5-8 Graphing Absolute Value Functions

Common Core State Standards

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs . . . **Also F-IF.C.7b**

MP 1, MP 2, MP 3, MP 4

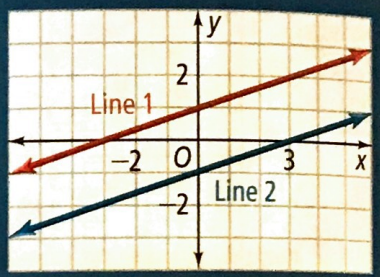
Objectives To graph an absolute value function
To translate the graph of an absolute value function

Remember what you have learned about families of functions.

SOLVE IT!

Getting Ready!

Write the equations of Line 1 and Line 2. How can you transform the equation of Line 1 into the equation of Line 2? How can you slide Line 1 in the coordinate plane so that it becomes Line 2? Explain.



MATHEMATICAL PRACTICES

In the Solve It you described how one line could be shifted to result in a second line. You can use a similar method to graph *absolute value functions*. An **absolute value function** has a V-shaped graph that opens up or down. The parent function for the family of absolute value functions is $y = |x|$.

A **translation** is a shift of a graph horizontally, vertically, or both. The result is a graph of the same size and shape, but in a different position.

VOCABULARY

Lesson Vocabulary
- absolute value function
- piecewise function
- step function
- translation

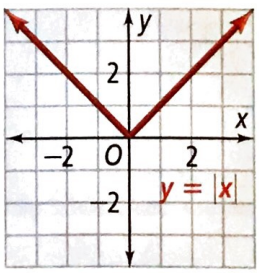
Essential Understanding You can quickly graph absolute value equations by shifting the graph of $y = |x|$.

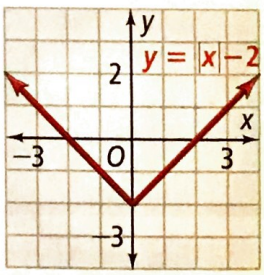
Problem 1 Describing Translations

Below are the graphs of $y = |x|$ and $y = |x| - 2$. How are the graphs related?

Plan

How can you compare the graphs? Look for the characteristics that you've studied with other graphs, such as shape, size, or individual points.

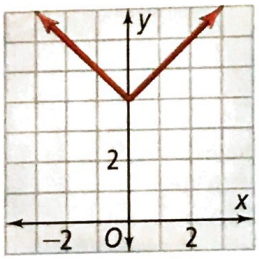




The graphs have the same shape. Notice each point on $y = |x| - 2$ is 2 units lower than the corresponding point on $y = |x|$. The graph of $y = |x| - 2$ is the graph of $y = |x|$ translated down 2 units.

Got It? **1. a.** How is the graph at the right related to the graph of $y = |x|$?

b. Reasoning What are the domain and range of each function in Problem 1?



The graph of $y = |x| + k$ is a translation of $y = |x|$. Let k be a positive number. Then $y = |x| + k$ translates the graph of $y = |x|$ up k units, while $y = |x| - k$ translates the graph of $y = |x|$ down k units.

Problem 2 Graphing a Vertical Translation

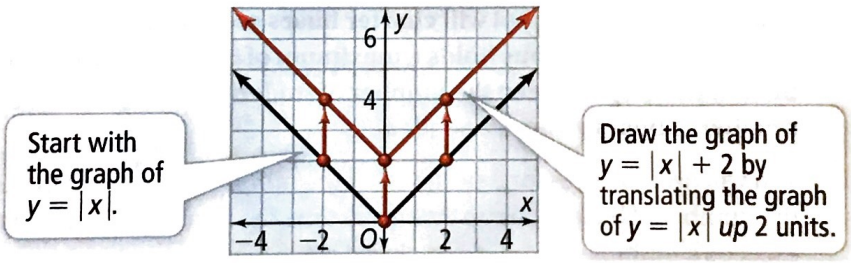
What is the graph of $y = |x| + 2$?

