

5-6

Parallel and Perpendicular Lines

Common Core State Standards

G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

MP 1, MP 2, MP 3, MP 4, MP 7

Objectives To determine whether lines are parallel, perpendicular, or neither
To write equations of parallel lines and perpendicular lines



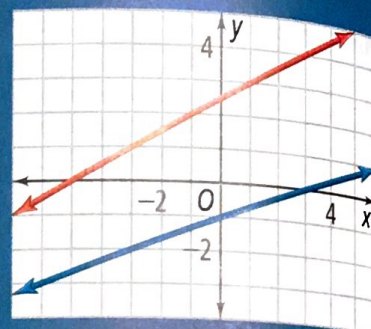
Shift your perspective to include the area outside the grid as well.



Getting Ready!

Copy the graph shown at the right. Can you draw a line that will not intersect either of the lines in the graph? If so, draw the line. If not, why not?

Can you draw a line that will intersect one of the lines in such a way that the intersection forms four congruent angles? If so, draw the line. If not, why not?



Two distinct lines in a coordinate plane either intersect or are *parallel*. **Parallel lines** are lines in the same plane that never intersect.

Essential Understanding You can determine the relationship between two lines by comparing their slopes and *y*-intercepts.



Lesson Vocabulary

- parallel lines
- perpendicular lines
- opposite reciprocals



Key Concept Slopes of Parallel Lines

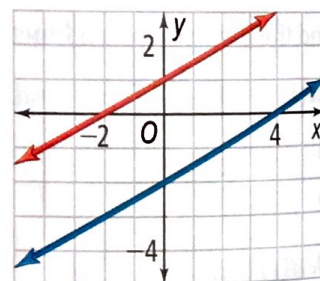
Words

Nonvertical lines are parallel if they have the same slope and different *y*-intercepts. Vertical lines are parallel if they have different *x*-intercepts.

Example

The graphs of $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x - 2$ are lines that have the same slope, $\frac{1}{2}$, and different *y*-intercepts. The lines are parallel.

Graph



You can use the fact that the slopes of parallel lines are the same to write the equation of a line parallel to a given line.



Problem 1 Writing an Equation of a Parallel Line

A line passes through $(12, 5)$ and is parallel to the graph of $y = \frac{2}{3}x - 1$. What equation represents the line in slope-intercept form?

Step 1 Identify the slope of the given line. The slope of the graph of $y = \frac{2}{3}x - 1$ is $\frac{2}{3}$. The parallel line has the same slope.

Step 2 Write an equation in slope-intercept form of the line through $(12, 5)$ with slope $\frac{2}{3}$.

$$y - y_1 = m(x - x_1) \quad \text{Start with point-slope form.}$$

$$y - 5 = \frac{2}{3}(x - 12) \quad \text{Substitute } (12, 5) \text{ for } (x_1, y_1) \text{ and } \frac{2}{3} \text{ for } m.$$

$$y - 5 = \frac{2}{3}x - \frac{2}{3}(12) \quad \text{Distributive Property}$$

$$y - 5 = \frac{2}{3}x - 8 \quad \text{Simplify.}$$

$$y = \frac{2}{3}x - 3 \quad \text{Add 5 to each side.}$$

The graph of $y = \frac{2}{3}x - 3$ passes through $(12, 5)$ and is parallel to the graph of $y = \frac{2}{3}x - 1$.



Got It? 1. A line passes through $(-3, -1)$ and is parallel to the graph of $y = 2x + 3$. What equation represents the line in slope-intercept form?

You can also use slope to determine whether two lines are *perpendicular*.

Perpendicular lines are lines that intersect to form right angles.

take note

Key Concept Slopes of Perpendicular Lines

Words

Two nonvertical lines are perpendicular if the product of their slopes is -1 . A vertical line and a horizontal line are also perpendicular.

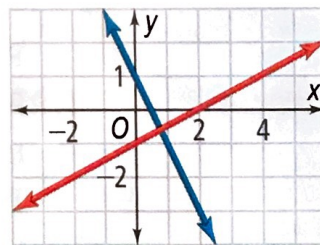
Example

The graph of $y = \frac{1}{2}x - 1$ has a slope of $\frac{1}{2}$.

The graph of $y = -2x + 1$ has a slope of -2 .

Since $\frac{1}{2}(-2) = -1$, the lines are perpendicular.

