5-1 Rate of Change and Slope

Common Core State Standards

F-LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **Also F-IF.B.6**

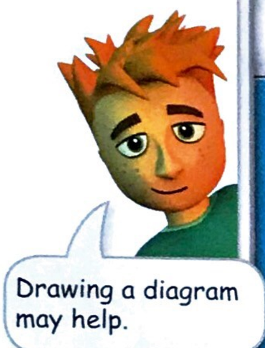
MP 1, MP 2, MP 3, MP 4

Objectives To find rates of change from tables
To find slope

SOLVE IT! **Getting Ready!**

The table shows the horizontal and vertical distances from the base of the mountain at several poles along the path of a ski lift. The poles are connected by cable. Between which two poles is the cable's path the steepest? How do you know?

Pole	Horizontal Distance	Vertical Distance
A	20	30
B	40	35
C	60	60
D	100	70



MATHEMATICAL PRACTICES

Lesson Vocabulary
- rate of change
- slope

Essential Understanding You can use ratios to show a relationship between changing quantities, such as vertical and horizontal change.

Rate of change shows the relationship between two changing quantities. When one quantity depends on the other, the following is true.

$$\text{rate of change} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

Problem 1 Finding Rate of Change Using a Table

Think

Does this problem look like one you've seen before?
Yes. In Lesson 2-6, you wrote rates and unit rates. The rate of change in Problem 1 is an example of a unit rate.

Marching Band **The table shows the distance a band marches over time. Is the rate of change in distance with respect to time constant? What does the rate of change represent?**

Distance Marched

Time (min)	Distance (ft)
1	260
2	520
3	780
4	1040

$$\text{rate of change} = \frac{\text{change in distance}}{\text{change in time}}$$

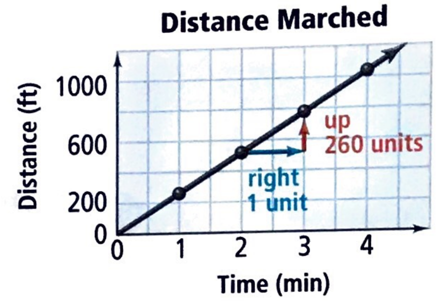
Calculate the rate of change from one row of the table to the next.

$$\frac{520-260}{2-1} = \frac{260}{1} \qquad \frac{780-520}{3-2} = \frac{260}{1} \qquad \frac{1040-780}{4-3} = \frac{260}{1}$$

The rate of change is constant and equals $\frac{260 \text{ ft}}{1 \text{ min}}$. It represents the distance the band marches per minute.

Got It? 1. In Problem 1, do you get the same rate of change if you use nonconsecutive rows of the table? Explain.

The graphs of the ordered pairs (time, distance) in Problem 1 lie on a line, as shown at the right. The relationship between time and distance is linear. When data are linear, the rate of change is constant.



Notice also that the rate of change found in Problem 1 is just the ratio of the vertical change (or *rise*) to the horizontal change (or *run*) between two points on the line. The rate of change is called the *slope* of the line.

$$\textbf{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

Problem 2 Finding Slope Using a Graph

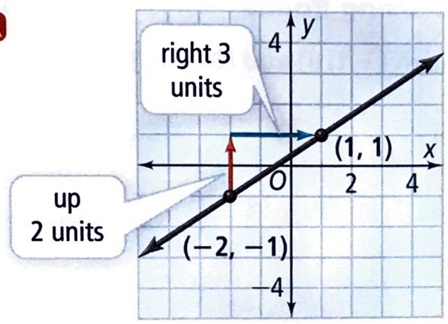
Plan

What do you need to find the slope?

You need to find the rise and run. You can use the graph to count units of rise and units of run.

What is the slope of each line?

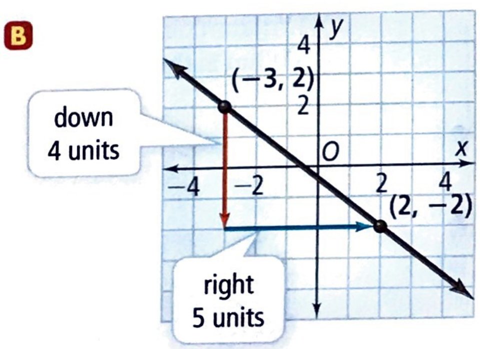
A



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{3}$$

The slope of the line is $\frac{2}{3}$.

