

5-3

Slope-Intercept Form

© Common Core State Standards

F-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. Also A-SSE.A.1a, A-CED.A.2, F-IF.B.4, F-LE.A.2

MP 1, MP 2, MP 3, MP 4

Objectives To write linear equations using slope-intercept form
To graph linear equations in slope-intercept form



Each point of the graph gives you information about the bamboo plant.



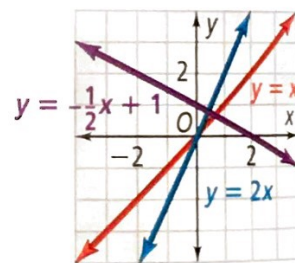
Getting Ready!

Bamboo can grow very quickly. The graph models the growth of a bamboo plant. Find the point where the line crosses the vertical axis. What does this point tell you about the bamboo plant? Find the slope of the line. What does the slope tell you about the bamboo plant? How do you know?



The function in the Solve It is a linear function, but it is not a direct variation. Direct variations are only part of the family of linear functions.

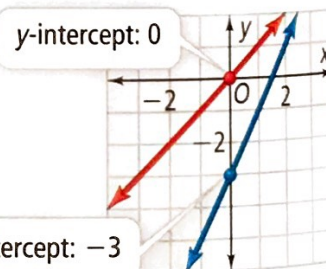
A family of functions is a group of functions with common characteristics. A **parent function** is the simplest function with these characteristics. The **linear parent function** is $y = x$ or $f(x) = x$. The graphs of three linear functions are shown at the right.



A **linear equation** is an equation that models a linear function. In a linear equation, the variables cannot be raised to a power other than 1. So $y = 2x$ is a linear equation, but $y = x^2$ and $y = 2^x$ are not. The graph of a linear equation contains all the ordered pairs that are solutions of the equation.

Graphs of linear functions may cross the y -axis at any point. A **y -intercept** of a graph is the y -coordinate of a point where the graph crosses the y -axis.

Essential Understanding You can use the slope and y -intercept of a line to write and graph an equation of the line.



Take note

Key Concept Slope-Intercept Form of a Linear Equation

The slope-intercept form of a linear equation of a nonvertical line is $y = mx + b$.

↑ ↑
slope y -intercept



Lesson Vocabulary

- parent function
- linear parent function
- linear equation
- y -intercept
- slope-intercept form

Problem 1 Identifying Slope and y-Intercept

What are the slope and y-intercept of the graph of $y = 5x - 2$?

$$y = mx + b \quad \text{Use slope-intercept form.}$$

slope

y-intercept

$$y = 5x + (-2) \quad \text{Think of } y = 5x - 2 \text{ as } y = 5x + (-2).$$

The slope is 5; the y-intercept is -2 .

- Got It?** 1. a. What are the slope and y-intercept of the graph of $y = -\frac{1}{2}x + \frac{2}{3}$?
 b. **Reasoning** How do the graph of the line and the equation in part (a) change if the y-intercept is moved down 3 units?

Problem 2 Writing an Equation in Slope-Intercept Form

What is an equation of the line with slope $-\frac{4}{5}$ and y-intercept 7?

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$y = -\frac{4}{5}x + 7 \quad \text{Substitute } -\frac{4}{5} \text{ for } m \text{ and } 7 \text{ for } b.$$

An equation for the line is $y = -\frac{4}{5}x + 7$.

- Got It?** 2. What is an equation of the line with slope $\frac{3}{2}$ and y-intercept -1 ?

Problem 3 Writing an Equation From a Graph

Multiple Choice Which equation represents the line shown?

(A) $y = -2x + 1$

(C) $y = \frac{1}{2}x - 2$

(B) $y = 2x + 1$

(D) $y = 2x - 2$

Find the slope. Two points on the line are $(0, -2)$ and $(2, 2)$.

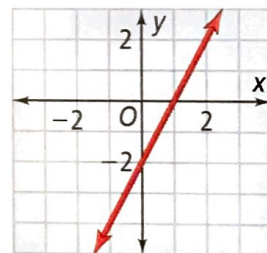
$$\text{slope} = \frac{2 - (-2)}{2 - 0} = \frac{4}{2} = 2$$

The y-intercept is -2 . Write an equation in slope-intercept form.

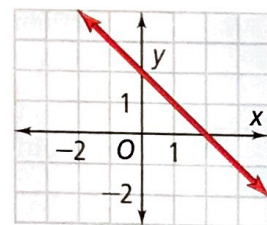
$$y = mx + b$$

$$y = 2x + (-2) \quad \text{Substitute } 2 \text{ for } m \text{ and } -2 \text{ for } b.$$

An equation for the line is $y = 2x - 2$. The correct answer is D.



