

5-4

Point-Slope Form

Common Core State Standards

F-LE.A.2 Construct linear . . . functions . . . given a graph, a description of a relationship, or two input-output pairs. Also A-CED.A.2, F-IF.B.4, F-IF.C.7a, F-LE.B.5

MP 1, MP 3, MP 4

Objective To write and graph linear equations using point-slope form

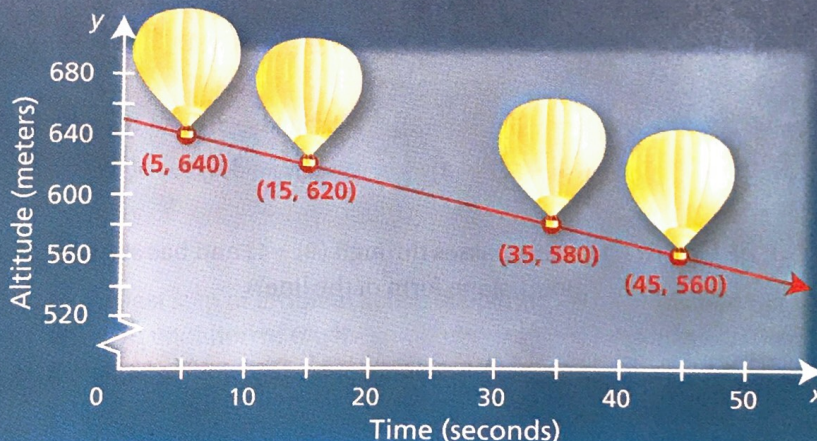


Think about this situation. The graph shows the altitude of the balloon with respect to time. It doesn't show the path of the balloon.



Getting Ready!

The red line shows the altitude of a hot-air balloon during its linear descent. What is an equation of the line in slope-intercept form? (Hint: What is the altitude of the balloon when it starts its descent at $x = 0$?)



You have learned how to write an equation of a line by using its y -intercept. In this lesson, you will learn how to write an equation *without* using the y -intercept.

Essential Understanding You can use the slope of a line and any point on the line to write and graph an equation of the line. Any two equations for the same line are equivalent.

take note

Key Concept Point-Slope Form of a Linear Equation

Definition

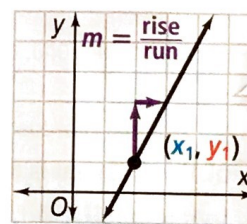
The **point-slope form** of an equation of a nonvertical line with slope m and through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Symbols

$$y - y_1 = m(x - x_1)$$

\uparrow \uparrow \uparrow
 y -coordinate **slope** **x -coordinate**

Graph



When you use $y - y_1 = m(x - x_1)$, (x_1, y_1) represents a *specific* point and (x, y) represents *any* point.

Here's Why It Works Given a point (x_1, y_1) on a line and the line's slope m , you can use the definition of slope to derive point-slope form.

$$\frac{y_2 - y_1}{x_2 - x_1} = m \quad \text{Use the definition of slope.}$$

$$\frac{y - y_1}{x - x_1} = m \quad \text{Let } (x, y) \text{ be any point on the line. Substitute } (x, y) \text{ for } (x_2, y_2).$$

$$\frac{y - y_1}{x - x_1} \cdot (x - x_1) = m(x - x_1) \quad \text{Multiply each side by } (x - x_1).$$

$$y - y_1 = m(x - x_1) \quad \text{Simplify the left side of the equation.}$$

Think

Since you know a point and the slope, use point-slope form.

Problem 1 Writing an Equation in Point-Slope Form

A line passes through $(-3, 6)$ and has slope -5 . What is an equation of the line?

$$y - y_1 = m(x - x_1) \quad \text{Use point-slope form.}$$

$$y_1 = 6 \quad m = -5 \quad x_1 = -3$$

$$y - 6 = -5[x - (-3)] \quad \text{Substitute } (-3, 6) \text{ for } (x_1, y_1) \text{ and } -5 \text{ for } m.$$

$$y - 6 = -5(x + 3) \quad \text{Simplify inside grouping symbols.}$$

Got It? 1. A line passes through $(8, -4)$ and has slope $\frac{2}{3}$. What is an equation in point-slope form of the line?

Plan

How does the equation help you make a graph?

Use the point from the equation. Use the slope from the equation to find another point. Graph using the two points.

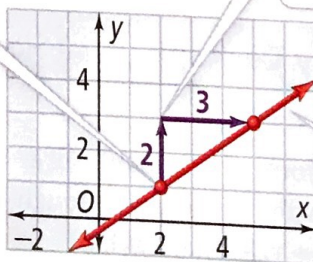
Problem 2 Graphing Using Point-Slope Form

What is the graph of the equation $y - 1 = \frac{2}{3}(x - 2)$?

The equation is in point-slope form, $y - y_1 = m(x - x_1)$. A point (x_1, y_1) on the line is $(2, 1)$, and the slope m is $\frac{2}{3}$.

