

5-5

Standard Form

Common Core State Standards

A-CED.A.2 Create equations in two or more variables... graph equations on coordinate axes... Also F-IF.A.4, F-IF.C.7a, F-LE.A.2, F-LE.B.5
MP 1, MP 2, MP 3, MP 4

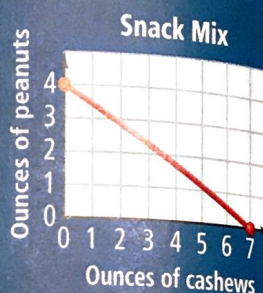
Objectives To graph linear equations using intercepts
To write linear equations in standard form



A point on the graph tells you (ounces of cashews, ounces of peanuts). What do you do with this information to find the grams of protein?

**Getting Ready!**

An athlete wants to make a snack mix of peanuts and cashews that will contain a certain amount of protein. Cashews have 4 g of protein per ounce, and peanuts have 7 g of protein per ounce. How many grams of protein will the athlete's mix contain? What do the points (7, 0) and (0, 4) represent? Explain.

**MATHEMATICAL PRACTICES**

In this lesson, you will learn to use intercepts to graph a line. Recall that a **y-intercept** is the **y**-coordinate of a point where a graph crosses the **y**-axis. The **x-intercept** is the **x**-coordinate of a point where a graph crosses the **x**-axis.

**Lesson Vocabulary**

- x-intercept
- standard form of a linear equation

Essential Understanding One form of a linear equation, called *standard form*, allows you to find intercepts quickly. You can use the intercepts to draw the graph.

**Key Concept Standard Form of a Linear Equation**

The **standard form of a linear equation** is $Ax + By = C$, where A , B , and C are real numbers, and A and B are not both zero.

**Problem 1 Finding x- and y-Intercepts**

What are the **x**- and **y**-intercepts of the graph of $3x + 4y = 24$?

Step 1 To find the **x**-intercept, substitute 0 for **y**. Solve for **x**.

$$\begin{aligned} 3x + 4y &= 24 \\ 3x + 4(0) &= 24 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

The **x**-intercept is 8.

Step 2 To find the **y**-intercept, substitute 0 for **x**. Solve for **y**.

$$\begin{aligned} 3x + 4y &= 24 \\ 3(0) + 4y &= 24 \\ 4y &= 24 \\ y &= 6 \end{aligned}$$

The **y**-intercept is 6.

Think

Why do you substitute 0 for **y** to find the **x**-intercept?

The **x**-intercept is the **x**-coordinate of a point on the **x**-axis. Any point on the **x**-axis has a **y**-coordinate of 0.

Given an equation in slope-intercept form or point-slope form, you can rewrite the equation in standard form using only integers.

Problem 4 Transforming to Standard Form

What is $y = -\frac{3}{7}x + 5$ written in standard form using integers?

$$y = -\frac{3}{7}x + 5$$

$$7y = 7\left(-\frac{3}{7}x + 5\right) \quad \text{Multiply each side by 7.}$$

$$7y = -3x + 35 \quad \text{Distributive Property}$$

$$3x + 7y = 35 \quad \text{Add } 3x \text{ to each side.}$$

Got It? 4. Write $y - 2 = -\frac{1}{3}(x + 6)$ in standard form using integers.

Plan

How can you get started?

You need to clear the fraction. So, multiply each side of the equation by the denominator of the fraction.

Problem 5 Using Standard Form as a Model

Online Shopping A media download store sells songs for \$1 each and movies for \$12 each. You have \$60 to spend. Write and graph an equation that describes the items you can purchase. What are three combinations of numbers of songs and movies you can purchase?

Relate cost of a song times number of songs plus cost of a movie times number of movies equals \$60

Define Let x = the number of songs purchased.

Let y = the number of movies purchased.

Write $1 \cdot x + 12 \cdot y = 60$

An equation for this situation is $x + 12y = 60$.