- Got It?** 3. a. What do you expect the slope of the line to be from looking at the graph? Explain.
 b. What is an equation of the line shown at the right?
 c. **Reasoning** Does the equation of the line depend on the points you use to find the slope? Explain.



Problem 4 Writing an Equation From Two Points

What equation in slope-intercept form represents the line that passes through the points (2, 1) and (5, -8)?

Know

The line passes through (2, 1) and (5, -8).

Need

An equation of the line

Plan

Use the two points to find the slope. Then use the slope and one point to solve for the y-intercept.

Think

Can you use either point to find the y-intercept?

Yes. You can substitute the slope and the coordinates of any point on the line into the form $y = mx + b$ and solve for b .

Step 1 Use the two points to find the slope.

$$\text{slope} = \frac{-8 - 1}{5 - 2} = \frac{-9}{3} = -3$$

Step 2 Use the slope and the coordinates of one of the points to find b .

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$1 = -3(2) + b \quad \text{Substitute } -3 \text{ for } m, 2 \text{ for } x, \text{ and } 1 \text{ for } y.$$

$$7 = b \quad \text{Solve for } b.$$

Step 3 Substitute the slope and y-intercept into the slope-intercept form.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$y = -3x + 7 \quad \text{Substitute } -3 \text{ for } m \text{ and } 7 \text{ for } b.$$

An equation of the line is $y = -3x + 7$.

Got It? 4. What equation in slope-intercept form represents the line that passes through the points (3, -2) and (1, -3)?

You can use the slope and y-intercept from an equation to graph a line.

Problem 5 Graphing a Linear Equation

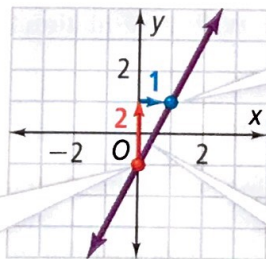
What is the graph of $y = 2x - 1$?

Plan

What information can you use?

The slope tells you the ratio of vertical change to horizontal change. Plot the y-intercept. Then use the slope to plot another point on the line.

Step 1 The y-intercept is -1. So plot a point at (0, -1).



Step 2 The slope is 2, or $\frac{2}{1}$. Move up 2 units and right 1 unit. Plot another point.

Step 3 Draw a line through the two points.

Got It? 5. What is the graph of each linear equation?
 a. $y = -3x + 4$ b. $y = 4x - 8$

Slope-intercept form is useful for modeling real-life situations where you are given a starting value (the y -intercept) and a rate of change (the slope).

At 0 meters, the pressure is 1 atm.

Problem 6 Modeling a Function STEM

Physics Water pressure can be measured in atmospheres (atm). Use the information in the diagram to write an equation that models the pressure y at a depth of x meters. What graph models the pressure?

Step 1 Identify the slope and the y -intercept.

The slope is the rate of change, 0.1 atm/m.

The y -intercept is the starting value, 1 atm.

Step 2 Substitute the slope and y -intercept into the slope-intercept form.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$y = 0.1x + 1 \quad \text{Substitute 0.1 for } m \text{ and 1 for } b.$$

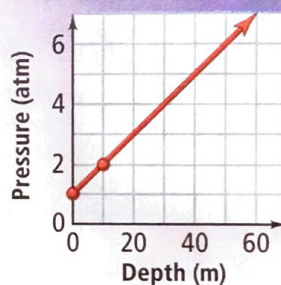
Step 3 Graph the equation.

The y -intercept is 1. Plot the point $(0, 1)$.

The slope is 0.1, which equals $\frac{1}{10}$. Plot a second point 1 unit above and 10 units to the right of the y -intercept. Then draw a line through the two points.

The pressure increases by 0.1 atm/m.

Pressure Underwater



Got It? 6. A plumber charges a \$65 fee for a repair plus \$35 per hour. Write an equation to model the total cost y of a repair that takes x hours. What graph models the total cost?



Lesson Check

Do you know HOW?

1. What is an equation of the line with slope 6 and y -intercept -4 ?
2. What equation in slope-intercept form represents the line that passes through the points $(-3, 4)$ and $(2, -1)$?
3. What is the graph of $y = 5x + 2$?

Do you UNDERSTAND?



4. **Vocabulary** Is $y = 5$ a linear equation? Explain.
5. **Reasoning** Is it *always*, *sometimes*, or *never* true that an equation in slope-intercept form represents a direct variation? Support your answer with examples.
6. **Writing** Describe two different methods you can use to graph the equation $y = 2x + 4$. Which method do you prefer? Explain.