Graph



Two numbers whose product is -1 are **opposite reciprocals**. So, the slopes of perpendicular lines are opposite reciprocals. To find the opposite reciprocal of $-\frac{3}{4}$, for example, first find the reciprocal, $-\frac{4}{3}$. Then write its opposite, $\frac{4}{3}$. Since $-\frac{3}{4} \cdot \frac{4}{3} = -1$, $\frac{4}{3}$ is the opposite reciprocal of $-\frac{3}{4}$.

Think

Why start with point-slope form? You know a point on the line. You can use what you know about parallel lines to find the slope. So, point-slope form is convenient to use.

Problem 2 Classifying Lines

Are the graphs of $4y = -5x + 12$ and $y = \frac{4}{5}x - 8$ *parallel*, *perpendicular*, or *neither*? Explain.

Step 1 Find the slope of each line by writing its equation in slope-intercept form, if necessary. Only the first equation needs to be rewritten.

$$4y = -5x + 12 \quad \text{Write the first equation.}$$

$$\frac{4y}{4} = \frac{-5x + 12}{4} \quad \text{Divide each side by 4.}$$

$$y = -\frac{5}{4}x + 3 \quad \text{Simplify.}$$

The slope of the graph of $y = -\frac{5}{4}x + 3$ is $-\frac{5}{4}$.

The slope of the graph of $y = \frac{4}{5}x - 8$ is $\frac{4}{5}$.

Step 2 The slopes are not the same, so the lines cannot be parallel. Multiply the slopes to see if they are opposite reciprocals.

$$-\frac{5}{4} \cdot \frac{4}{5} = -1$$

The slopes are opposite reciprocals, so the lines are perpendicular.

Got It? 2. Are the graphs of the equations *parallel*, *perpendicular*, or *neither*? Explain.
a. $y = \frac{3}{4}x + 7$ and $4x - 3y = 9$ b. $6y = -x + 6$ and $y = -\frac{1}{6}x + 6$

Problem 3 Writing an Equation of a Perpendicular Line

Multiple Choice Which equation represents the line that passes through (2, 4) and is perpendicular to the graph of $y = \frac{1}{3}x - 1$?

(A) $y = \frac{1}{3}x + 10$ (B) $y = 3x + 10$ (C) $y = -3x - 2$ (D) $y = -3x + 10$

Step 1 Identify the slope of the graph of the given equation. The slope is $\frac{1}{3}$.

Step 2 Find the opposite reciprocal of the slope from Step 1. The opposite reciprocal of $\frac{1}{3}$ is -3 . So, the perpendicular line has a slope of -3 .

Step 3 Use point-slope form to write an equation of the perpendicular line.

$$y - y_1 = m(x - x_1) \quad \text{Write point-slope form.}$$

$$y - 4 = -3(x - 2) \quad \text{Substitute (2, 4) for } (x_1, y_1) \text{ and } -3 \text{ for } m.$$

$$y - 4 = -3x + 6 \quad \text{Distributive Property}$$

$$y = -3x + 10 \quad \text{Add 4 to each side.}$$

The equation is $y = -3x + 10$. The correct answer is D.

Got It? 3. A line passes through (1, 8) and is perpendicular to the graph of $y = 2x + 1$. What equation represents the line in slope-intercept form?

Think

Why write each equation in slope-intercept form?

You can easily identify the slope of an equation in slope-intercept form. Just look at the coefficient of x .

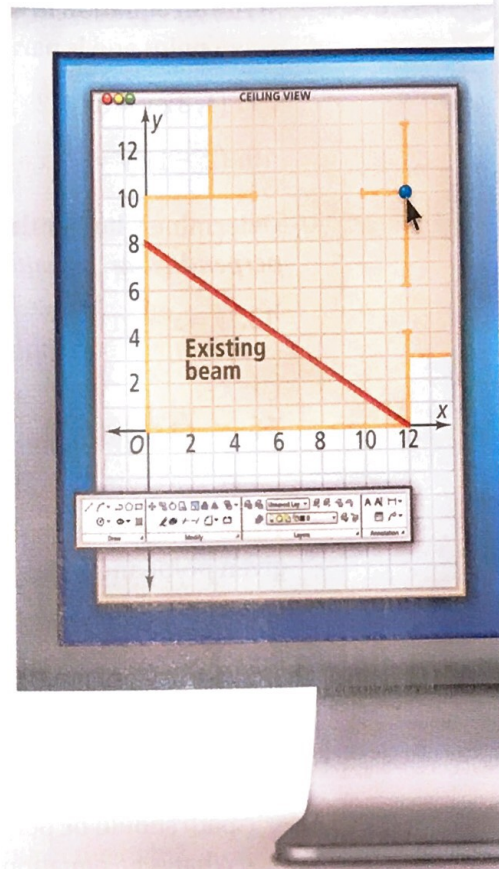
Think

How do you know you have found the opposite reciprocal?