Find the intercepts.

$$x + 12y = 60$$

$$x + 12y = 60$$

$$x + 12(0) = 60$$

$$0 + 12y = 60$$

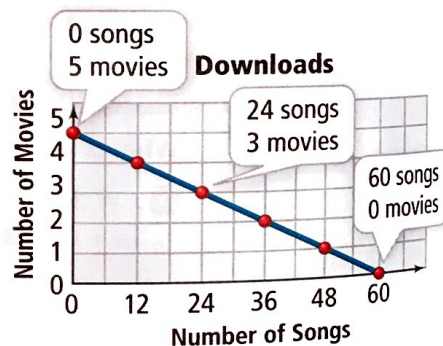
$$x = 60$$

$$y = 5$$

Use the intercepts to draw the graph. Only points in the first quadrant make sense.

The intercepts give you two combinations of songs and movies. Use the graph to identify a third combination. Each of the red points is a possible solution.



Check for Reasonableness You cannot buy a fraction of a song or movie. The graph is a line, but only points with integer coordinates are solutions.



Think

Is there another way to find solutions?

You can guess and check by substituting values for one variable and solving for the other. Then check if your solution makes sense in the context of the problem. Graphing is the quickest way to see all the solutions.

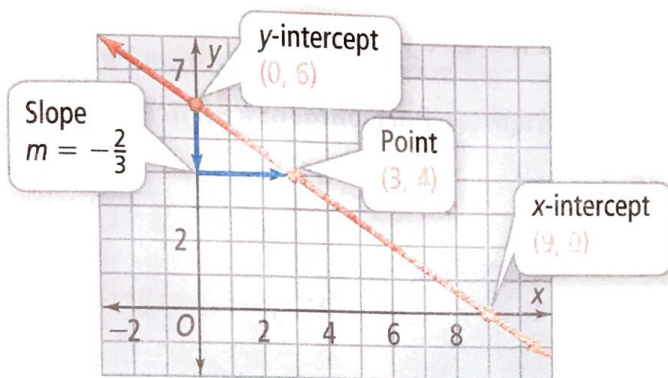
-   **Got It?** 5. a. In Problem 5, suppose the store charged \$15 for each movie. What equation describes the numbers of songs and movies you can purchase for \$60?
- b. **Reasoning** What domain and range are reasonable for the equation in part (a)? Explain.

Take note

Concept Summary Linear Equations

You can describe any line using one or more of these forms of a linear equation. Any two equations for the same line are equivalent.

Graph



Forms

Slope-Intercept Form

$$y = mx + b$$

$$y = -\frac{2}{3}x + 6$$

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 3)$$

Standard Form

$$Ax + By = C$$

$$2x + 3y = 18$$



Lesson Check

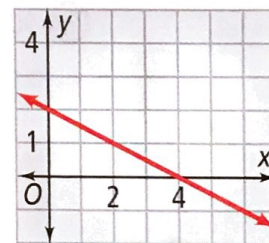
Do you know HOW?

1. What are the x - and y -intercepts of the graph of $3x - 4y = 9$?
2. What is the graph of $5x + 4y = 20$?
3. Is the graph of $y = -0.5$ a *horizontal line*, a *vertical line*, or *neither*?
4. What is $y = \frac{1}{2}x + 3$ written in standard form using integers?
5. A store sells gift cards in preset amounts. You can purchase gift cards for \$10 or \$25. You have spent \$285 on gift cards. Write an equation in standard form to represent this situation. What are three combinations of gift cards you could have purchased?

Do you UNDERSTAND?



-  6. **Vocabulary** Tell whether each linear equation is in *slope-intercept form*, *point-slope form*, or *standard form*.
- a. $y + 5 = -(x - 2)$
 - b. $y = -2x + 5$
 - c. $y - 10 = -2(x - 1)$
 - d. $2x + 4y = 12$
-  7. **Reasoning** Which form would you use to write an equation of the line at the right: *slope-intercept form*, *point-slope form*, or *standard form*? Explain.