Know
- The equation of an absolute value function
- The graph of $y = |x|$

Need
The graph of the function

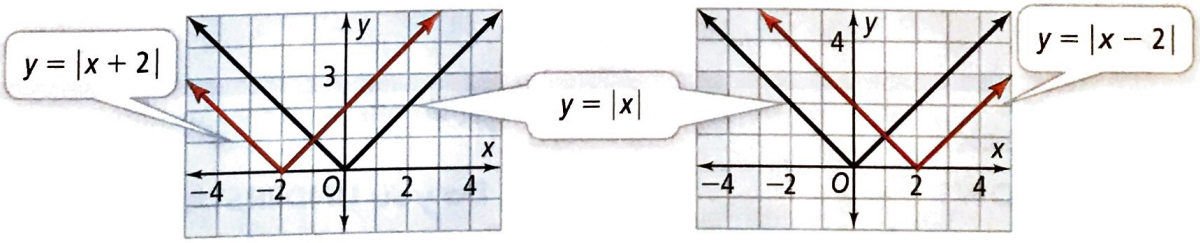
Plan
Identify the direction and amount of the translation. Translate the *y*-intercept point and one point on each side of it. Draw the graph.

Think
Why start with the graph of $y = |x|$?
Since $y = |x|$ is the parent function of $y = |x| + 2$, you can start with the graph of $y = |x|$ and shift it up.



Got It? **2.** What is the graph of $y = |x| - 7$?

The graphs below show what happens when you graph $y = |x + 2|$ and $y = |x - 2|$.



For a positive number h, $y = |x + h|$ translates the graph of $y = |x|$ left h units, and $y = |x - h|$ translates the graph of $y = |x|$ right h units.

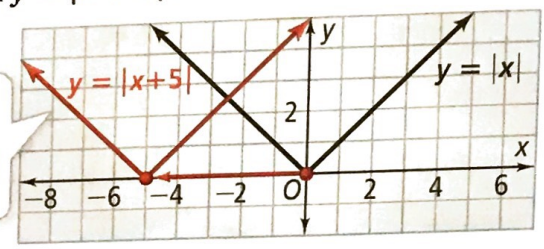
Problem 3 Graphing a Horizontal Translation

What is the graph of $y = |x + 5|$?

Think

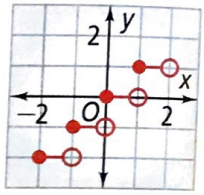
How can you check that the graph is correct?
You can use the equation to check that points on the graph are solutions.

Draw the graph of $y = |x + 5|$ by translating $y = |x|$ *left* 5 units.



Got It? **3.** What is the graph of $y = |x - 5|$?

The absolute value function is an example of a piecewise function. A **piecewise function** is a function that has different rules for different parts of its domain. For example, when $x \geq 0$, $|x| = x$. When $x < 0$, $|x| = -x$. Another example of a piecewise function is a step function. A **step function** is a function that pairs every number in an interval with a single value. The graph of a step function can look like the steps of a staircase.

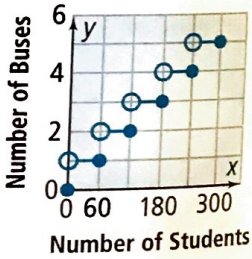


Each piece of the graph is a horizontal segment that is missing its right endpoint, indicated by an open circle.

Problem 4 Graphing a Step Function

Transportation **A school will charter buses so that the student body can attend a football game. Each bus holds a maximum of 60 students. Make a graph that models the relationship between the number of students x that go to the game by bus and the number of buses y that are needed.**

You will need 0 buses for 0 students. As the number of students increases, the number of buses goes up by 1 every time the number of students exceeds a multiple of 60. Draw a closed circle when the endpoints are part of the graph, and then draw an open point when they are not.



Got It? **4.** Make a graph that models the relationship between the number of students x that go to the game by bus and the number of buses y that are needed if each bus holds a maximum of 50 students.

Lesson Check

Do you know HOW?

1. How is the graph of $y = |x| - 8$ different from the graph of $y = |x|$? How is it the same?
2. What is the equation for the translation of $y = |x|$ 9 units up?
3. What is the graph of $y = |x + 7|$?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Compare and Contrast** How are the graphs of $y = |x| - 4$ and $y = |x - 4|$ the same? How are they different?
5. **Error Analysis** A student is graphing the equation $y = |x - 10|$ and translates the graph of $y = |x|$ 10 units left. Describe the student's error.

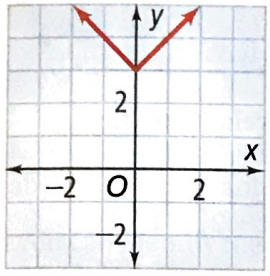
Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Describe how each graph is related to the graph of $y = |x|$. See Problem 1.

6.