Practice and Problem-Solving Exercises

A Practice

Find the slope and y-intercept of the graph of each equation.

7. $y = 3x + 1$

8. $y = -x + 4$

9. $y = 2x - 5$

10. $y = -3x + 2$

11. $y = 5x - 3$

12. $y = -6x$

13. $y = 4$

14. $y = -0.2x + 3$

15. $y = \frac{1}{4}x - \frac{1}{3}$

Write an equation in slope-intercept form of the line with the given slope m and y-intercept b .

16. $m = 1, b = -1$

17. $m = 3, b = 2$

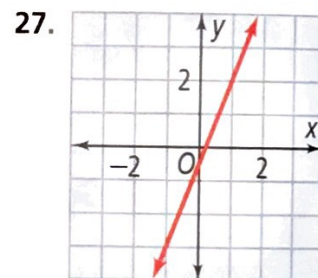
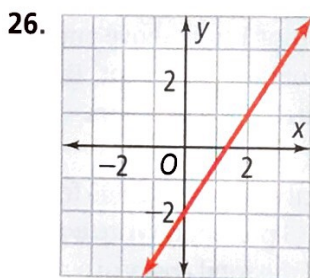
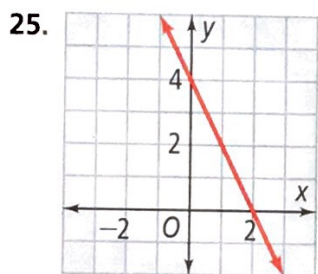
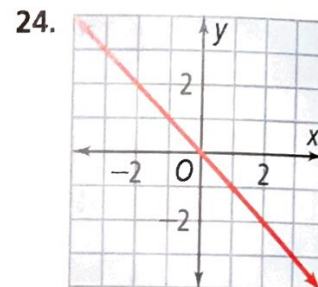
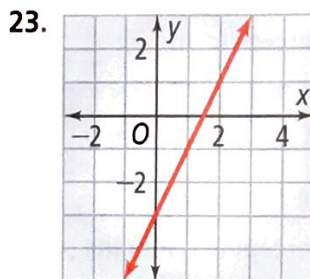
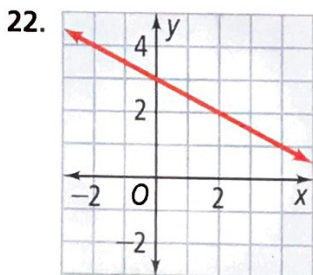
18. $m = \frac{1}{2}, b = -\frac{1}{2}$

19. $m = 0.7, b = -2$

20. $m = -0.5, b = 1.5$

21. $m = -2, b = \frac{8}{5}$

Write an equation in slope-intercept form of each line.



Write an equation in slope-intercept form of the line that passes through the given points.

28. $(0, 3)$ and $(2, 5)$

29. $(-2, 4)$ and $(3, -1)$

30. $(-3, 3)$ and $(1, 2)$

Graph each equation.

31. $y = x + 5$

32. $y = 3x + 4$

33. $y = -2x + 1$

34. **Retail Sales** Suppose you have a \$5-off coupon at a fabric store. You buy fabric that costs \$7.50 per yard. Write an equation that models the total amount of money y you pay if you buy x yards of fabric. What is the graph of the equation?

35. **Temperature** The temperature at sunrise is 65°F . Each hour during the day, the temperature rises 5°F . Write an equation that models the temperature y , in degrees Fahrenheit, after x hours during the day. What is the graph of the equation?

See Problem 1.

See Problem 2.

See Problem 3.

See Problem 4.

See Problem 5.

See Problem 6.

36. Using the tables below, predict whether the two graphs will intersect. Plot the points and sketch the lines. Do the two lines appear to intersect? Explain.

x	y
-2	9
-1	7
0	5
1	3
2	1

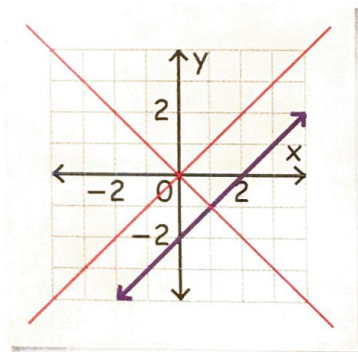
x	y
-2	-18
-1	-14
0	-10
1	-6
2	-2

Find the slope and y-intercept of the graph of each equation.