Multiply the two numbers together as a check. If the product is -1 , the numbers are opposite reciprocals: $\frac{1}{3}(-3) = -1$.

Problem 4 Solving a Real-World Problem STEM

Architecture An architect uses software to design the ceiling of a room. The architect needs to enter an equation that represents a new beam. The new beam will be perpendicular to the existing beam, which is represented by the red line. The new beam will pass through the corner represented by the blue point. What is an equation that represents the new beam?



Step 1 Use the slope formula to find the slope of the red line that represents the existing beam.

$$\begin{aligned} m &= \frac{4 - 6}{6 - 3} && \text{Points } (3, 6) \text{ and } (6, 4) \\ & && \text{are on the red line.} \\ &= -\frac{2}{3} && \text{Simplify.} \end{aligned}$$

The slope of the line that represents the existing beam is $-\frac{2}{3}$.

Step 2 Find the opposite reciprocal of the slope from Step 1. The opposite reciprocal of $-\frac{2}{3}$ is $\frac{3}{2}$.

Step 3 Use point-slope form to write an equation. The slope of the line that represents the new beam is $\frac{3}{2}$. It will pass through $(12, 10)$. An equation that represents the new beam is $y - 10 = \frac{3}{2}(x - 12)$ or, in slope-intercept form, $y = \frac{3}{2}x - 8$.

Got It? 4. What equation could the architect enter to represent a second beam whose graph will pass through the corner at $(0, 10)$ and be parallel to the existing beam? Give your answer in slope-intercept form.

Lesson Check

Do you know HOW?

1. Which equations below have graphs that are parallel to one another? Which have graphs that are perpendicular to one another?

$$y = -\frac{1}{6}x \quad y = 6x \quad y = 6x - 2$$

2. What is an equation of the line that passes through $(3, -1)$ and is parallel to $y = -4x + 1$? Give your answer in slope-intercept form.
3. What is an equation of the line that passes through $(2, -3)$ and is perpendicular to $y = x - 5$? Give your answer in slope-intercept form.

Do you UNDERSTAND? M MATHEMATICAL PRACTICES

- C 4. **Vocabulary** Tell whether the two numbers in each pair are opposite reciprocals.
- a. $-2, \frac{1}{2}$ b. $\frac{1}{4}, 4$ c. $5, -5$
- C 5. **Open-Ended** Write equations of two parallel lines.
- C 6. **Compare and Contrast** How is determining if two lines are parallel similar to determining if they are perpendicular? How are the processes different?

Plan
Have you seen a problem like this before? Yes. You wrote the equation of a perpendicular line in Problem 3. Follow the same steps here after you calculate the slope of the line from the graph.

Practice and Problem-Solving Exercises

A Practice

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of the given equation.

7. $(1, 3)$; $y = 3x + 2$ 8. $(2, -2)$; $y = -x - 2$ 9. $(1, -3)$; $y + 2 = 4(x - 1)$
 10. $(2, -1)$; $y = -\frac{3}{2}x + 6$ 11. $(0, 0)$; $y = \frac{2}{3}x + 1$ 12. $(4, 2)$; $x = -3$

See Problem 1.

Determine whether the graphs of the given equations are *parallel*, *perpendicular*, or *neither*. Explain.

13. $y = x + 11$ 14. $y = \frac{3}{4}x - 1$ 15. $y = -2x + 3$
 $y = -x + 2$ $y = \frac{3}{4}x + 29$ $2x + y = 7$
 16. $y - 4 = 3(x + 2)$ 17. $y = -7$ 18. $y = 4x - 2$
 $2x + 6y = 10$ $x = 2$ $-x + 4y = 0$

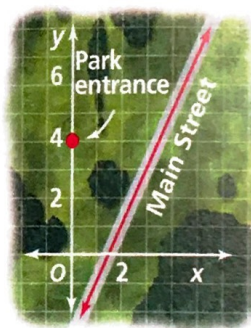
See Problem 2.

Write an equation in slope-intercept form of the line that passes through the given point and is perpendicular to the graph of the given equation.

19. $(0, 0)$; $y = -3x + 2$ 20. $(-2, 3)$; $y = \frac{1}{2}x - 1$ 21. $(1, -2)$; $y = 5x + 4$
 22. $(-3, 2)$; $x - 2y = 7$ 23. $(5, 0)$; $y + 1 = 2(x - 3)$ 24. $(1, -6)$; $x - 2y = 4$

See Problem 3.