B

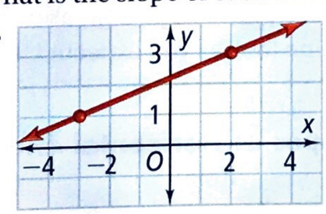


$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-4}{5} = -\frac{4}{5}$$

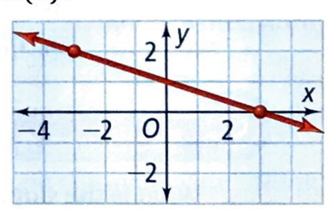
The slope of the line is $-\frac{4}{5}$.

Got It? **2.** What is the slope of each line in parts (a) and (b)?

a.



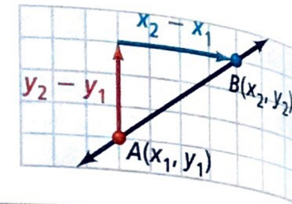
b.



c. **Reasoning** In part (A) of Problem 2, pick two new points on the line to find the slope. Do you get the same slope?

Notice that the line in part (A) of Problem 2 has a positive slope and slants upward from left to right. The line in part (B) of Problem 2 has a negative slope and slopes downward from left to right.

You can use any two points on a line to find its slope. Use subscripts to distinguish between the two points. In the diagram, (x_1, y_1) are the coordinates of point A, and (x_2, y_2) are the coordinates of point B. To find the slope of $\overleftrightarrow{AB}$, you can use the *slope formula*.



take note

Key Concept The Slope Formula

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0$$

The x-coordinate you use first in the denominator must belong to the same ordered pair as the y-coordinate you use first in the numerator.

Problem 3 Finding Slope Using Points

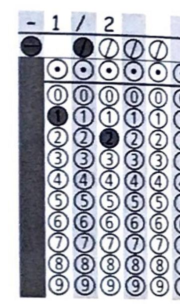
GRIDDED RESPONSE

What is the slope of the line through $(-1, 0)$ and $(3, -2)$?

Plan

Does it matter which point is (x_1, y_1) and which is (x_2, y_2)?
No. You can pick either point for (x_1, y_1) in the slope formula. The other point is then (x_2, y_2).

Think	Write
You need the slope, so start with the slope formula.	$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute $(-1, 0)$ for (x_1, y_1) and $(3, -2)$ for (x_2, y_2).	$= \frac{-2 - 0}{3 - (-1)}$
Simplify to find the answer to place on the grid.	$= \frac{-2}{4} = -\frac{1}{2}$



Got It? 3. a. What is the slope of the line through $(1, 3)$ and $(4, -1)$?

b. Reasoning Plot the points in part (a) and draw a line through them. Does the slope of the line look as you expected it to? Explain.

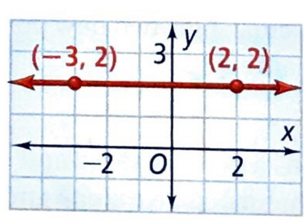
Problem 4 Finding Slopes of Horizontal and Vertical Lines

What is the slope of each line?

Think

Can you generalize these results?
Yes. All points on a horizontal line have the same y-value, so the slope is always zero. Finding the slope of a vertical line always leads to division by zero. The slope is always undefined.

A

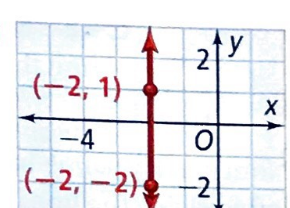


Let $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (2, 2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{2 - (-3)} = \frac{0}{5} = 0$$

The slope of the horizontal line is 0.

B



Let $(x_1, y_1) = (-2, -2)$ and $(x_2, y_2) = (-2, 1)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{-2 - (-2)} = \frac{3}{0}$$

Division by zero is undefined. The slope of the vertical line is undefined.

Got It? **4.** What is the slope of the line through the given points?

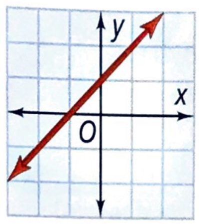
a. (4, −3), (4, 2)   **b.** (−1, −3), (5, −3)

The following summarizes what you have learned about slope.

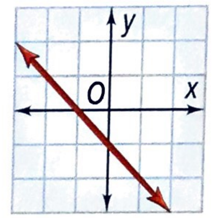
take note

Concept Summary Slopes of Lines

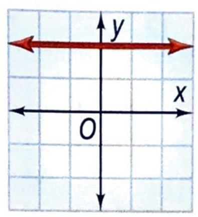
A line with positive slope slants upward from left to right.



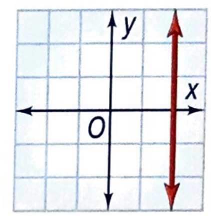
A line with negative slope slants downward from left to right.



A line with a slope of 0 is horizontal.



A line with an undefined slope is vertical.