37. $y - 2 = -3x$ 38. $y + \frac{1}{2}x = 0$ 39. $y - 9x = \frac{1}{2}$ 40. $2y - 6 = 3x$
 41. $-2y = 6(5 - 3x)$ 42. $y - d = cx$ 43. $y = (2 - a)x + a$ 44. $2y + 4n = -6x$

- © 45. **Think About a Plan** Polar bears are listed as a threatened species. In 2005, there were about 25,000 polar bears in the world. If the number of polar bears declines by 1000 each year, in what year will polar bears become extinct?
- What equation models the number of polar bears?
 - How can graphing the equation help you solve the problem?

- © 46. **Error Analysis** A student drew the graph at the right for the equation $y = -2x + 1$. What error did the student make? Draw the correct graph.



- © 47. **Computers** A computer repair service charges \$50 for diagnosis and \$35 per hour for repairs. Let x be the number of hours it takes to repair a computer. Let y be the total cost of the repair.
- Write an equation in slope-intercept form that relates x and y .
 - Graph the equation.
 - Reasoning** Explain why you should draw the line only in Quadrant I.

Use the slope and y-intercept to graph each equation.

48. $y = 7 - 3x$ 49. $2y + 4x = 0$ 50. $3y + 6 = -2x$
 51. $y + 2 = 5x - 4$ 52. $4x + 3y = 2x - 1$ 53. $-2(3x + 4) + y = 0$

Write a recursive formula and an explicit formula in slope-intercept form that model each arithmetic sequence. How does the recursive formula relate to the slope-intercept form?

54. 3, 5, 7, 9, ... 55. -1, 3, 7, 11, ... 56. 0.7, 0.3, -0.1, -0.5, ...

- © 57. **Writing** Describe two ways you can determine whether an equation is linear.

58. **Hobbies** Suppose you are doing a 5000-piece puzzle. You have already placed 175 pieces. Every minute you place 10 more pieces.

- Write an equation in slope-intercept form to model the number of pieces placed. Graph the equation.
- After 50 more minutes, how many pieces will you have placed?

Find the value of a such that the graph of the equation has the given slope m .

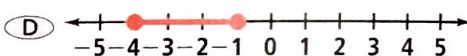
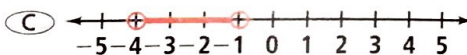
59. $y = 2ax + 4$, $m = -1$ 60. $y = -\frac{1}{2}ax - 5$, $m = \frac{5}{2}$ 61. $y = \frac{3}{4}ax + 3$, $m = \frac{9}{16}$

- 62. Sailing** A sailboat begins a voyage with 145 lb of food. The crew plans to eat a total of 15 lb of food per day.
- Write an equation in slope-intercept form relating the remaining food supply y to the number of days x .
 - Graph your equation.
 - The crew plans to have 25 lb of food remaining when they end their voyage. How many days does the crew expect their voyage to last?

Standardized Test Prep

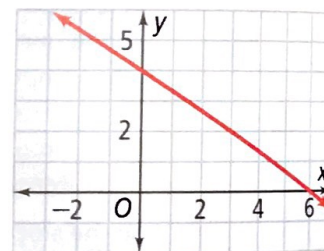
SAT/ACT

- 63.** Which equation represents the line that has slope 5 and passes through the point $(0, -2)$?
- (A) $y = x - 2$ (B) $y = 5x - 2$ (C) $y = -2x - 5$ (D) $y = 5x$
- 64.** What is the slope of the line that passes through the points $(-5, 3)$ and $(1, 7)$?
- (F) $-\frac{5}{3}$ (G) $-\frac{2}{3}$ (H) $\frac{2}{3}$ (I) $\frac{3}{2}$
- 65.** Which number line shows the solution of $|2x + 5| \leq 3$?



- 66.** Which equation represents the graph at the right?

(F) $y = -\frac{3}{2}x + 4$ (H) $y = -\frac{2}{3}x + 4$
 (G) $y = -4x + \frac{3}{2}$ (I) $y = 4x - \frac{2}{3}$



Short Response

- 67.** If a , b , and c are real numbers, $a \neq 0$, and $b > c$, is the statement $ab > ac$ always, sometimes, or never true? Explain.

Mixed Review

Suppose y varies directly with x . Write a direct variation equation that relates x and y . Then find the value of y when $x = 10$.

See Lesson 5-2.

68. $y = 5$ when $x = 1$.

69. $y = 8$ when $x = 4$.

70. $y = 9$ when $x = 3$.

Solve each equation. Justify each step.

See Lesson 2-2.

71. $21 = -2t + 3$

72. $\frac{q}{3} - 3 = 6$

73. $8x + 5 = 61$

Get Ready! To prepare for Lesson 5-4, do Exercises 74-77.

Simplify each expression.

See Lesson 1-7.

74. $-3(x - 5)$

75. $5(x + 2)$

76. $-\frac{4}{9}(x - 6)$

77. $1.5(x + 12)$