25. Urban Planning A path for a new city park will connect the park entrance to Main Street. The path should be perpendicular to Main Street. What is an equation that represents the path?



See Problem 4.

26. Bike Path A bike path is being planned for the park in Exercise 25. The bike path will be parallel to Main Street and will pass through the park entrance. What is an equation of the line that represents the bike path?

B Apply

27. Identify each pair of parallel lines. Then identify each pair of perpendicular lines.

- line a : $y = 3x + 3$ line b : $x = -1$ line c : $y - 5 = \frac{1}{2}(x - 2)$
 line d : $y = 3$ line e : $y + 4 = -2(x + 6)$ line f : $9x - 3y = 5$

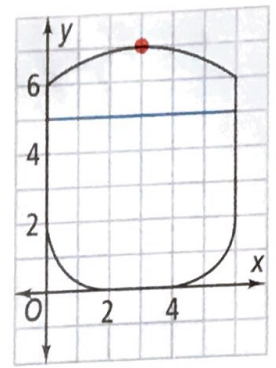
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

28. A horizontal line is parallel to the x -axis.
 29. Two lines with positive slopes are parallel.
 30. Two lines with the same slope and different y -intercepts are perpendicular.
 31. **Reasoning** For an arithmetic sequence, the first term is $A(1) = 3$. Each successive term adds 2 to the previous term. Another arithmetic sequence has the rule $B(n) = 5 + (n - 1)d$, where n is the term number and d is the common difference. If the graphs of the two sequences are parallel, what is the value of d ? Explain.

32. **Reasoning** Will the graph of the line represented by the table intersect the graph of $y = 4x + 5$? Explain.

x	-1	0	1	2
y	-1	3	7	11

33. **Think About a Plan** A designer is creating a new logo, as shown at the right. The designer wants to add a line to the logo that will be perpendicular to the blue line and pass through the red point. What equation represents the new line?
- What is the slope of the blue line?
 - What is the slope of the new line?



34. **Reasoning** For what value of k are the graphs of $12y = -3x + 8$ and $6y = kx - 5$ parallel? For what value of k are they perpendicular?

35. **Agriculture** Two farmers use combines to harvest corn from their fields. One farmer has 600 acres of corn, and the other has 1000 acres of corn. Each farmer's combine can harvest 100 acres per day. Write two equations for the number of acres y of corn *not* harvested after x days. Are the graphs of the equations *parallel*, *perpendicular*, or *neither*? How do you know?



36. **Geometry** In a rectangle, opposite sides are parallel and adjacent sides are perpendicular. Figure $ABCD$ has vertices $A(-3, 3)$, $B(-1, -2)$, $C(4, 0)$, and $D(2, 5)$. Show that $ABCD$ is a rectangle.
37. **Geometry** A right triangle has two sides that are perpendicular to each other. Triangle PQR has vertices $P(4, 3)$, $Q(2, -1)$, and $R(0, 1)$. Determine whether PQR is a right triangle. Explain your reasoning.

Standardized Test Prep

SAT/ACT

38. Which equation represents the graph of a line parallel to the line at the right?

(A) $y = \frac{1}{2}x + 5$

(C) $y = -2x + 4$

(B) $y = 2x - 6$

(D) $y = -\frac{1}{2}x - 2$

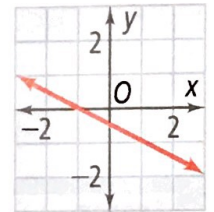
39. What is the solution of $(5x - 1) + (-2x + 7) = 9$?

(F) $\frac{3}{7}$

(G) 1

(H) 3

(I) 5



Short Response

40. Sal's Supermarket sells cases of twenty-four 12-oz bottles of water for \$15.50. Shopper's World sells 12-packs of 12-oz bottles of water for \$8.15. Which store has the better price per bottle? Explain.

Mixed Review

Graph each equation using x - and y -intercepts.

See Lesson 5-5.

41. $x + y = 8$

42. $2x + y = -3$

43. $x - 3y = -6$

Get Ready! To prepare for Lesson 5-7, do Exercises 44-47.

Write an equation in slope-intercept form of the line that passes through the given points.

See Lesson 5-3.

44. $(1, 1)$, $(3, 7)$

45. $(2, 5)$, $(12, 1)$

46. $(0.5, 2)$, $(4.5, 3)$

47. $(13, 20)$, $(6, 60)$