Practice and Problem-Solving Exercises

A Practice

Find the x - and y -intercepts of the graph of each equation.

8. $x + y = 9$

9. $x - 2y = 2$

11. $3x - 5y = -20$

12. $7x - y = 21$

Draw a line with the given intercepts.

14. x -intercept: 3
 y -intercept: 5

15. x -intercept: -1
 y -intercept: -4

16. x -intercept: 4
 y -intercept: -3

Graph each equation using x - and y -intercepts.

17. $x + y = 4$

18. $x + y = -3$

19. $x - y = -8$

20. $-2x + y = 8$

21. $-4x + y = -12$

22. $6x - 2y = 18$

For each equation, tell whether its graph is a *horizontal* or a *vertical* line.

23. $y = -4$

24. $x = 3$

25. $y = \frac{7}{4}$

26. $x = -1.8$

Graph each equation.

27. $y = 6$

28. $x = -3$

29. $y = -2$

30. $x = 7$

Write each equation in standard form using integers.

31. $y = 2x + 5$

32. $y + 3 = 4(x - 1)$

33. $y - 4 = -2(x - 3)$

34. $y = \frac{1}{4}x - 2$

35. $y = -\frac{2}{3}x - 1$

36. $y + 2 = \frac{2}{3}(x + 4)$

37. **Video Games** In a video game, you earn 5 points for each jewel you find. You earn 2 points for each star you find. Write and graph an equation that represents the numbers of jewels and stars you must find to earn 250 points. What are three combinations of jewels and stars you can find that will earn you 250 points?

38. **Clothing** A store sells T-shirts for \$12 each and sweatshirts for \$15 each. You plan to spend \$120 on T-shirts and sweatshirts. Write and graph an equation that represents this situation. What are three combinations of T-shirts and sweatshirts you can buy for \$120?

B Apply

39. **Writing** The three forms of linear equations you have studied are slope-intercept form, point-slope form, and standard form. Explain when each form is most useful.

40. **Think About a Plan** You are preparing a fruit salad. You want the total carbohydrates from pineapple and watermelon to equal 24 g. Pineapple has 3 g of carbohydrates per ounce and watermelon has 2 g of carbohydrates per ounce. What is a graph that shows all possible combinations of ounces of pineapple and ounces of watermelon?

- Can you write an equation to model the situation?
- What domain and range are reasonable for the graph?

See Problem 1.

10. $-3x + 3y = 7$

13. $-5x + 3y = -7.5$


See Problem 2.

See Problem 3.

See Problem 4.

See Problem 5.

- © 41. **Compare and Contrast** Graph $3x + y = 6$, $3x - y = 6$, and $-3x + y = 6$. How are the graphs similar? How are they different?
- © 42. **Reasoning** What are the slope and y -intercept of the graph of $Ax + By = C$?
- © 43. **Error Analysis** A student says the equation $y = 4x + 1$ can be written in standard form as $4x - y = 1$. Describe and correct the student's error.
- © 44. **Reasoning** The coefficients of x and y in the standard form of a linear equation cannot both be zero. Explain why.

 **Graphing Calculator** Use a graphing calculator to graph each equation. Make a sketch of the graph. Include the x - and y -intercepts.

45. $2x - 8y = -16$

46. $-3x - 4y = 0$

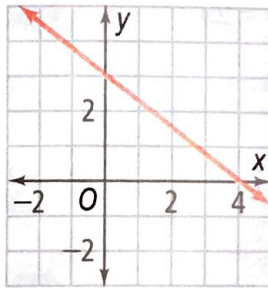
47. $x + 3.5y = 7$

48. $-x + 2y = -8$

49. $3x + 3y = -15$

50. $4x - 6y = 9$

51. **Compare and Contrast** The graph below represents one function, and the table represents a different function. How are the functions similar? How are they different?



| | | | | | |
|-----|----|----|---|---|---|
| x | -4 | -2 | 0 | 2 | 4 |
| y | 5 | 4 | 3 | 2 | 1 |

Find the x - and y -intercepts of the line that passes through the given points.