Step 1 Graph a point at $(2, 1)$.

Step 2 Use the slope, $\frac{2}{3}$. Go up 2 units and right 3 units. Draw a point.



Step 3 Draw a line through the two points.

Got It? 2. What is the graph of the equation $y + 7 = -\frac{4}{5}(x - 4)$?

You can write the equation of a line given any two points on the line. First use the two given points to find the slope. Then use the slope and one of the points to write the equation.

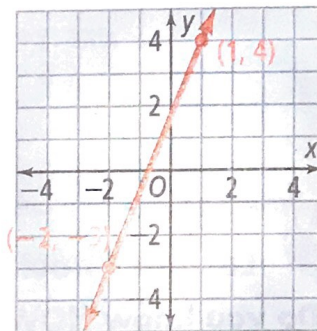
Plan

How does the graph help you write an equation?

You can use two points on the line to find the slope. Then use point-slope form.

Problem 3 Using Two Points to Write an Equation

What is an equation of the line at the right?



Think

You need the slope m , so start with the slope formula.

Write

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use the given points to find the slope.

$$m = \frac{-3 - 4}{-2 - 1} = \frac{-7}{-3} = \frac{7}{3}$$

Use point-slope form.

$$y - y_1 = m(x - x_1)$$

Use either given point for (x_1, y_1) . For example, you can use $(1, 4)$.

$$y - 4 = \frac{7}{3}(x - 1)$$

- Got It?** 3. a. In the last step of Problem 3, use the point $(-2, -3)$ instead of $(1, 4)$ to write an equation of the line.
- b. **Reasoning** Rewrite the equations in Problem 3 and part (a) in slope-intercept form. Compare the two rewritten equations. What can you conclude?

Problem 4 Using a Table to Write an Equation

Recreation The table shows the altitude of a hot-air balloon during its linear descent. What equation in slope-intercept form gives the balloon's altitude at any time? What do the slope and y -intercept represent?

$$m = \frac{590 - 640}{30 - 10} = -2.5 \quad \text{Use two points, such as } (10, 640) \text{ and } (30, 590), \text{ to find the slope.}$$

$$y - y_1 = m(x - x_1) \quad \text{Use point-slope form.}$$

$$y - 640 = -2.5(x - 10) \quad \text{Use the data point } (10, 640) \text{ and the slope } -2.5.$$

$$y = -2.5x + 665 \quad \text{Rewrite in slope-intercept form.}$$

Hot-Air Balloon Descent

Time, x (s)	Altitude, y (m)
10	640
30	590
70	490
90	440

The slope -2.5 represents the rate of descent of the balloon in meters per second. The y -intercept 665 represents the initial altitude of the balloon in meters.

Plan

How does the table help you write an equation?

The table gives four points. You can use any two of the points to find the slope. Then use point-slope form.



- Got It?** 4. a. The table shows the number of gallons of water y in a tank after x hours. The relationship is linear. What is an equation in point-slope form that models the data? What does the slope represent?
- b. **Reasoning** Write the equation from part (a) in slope-intercept form. What does the y -intercept represent?

Volume of Water in Tank

Time, x (h)	Water, y (gal)
2	3320
3	4570
5	7070
8	10,820



Lesson Check

Do you know HOW?

- What are the slope and one point on the graph of $y - 12 = \frac{4}{9}(x + 7)$?
- What is an equation of the line that passes through the point $(3, -8)$ and has slope -2 ?
- What is the graph of the equation $y - 4 = 3(x + 2)$?
- What is an equation of the line that passes through the points $(-1, -2)$ and $(2, 4)$?

Do you UNDERSTAND?



- Vocabulary** What features of the graph of the equation $y - y_1 = m(x - x_1)$ can you identify?
- Reasoning** Is $y - 4 = 3(x + 1)$ an equation of a line through $(-2, 1)$? Explain.
- Reasoning** Can any equation in point-slope form also be written in slope-intercept form? Give an example to explain.



Practice and Problem-Solving Exercises



A Practice

Write an equation in point-slope form of the line that passes through the given point and with the given slope m .

- $(3, -4); m = 6$
- $(4, 2); m = -\frac{5}{3}$
- $(-2, -7); m = \frac{4}{5}$
- $(4, 0); m = -1$

◀ See Problem 1.

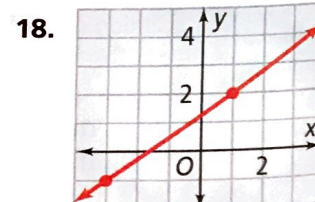
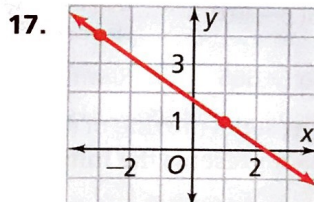
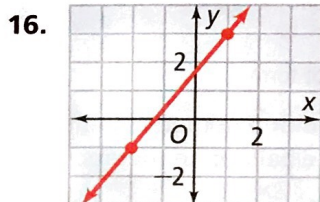
Graph each equation.

