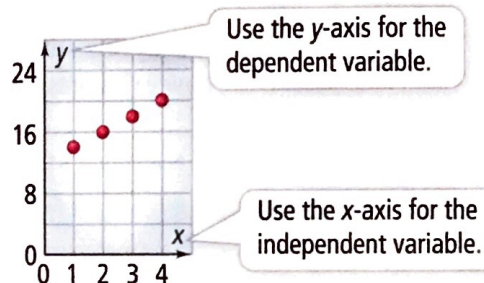
- Step 2** Look for a pattern in the table. Describe the pattern in words so you can write an equation to represent the relationship.

Words Multiply the number of rectangles in each figure by 2 to get the total length of the top and bottom sides of the combined figure. Then add $2(6)$, or 12, for the total length of the left and right sides of the combined figure to get the entire perimeter.

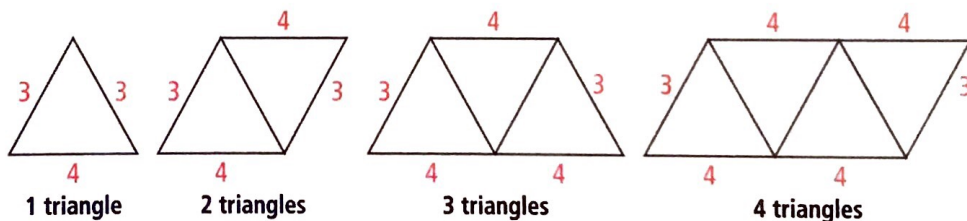
Equation $y = 2x + 12$

- Step 3** Use the table to make a graph.

With a graph, you can see a pattern formed by the relationship between the number of rectangles and the perimeter of the combined figure.



- Got It?** 1. a. In the diagram below, what is the relationship between the number of triangles and the perimeter of the figure they form? Represent this relationship using a table, words, an equation, and a graph.

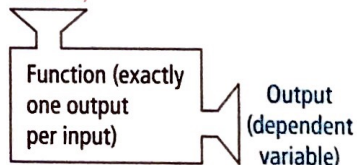


- b. **Reasoning** Suppose you know the perimeter of n triangles. What would you do to find the perimeter of $n + 1$ triangles?
c. How does your answer to part (b) relate to the equation you wrote in part (a)?

You can describe the relationship in Problem 1 by saying that the perimeter is a function of the number of rectangles. A **function** is a relationship that pairs each input value with exactly one output value.

You have seen that one way to represent a function is with a graph. A **linear function** is a function whose graph is a nonvertical line or part of a nonvertical line.

Input
(independent
variable)



Problem 2 Representing a Linear Function

Photography The table shows the relationship between the number of photos x you take and the amount of memory y in megabytes (MB) left on your camera's memory chip. Is the relationship a linear function? Describe the relationship using words, an equation, and a graph.

Camera Memory

Number of Photos, x	Memory (MB), y
0	512
1	509
2	506
3	503

Know

The amount of memory left given the number of pictures taken, as shown in the table

Need

Other representations that describe the relationship

Plan

Look for a pattern that you can describe in words to write an equation. Make a graph to show the pattern.

Think

How can you tell whether a relationship in a table is a function?

If each input is paired with exactly one output, then the relationship is a function.

The amount y of memory left is uniquely determined by the number x of photos you take. You can see this in the table above, where each input value of x corresponds to exactly one output value of y . So y is a function of x . To describe the relationship, look at how y changes for each change in x in the table below.

Camera Memory

Memory is 512 MB before any photos are taken.

The independent variable x increases by 1 each time.

+1
+1
+1

Number of Photos, x	Memory (MB), y
0	512
1	509
2	506
3	503

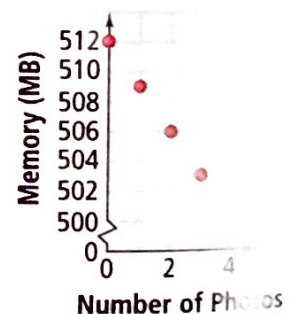
-3
-3
-3

The dependent variable y decreases by 3 each time x increases by 1.

Words The amount of memory left on the chip is 512 minus the quantity 3 times the number of photos taken.

Equation $y = 512 - 3x$

Graph You can use the table to make a graph. The points lie on a line, so the relationship between the number of photos taken and the amount of memory remaining is a linear function.



- Got It?** 2. a. Is the relationship in the table below a linear function? Describe the relationship using words, an equation, and a graph.

Input, x	0	1	2	3
Output, y	8	10	12	14

- b. **Reasoning** Does the set of ordered pairs $(0, 2)$, $(1, 4)$, $(3, 5)$, and $(1, 8)$ represent a linear function? Explain.

Lesson Check

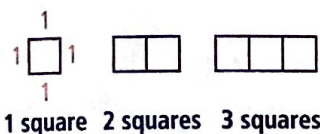
Do you know HOW?

1. Graph each set of ordered pairs. Use words to describe the pattern shown in the graph.

- $(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)$
- $(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)$
- $(3, 0), (3, 1), (3, 2), (3, 3), (3, 4)$

2. Use the diagram below.

Copy and complete the table showing the relationship between the number of squares and the perimeter of the figure they form.

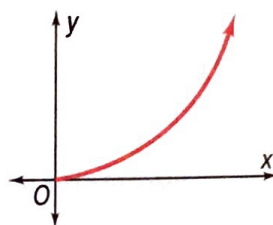


Number of Squares	Perimeter
1	4
2	6
3	■
4	■
10	■
■	62
n	■

Do you UNDERSTAND?