52. $(-6, 4), (3, -5)$

53. $(-5, -5), (4, -2)$

54. $(-7, 6), (-4, 11)$

55. $(-2, 8), (4, 2)$

56. $(3, -8), (-4, 13)$

57. $(5, 0.4), (-1, -2)$

58. **Sports** The scoreboard for a football game is shown at the right. All of the points the home team scored came from field goals worth 3 points and touchdowns with successful extra-point attempts worth 7 points. Write and graph a linear equation that represents this situation. List every possible combination of field goals and touchdowns the team could have scored.



59. **Geometry** Graph $x + 4y = 8$, $4x - y = -1$, $x + 4y = -12$, and $4x - y = 20$ in the same coordinate plane. What figure do the four lines appear to form?

Write an equation of each line in standard form.

60. The line contains the point $(-4, -7)$ and has the same slope as the graph of $y + 3 = 5(x + 4)$.

61. The line has the same slope as $4x - y = 5$ and the same y -intercept as the graph of $3y - 13x = 6$.

62. a. Graph $2x + 3y = 6$, $2x + 3y = 12$, and $2x + 3y = 18$ in the same coordinate plane.
 b. How are the lines from part (a) related?
 c. As C increases, what happens to the graph of $2x + 3y = C$?
- © 63. a. **Fundraising** Suppose your school is having a talent show to raise money for new band supplies. You think that 200 students and 150 adults will attend. It will cost \$200 to put on the talent show. What is an equation that describes the ticket prices you can set for students and adults to raise \$1000?
 b. **Open-Ended** Graph your equation. What are three possible prices you could set for student and adult tickets?

Standardized Test Prep

SAT/ACT

64. What is $y = -\frac{3}{4}x + 2$ written in standard form using integers?
 (A) $\frac{3}{4}x + y = 2$ (B) $3x + 4y = 2$ (C) $3x + 4y = 8$ (D) $-3x - 4y = 8$
65. Which of the following is an equation of a horizontal line?
 (F) $3x + 6y = 0$ (G) $2x + 7 = 0$ (H) $-3y = 29$ (I) $x - 2y = 4$
66. Which equation models a line with the same y -intercept but half the slope of the line $y = 6 - 8x$?
 (A) $y = -4x + 3$ (B) $y = 6 - 4x$ (C) $y = 3 - 8x$ (D) $y = -16x + 6$
67. What is the solution of $\frac{7}{2}x - 19 = -13 + 2x$?
 (F) -9 (G) -4 (H) 4 (I) 9

Short Response

68. The drama club plans to attend a professional production. Between 10 and 15 students will go. Each ticket costs \$25 plus a \$2 surcharge. There is a one-time handling fee of \$3 for the entire order. What is a linear function that models this situation? What domain and range are reasonable for the function?

Mixed Review

Write an equation in point-slope form of the line that passes through the given points. Then write the equation in slope-intercept form.

69. $(5, -1), (-3, 4)$

70. $(0, -2), (3, 2)$

71. $(-2, -1), (1, 2)$

◀ See Lesson 5-4.

Solve each compound inequality. Graph your solution.

72. $-6 < 3t \leq 9$

73. $-9.5 < 3 - y \leq 1.3$

74. $3x + 1 > 10$ or $5x + 3 \leq -2$

◀ See Lesson 3-6.

Get Ready! To prepare for Lesson 5-6, do Exercises 75-77.

Find the slope of the line that passes through each pair of points.

75. $(0, -4), (2, 0)$

76. $(5, 5), (3, -1)$

77. $(-4, 2), (5, 2)$

◀ See Lesson 5-1.