Lesson Check

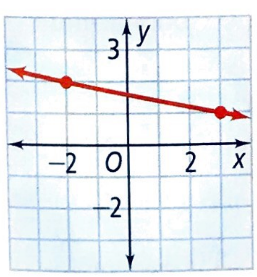
Do you know HOW?

1. Is the rate of change in cost constant with respect to the number of pencils bought? Explain.

Cost of Pencils

Number of Pencils	1	4	7	12
Cost ($)	0.25	1	1.75	3

2. What is the slope of the line?



3. What is the slope of the line through (−1, 2) and (2, −3)?

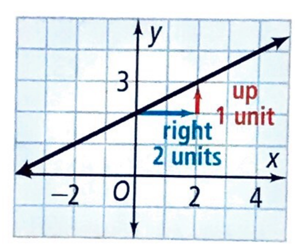
Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** What characteristic of a graph represents the rate of change? Explain.

5. **Open-Ended** Give an example of a real-world situation that you can model with a horizontal line. What is the rate of change for the situation? Explain.

6. **Compare and Contrast** How does finding a line's slope by counting units of vertical and horizontal change on a graph compare with finding it using the slope formula?

7. **Error Analysis** A student calculated the slope of the line at the right to be 2. Explain the mistake. What is the correct slope?



Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Determine whether each rate of change is constant. If it is, find the rate of change and explain what it represents.

See Problem 1.

8. Turtle Walking

Time (min)	Distance (m)
1	6
2	12
3	15
4	21

9. Hot Dogs and Buns

Hot Dogs	Buns
1	1
2	2
3	3
4	4

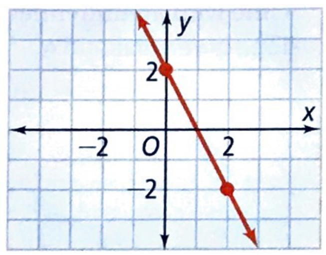
10. Airplane Descent

Time (min)	Elevation (ft)
0	30,000
2	29,000
5	27,500
12	24,000

Find the slope of each line.

See Problem 2.

11.



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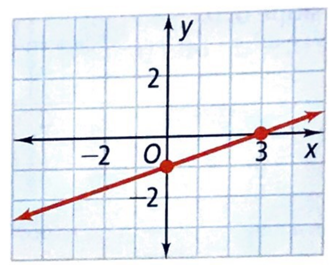

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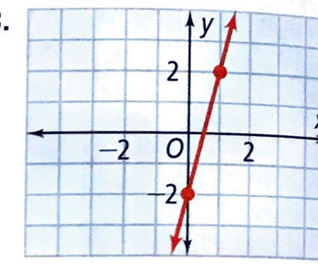

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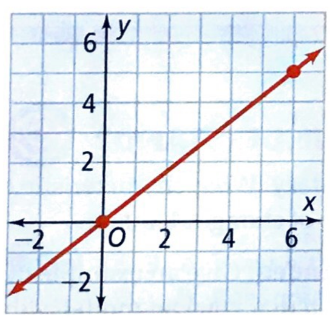

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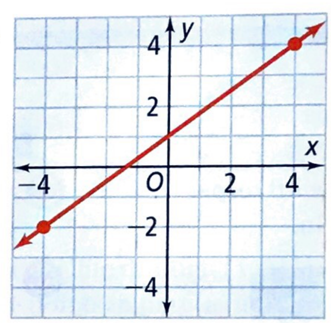

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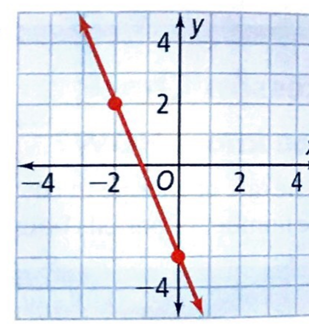


Find the slope of the line that passes through each pair of points.

See Problem 3.

17. (0, 0), (3, 3)

18. (1, 3), (5, 5)

19. (4, 4), (5, 3)

20. (0, −1), (2, 3)

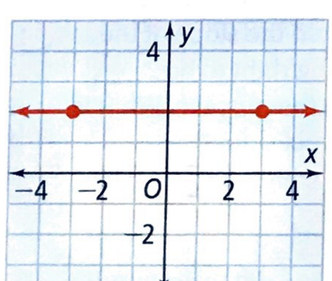
21. (−6, 1), (4, 8)

22. (2, −3), (5, −4)

Find the slope of each line.

See Problem 4.

23.



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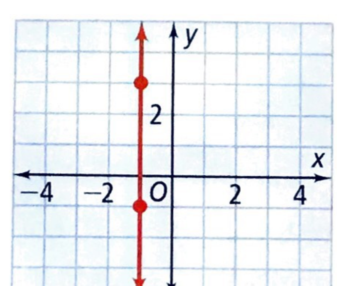


25.