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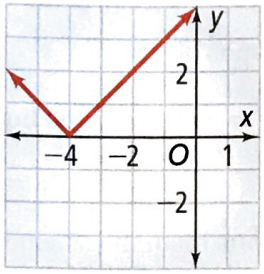

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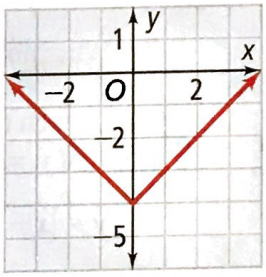


Graph each function by translating $y = |x|$. See Problem 2.

9. $y = |x| - 3$

10. $y = |x| + 7$

11. $y = |x| + 3$

12. $y = |x| - 6$

13. $y = |x| + 6$

14. $y = |x| - 2.5$

Graph each function by translating $y = |x|$. See Problem 3.

15. $y = |x - 3|$

16. $y = |x + 3|$

17. $y = |x - 1|$

18. $y = |x + 6|$

19. $y = |x - 7|$

20. $y = |x + 2.5|$

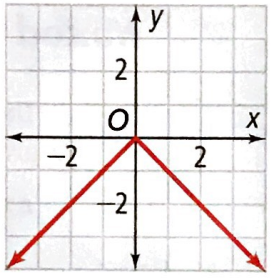
21. Postage The table lists postage for letters weighing as much as 3 oz. You want to mail a letter that weighs 2.7 oz. Graph the step function. How much will you pay in postage? See Problem 4.

First-Class Postage

Weight x	Price y
$0 <$ Weight < 1 oz	\$.44
1 oz $\leq$ Weight < 2 oz	\$.61
2 oz $\leq$ Weight ≤ 3 oz	\$.78

B Apply

At the right is the graph of $y = -|x|$. Graph each function by translating $y = -|x|$.



22. $y = -|x| + 3$

23. $y = -|x| - 3$

24. $y = -|x + 3|$

25. $y = -|x - 3|$

Write an equation for each translation of $y = -|x|$.

26. 2 units up

27. 2.25 units left

28. 15 units down

29. 4 units right

30. Writing Explain how the relationship between $y = |x|$ and $y = |x| + k$ is similar to the relationship between $y = mx$ and $y = mx + b$.

31. Think About a Plan What point(s) do the graphs of $y = |x - 2|$ and $y = |x + 4|$ have in common?

- How are these graphs related?
- Could a graph or a table help you solve this problem?

32. What point(s) do the graphs of $y = -|x| + 7$ and $y = |x - 3|$ have in common?

Graph each translation of $y = |x|$. Describe how the graph is related to the graph of $y = |x|$.

33. $y = |x - 1| + 2$

34. $y = |x + 2| - 1$

35. a. Graph $y = |x - 2| + 3$.

b. The *vertex* of an absolute value function is the point at which its graph changes direction. What is the vertex of the graph of $y = |x - 2| + 3$?

c. Reasoning What relationship do you see between the vertex and the equation? What is the vertex of the graph of $y = |x - h| + k$?

Challenge

36. a. Graph $y = |2x|$ by making a table of values.

b. Translate $y = |2x|$ to graph $y = |2x| + 3$.

c. Translate $y = |2x|$ to graph $y = |2(x - 1)|$.

d. Translate $y = |2x|$ to graph $y = |2(x - 1)| + 3$.

37. Graph $y = -|x + 4| - 7$.

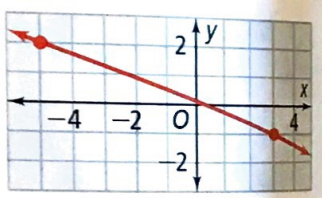
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

38. For $f(x) = 5x - 7$, what value of x gives $f(x) = -3$?

39. What is the slope of the line at the right?



40. What is the value of $f(x) = x^2 - 4x + 6$ when $x = -3$?

41. What is the x-intercept of the line $y = -4x + 2$?

Mixed Review

The data below follow a linear model. Write an equation of a trend line or use a graphing calculator to find an equation of the line of best fit.

See Lesson 5-7.

42.

Year	1	2	3	4
Price	$5.30	$5.57	$5.82	$6.05

43.

Ounces	8	12	16	20
Calories	100	151	202	250

Get Ready! To prepare for Lesson 6-1, do Exercises 44–47.

Graph each equation.

See Lesson 5-3.

44. $y = 2x - 1$

45. $y = -3x + 5$

46. $y = \frac{1}{3}x + 2$

47. $y = -\frac{5}{2}x - 7$