6-1 Solving Systems by Graphing

Common Core State Standards

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MP 1, MP 2, MP 3, MP 4

Objectives To solve systems of equations by graphing
To analyze special systems

There is more than one way to find the answer. Start with a plan.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

Two professional downhill skiers are racing at the speeds shown in the diagram. Skier 1 starts 5 s before Skier 2. The course is 5000 ft long. Will Skier 2 pass Skier 1? How do you know?

Lesson Vocabulary

- system of linear equations
- solution of a system of linear equations
- consistent
- independent
- dependent
- inconsistent

You can model the problem in the Solve It with two linear equations. Two or more linear equations form a **system of linear equations.** Any ordered pair that makes *all* of the equations in a system true is a **solution of a system of linear equations.**

Essential Understanding You can use systems of linear equations to model problems. Systems of equations can be solved in more than one way. One method is to graph each equation and find the intersection point, if one exists.

Problem 1 Solving a System of Equations by Graphing

What is the solution of the system? Use a graph. $y = x + 2$, $y = 3x - 2$

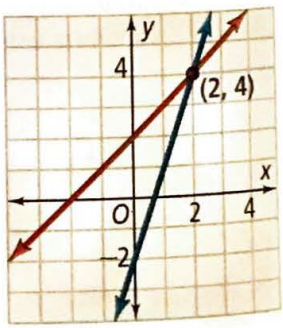
Think

How does graphing each equation help you find the solution?
A line represents the solutions of *one* linear equation. The intersection point is a solution of *both* equations.

Graph both equations in the same coordinate plane.

$y = x + 2$ The slope is 1. The y-intercept is 2.

$y = 3x - 2$ The slope is 3. The y-intercept is -2.



Find the point of intersection. The lines appear to intersect at (2, 4). Check to see if (2, 4) makes both equations true.

$y = x + 2$
$4 \stackrel{?}{=} 2 + 2$
$4 = 4$ ✔

Substitute (2, 4) for (x, y).

$y = 3x - 2$
$4 \stackrel{?}{=} 3(2) - 2$
$4 = 4$ ✔

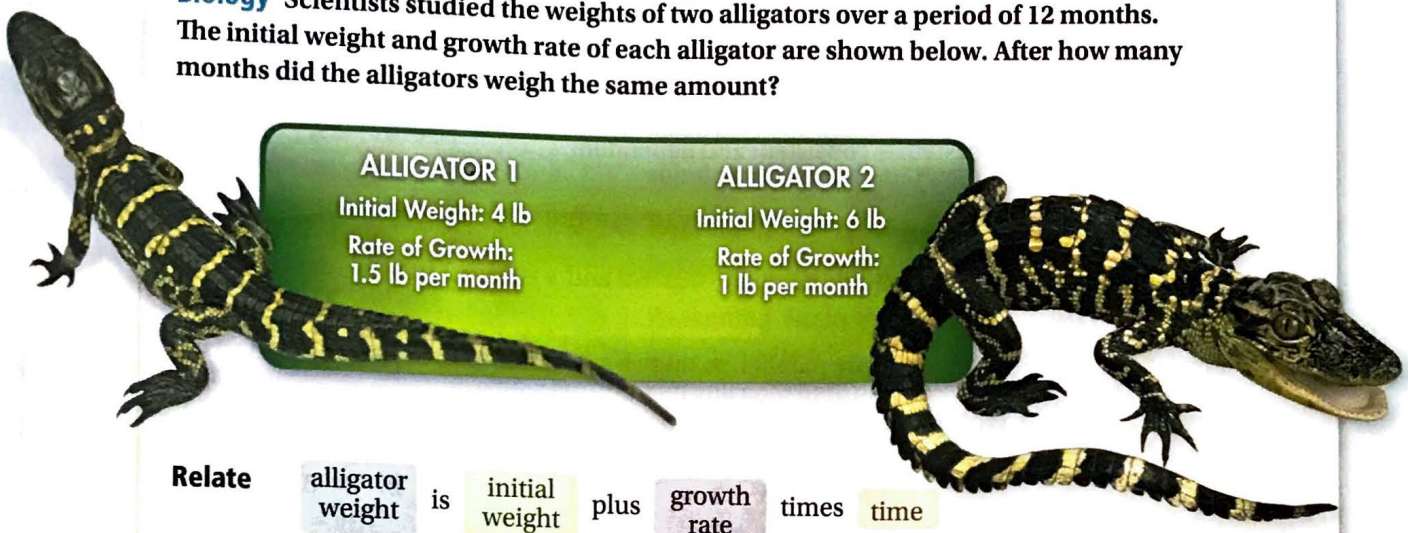
The solution of the system is (2, 4).

Got It? 1. What is the solution of the system? Use a graph. Check your answer.

$$y = 2x + 4$$
$$y = x + 2$$

Problem 2 Writing a System of Equations STEM

Biology **Scientists studied the weights of two alligators over a period of 12 months. The initial weight and growth rate of each alligator are shown below. After how many months did the alligators weigh the same amount?**



Relate alligator weight is initial weight plus growth rate times time

Define Let w = alligator weight.
Let t = time in months.