- $y + 3 = 2(x - 1)$
- $y - 1 = -3(x + 2)$
- $y + 5 = -(x + 2)$
- $y - 2 = \frac{4}{9}(x - 3)$

◀ See Problem 2.

Write an equation in point-slope form for each line.

◀ See Problem 3.



Write an equation in point-slope form of the line that passes through the given points. Then write the equation in slope-intercept form.

See Problem 4.

19. $(1, 4), (-1, 1)$

20. $(2, 4), (-3, -6)$

21. $(-6, 6), (3, 3)$

Model the data in each table with a linear equation in slope-intercept form. Then tell what the slope and y -intercept represent.

22.

Time Painting, x (days)	Volume of Paint, y (gal)
2	56
3	44
5	20

23.

Time Worked, x (h)	Wages Earned, y (\$)
1	8.50
3	25.50
6	51.00

B Apply

Graph the line that passes through the given point and has the given slope m .

24. $(-3, -2); m = 2$

25. $(6, -1); m = -\frac{5}{3}$

26. $(-3, 1); m = \frac{1}{3}$

© 27. **Think About a Plan** The relationship of degrees Fahrenheit ($^{\circ}\text{F}$) and degrees Celsius ($^{\circ}\text{C}$) is linear. When the temperature is 50°F , it is 10°C . When the temperature is 77°F , it is 25°C . Write an equation giving the Celsius temperature C in terms of the Fahrenheit temperature F . What is the Celsius temperature when it is 59°F ?

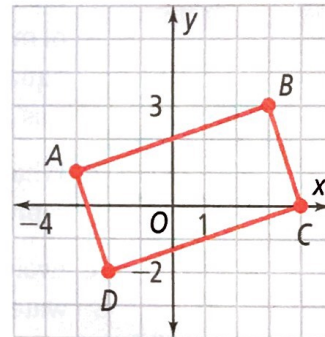
- How can point-slope form help you write the equation?
- What are two points you can use to find the slope?

© 28. a. **Geometry** Figure $ABCD$ is a rectangle. Write equations in point-slope form of the lines containing the sides of $ABCD$.

b. **Reasoning** Make a conjecture about the slopes of parallel lines.

c. Use your conjecture to write an equation of the line that passes through $(0, -4)$ and is parallel to $y - 9 = -7(x + 3)$.

STEM 29. **Boiling Point** The relationship between altitude and the boiling point of water is linear. At an altitude of 8000 ft, water boils at 197.6°F . At an altitude of 4500 ft, water boils at 203.9°F . Write an equation giving the boiling point b of water, in degrees Fahrenheit, in terms of the altitude a , in feet. What is the boiling point of water at 2500 ft?



30. Using a graphing calculator, graph $f(x) = 3x + 2$.

a. If $f(x) = 3x + 2$ and $g(x) = 4f(x)$, write the equation for $g(x)$. Graph $g(x)$ and compare it to the graph of $f(x)$.

b. If $f(x) = 3x + 2$ and $h(x) = f(4x)$, write the equation for $h(x)$. Graph $h(x)$ and compare it to the graph of $f(x)$.

c. Compare how multiplying a function by a number and multiplying the x value of a function by a number change the graphs of the functions.

31. Using a graphing calculator, graph $f(x) = 2x - 5$.

a. If $f(x) = 2x - 5$ and $j(x) = f(x) + 3$, write the equation for $j(x)$. Graph $j(x)$ and compare it to the graph of $f(x)$.

b. If $f(x) = 2x - 5$ and $k(x) = f(x + 3)$, write the equation for $k(x)$. Graph $k(x)$ and compare it to the graph of $f(x)$.

c. Compare how adding a number to a function and adding a number to the x value of a function change the graphs of the functions.

Challenge

32. Forestry A forester plants a tree and measures its circumference yearly over the next four years. The table shows the forester's measurements.

Tree Growth				
Time (yr)	1	2	3	4
Circumference (in.)	2	4	6	8

- Show that the data are linear, and write an equation that models the data.
- Predict the circumference of the tree after 10 yr.
- The circumference of the tree after 10 yr was actually 43 in. After four more years, the circumference was 49 in. Based on this new information, does the relationship between time and circumference continue to be linear? Explain.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2, MP 4

In the Apply What You've Learned in Lesson 5-1, you showed that there is a linear relationship between the number of marbles in the glass on page 293 and the height of the water.

- Choose two points from the table at the right, which shows the height of the water in the glass when different numbers of marbles are dropped into it. Use the points to write an equation in slope-intercept form that gives the water height y as a function of the number of marbles x in the glass.
- What does the y -intercept of the graph of your equation represent? What does the slope represent?
- Evaluate the function from part (a) when $x = 40$. Does the water height given by the function make sense? Explain. What does the function value tell you about what would happen if you dropped 40 marbles in the glass?

Number of Marbles, x	Height of Water (cm), y
3	6.9
5	7.5
9	8.7
14	10.2
17	11.1
23	12.9