- Vocabulary** The amount of toothpaste in a tube decreases each time you brush your teeth. Identify the independent and dependent variables in this relationship.
- Reasoning** Tell whether each set of ordered pairs in Exercise 1 represents a function. Justify your answers.
- Reasoning** Does the graph below represent a linear function? Explain.



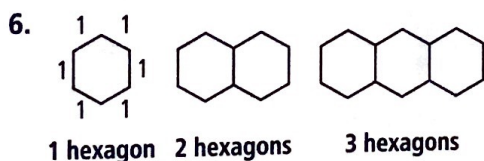
Practice and Problem-Solving Exercises



A Practice

For each diagram, find the relationship between the number of shapes and the perimeter of the figure they form. Represent this relationship using a table, words, an equation, and a graph.

See Problem 1.



For each table, determine whether the relationship is a linear function. Then represent the relationship using words, an equation, and a graph.

See Problem 2.

8.

x	y
0	5
1	8
2	11
3	14

9.

x	y
0	-3
1	2
2	7
3	12

10.

x	y
0	43
1	32
2	21
3	10

For each table, determine whether the relationship is a linear function. Then represent the relationship using words, an equation, and a graph.

11. **Mountain Climbing**

Number of Hours Climbing, x	Elevation (ft), y
0	1127
1	1219
2	1311
3	1403

12. **Grocery Bill**

Number of Soup Cans, x	Total Bill, y
0	\$52.07
1	\$53.36
2	\$54.65
3	\$55.94

13. **Gas in Tank**

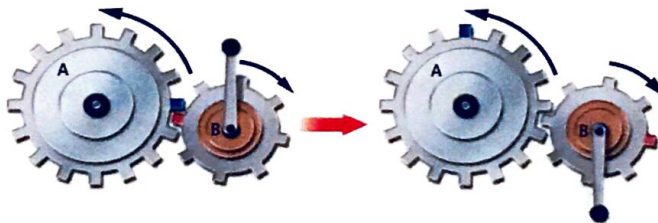
Miles Traveled, x	Gallons of Gas, y
0	11.2
17	10.2
34	9.2
51	8.2

B Apply

14. **Gardening** You can make 5 gal of liquid fertilizer by mixing 8 tsp of powdered fertilizer with water. Represent the relationship between the teaspoons of powder used and the gallons of fertilizer made using a table, an equation, and a graph. Is the amount of fertilizer made a function of the amount of powder used? Explain.

© 15. **Reasoning** Graph the set of ordered pairs $(-2, -3)$, $(0, -1)$, $(1, 0)$, $(3, 2)$, and $(4, 4)$. Determine whether the relationship is a linear function. Explain how you know.

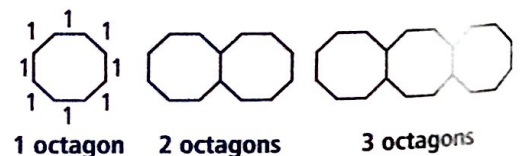
© 16. **Think About a Plan** Gears are common parts in many types of machinery. In the diagram below, Gear A turns in response to the cranking of Gear B. Describe the relationship between the number of turns of Gear B and the number of turns of Gear A. Use words, an equation, and a graph.



- What are the independent and dependent variables?
- How much must you turn Gear B to get Gear A to go around once?

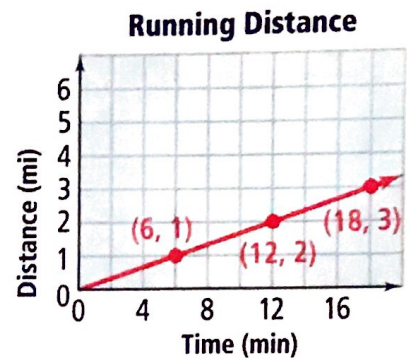
STEM 17. **Electric Car** An automaker makes a car that can travel 40 mi on its charged battery before it begins to use gas. Then the car travels 50 mi per gallon of gas used. Represent the relationship between the amount of gas used and the distance traveled using a table, an equation, and a graph. Is total distance traveled a function of the amount of gas used? What are the independent and dependent variables? Explain.

© 18. **Reasoning** Suppose you know the perimeter of n octagons arranged as shown. What would you do to find the perimeter if 1 more octagon was added?





- 19. Athletics** The graph at the right shows the distance a runner has traveled as a function of the amount of time (in minutes) she has been running. Draw a graph that shows the time she has been running as a function of the distance she has traveled.
- 20. Movies** When a movie on film is projected, a certain number of frames pass through the projector per minute. You say that the length of the movie in minutes is a function of the number of frames. Someone else says that the number of frames is a function of the length of the movie. Can you both be right? Explain.



Apply What You've Learned



Look back at the table on page 233 showing the number of months and number of subscribers for Jayden's blog.

- What is the relationship between the number of months and the number of subscribers? Explain.
- Which column in the table represents the independent variable, and which column represents the dependent variable? Explain.
- Is this relationship a function? Explain.
- Look for a pattern in the table, and then write an equation to represent the relationship you find.
- Use the pattern you found in part (d) to expand the table to 8 months. Then use your equation from part (d) to find the number of subscribers for the 8th month and compare your result with the value in the table.
- Which method would be best to find the number of subscribers after a year? Explain.