B Apply

Without graphing, tell whether the slope of a line that models each linear relationship is *positive, negative, zero,* or *undefined.* Then find the slope.

26. The length of a bus route is 4 mi long on the sixth day and 4 mi long on the seventeenth day.

27. A babysitter earns \$9 for 1 h and \$36 for 4 h.

28. A student earns a 98 on a test for answering one question incorrectly and earns a 90 for answering five questions incorrectly.

29. The total cost, including shipping, for ordering five uniforms is \$66. The total cost, including shipping, for ordering nine uniforms is \$114.

State the independent variable and the dependent variable in each linear relationship. Then find the rate of change for each situation.

30. Snow is 0.02 m deep after 1 h and 0.06 m deep after 3 h.

31. The cost of tickets is \$36 for three people and \$84 for seven people.

32. A car is 200 km from its destination after 1 h and 80 km from its destination after 3 h.

Use the slope formula to find the slope of the line that passes through each pair of points. Then plot the points and sketch the line that passes through them. Does the slope you found using the formula match the direction of the line you sketched?

33. $(-2, 1), (7, 1)$

34. $(4.25, 0), (3.5, 3)$

35. $\left(-\frac{1}{2}, \frac{4}{7}\right), \left(8, \frac{4}{7}\right)$

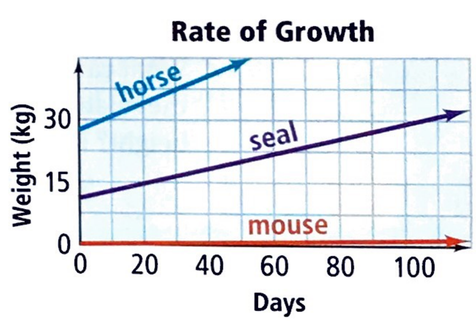
36. $(-5, 0.124), (-5, -0.584)$

37. $(-42.25, 5.2), (3.25, 3)$

38. $\left(-2, \frac{2}{11}\right), \left(-2, \frac{7}{13}\right)$

39. Think About a Plan The graph shows the average growth rates for three different animals. Which animal's growth shows the fastest rate of change? The slowest rate of change?

- How can you use the graph to find the rates of change?
- Are your answers reasonable?



40. Open-Ended Find two points that lie on a line with slope -9.

41. Profit John's business made \$4500 in January and \$8600 in March. What is the rate of change in his profit for this time period?

Each pair of points lies on a line with the given slope. Find *x* or *y*.

42. $(2, 4), (x, 8)$; slope $= -2$

43. $(4, 3), (5, y)$; slope $= 3$

44. $(2, 4), (x, 8)$; slope $= -\frac{1}{2}$

45. $(3, y), (1, 9)$; slope $= -\frac{5}{2}$

46. $(-4, y), (2, 4y)$; slope $= 6$

47. $(3, 5), (x, 2)$; undefined slope

48. Reasoning Is it true that a line with slope 1 always passes through the origin? Explain your reasoning.

49. **Arithmetic Sequences** Use the arithmetic sequence 10, 15, 20, 25, . . .

a. Find the common difference of the sequence.

b. Let x = the term number, and let y = the corresponding term of the sequence. Graph the ordered pairs (x, y) for the first eight terms of the sequence. Draw a line through the points.

c. **Reasoning** How is the slope of a line from part (b) related to the common difference of the sequence?

Challenge

Do the points in each set lie on the same line? Explain your answer.

50. $A(1, 3), B(4, 2), C(-2, 4)$

51. $G(3, 5), H(-1, 3), I(7, 7)$

52. $D(-2, 3), E(0, -1), F(2, 1)$

53. $P(4, 2), Q(-3, 2), R(2, 5)$

54. $G(1, -2), H(-1, -5), I(5, 4)$

55. $S(-3, 4), T(0, 2), X(-3, 0)$

Find the slope of the line that passes through each pair of points.

56. $(a, -b), (-a, -b)$

57. $(-m, n), (3m, -n)$

58. $(2a, b), (c, 2d)$

PERFORMANCE TASK

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1

The table at the right shows the height of water in a glass when different numbers of marbles are dropped into it.

a. Find the rate of change in the water height with respect to the number of marbles from one row in the table to the next. What do you notice?

b. For each marble you add to the glass, how much does the water level rise?

c. What can you conclude about the type of function that models the relationship between the number of marbles and the water height? Explain.

Number of Marbles, x	Height of Water (cm), y
3	6.9
5	7.5
9	8.7
14	10.2
17	11.1
23	12.9