Write Alligator 1: $w = 4 + 1.5 \cdot t$
Alligator 2: $w = 6 + 1 \cdot t$

Think
Is there another way to solve this problem?
Yes. You can *make a table*. Show the weight of each alligator after 1 month, 2 months, and so on.

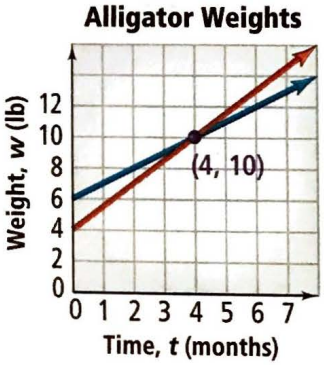
Graph both equations in the same coordinate plane.

$w = 4 + 1.5t$ The slope is 1.5. The w-intercept is 4.

$w = 6 + t$ The slope is 1. The w-intercept is 6.

The lines intersect at (4, 10).

After 4 months, both alligators weighed 10 lb.



Got It? 2. One satellite radio service charges $10 per month plus an activation fee of $20. A second service charges $11 per month plus an activation fee of $15. In what month was the cost of the service the same?

A system of equations that has at least one solution is **consistent.** A consistent system can be either *independent* or *dependent.*

A consistent system that is **independent** has exactly one solution. For example, the systems in Problems 1 and 2 are consistent and independent. A consistent system that is **dependent** has infinitely many solutions.

A system of equations that has no solution is **inconsistent.**

Problem 3 Systems With Infinitely Many Solutions or No Solution

Think

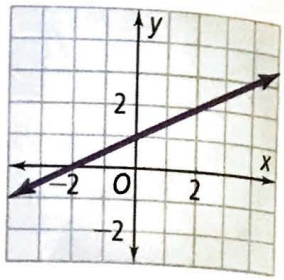
If two equations have the same slope and y-intercept, their graphs will be the same line. If two equations have the same slope but different y-intercepts, their graphs will be parallel lines.

What is the solution of each system? Use a graph.

A $2y - x = 2$

$y = \frac{1}{2}x + 1$

Graph the equations $2y - x = 2$ and $y = \frac{1}{2}x + 1$ in the same coordinate plane.

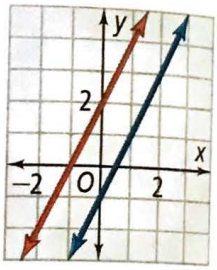


The equations represent the same line. Any point on the line is a solution of the system, so there are infinitely many solutions. The system is consistent and dependent.

B $y = 2x + 2$

$y = 2x - 1$

Graph the equations $y = 2x + 2$ and $y = 2x - 1$ in the same coordinate plane.



The lines are parallel, so there is no solution. The system is inconsistent.

Got It? **3.** What is the solution of each system in parts (a) and (b)? Use a graph. Describe the number of solutions.

a. $y = -x - 3$

$y = -x + 5$

b. $y = 3x - 3$

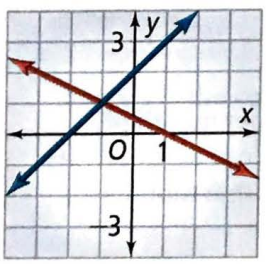
$3y = 9x - 9$

c. Reasoning Before graphing the equations, how can you determine whether a system of equations has exactly one solution, infinitely many solutions, or no solution?

Take Note

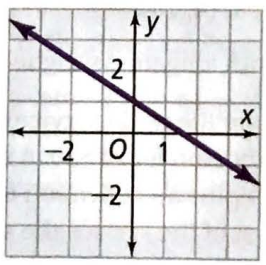
Concept Summary Systems of Linear Equations

One solution



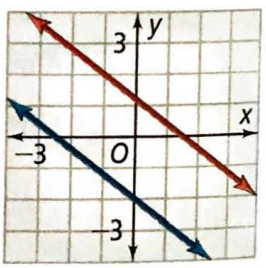
The lines intersect at one point. The lines have different slopes. The equations are consistent and independent.

Infinitely many solutions



The lines are the same. The lines have the same slope and y-intercept. The equations are consistent and dependent.

No solution



The lines are parallel. The lines have the same slope and different y-intercepts. The equations are inconsistent.

Lesson Check

Do you know HOW?

Solve each system by graphing.

1. $y = x + 7$
   $y = 2x + 1$

2. $y = \frac{1}{2}x + 6$
   $y = x - 2$

3. $y = -3x - 3$
   $y = 2x + 2$

4. $y = -x - 4$
   $4x - y = -1$

5. **Concert Tickets** Tickets for a concert cost \$10 each if you order them online, but you must pay a service charge of \$8 per order. The tickets are \$12 each if you buy them at the door on the night of the concert.
   a. Write a system of equations to model the situation. Let c be the total cost. Let t be the number of tickets.
   b. Graph the equations and find the intersection point. What does this point represent?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** Match each type of system with the number of solutions the system has.

A. inconsistent	I. exactly one
B. consistent and dependent	II. infinitely many
C. consistent and independent	III. no solution

7. **Writing** Suppose you graph a system of linear equations. If a point is on only one of the lines, is it a solution of the system? Explain.

8. **Reasoning** Can a system of two linear equations have exactly two solutions? Explain.

9. **Reasoning** Suppose you find that two linear equations are true when $x = -2$ and $y = 3$. What can you conclude about the graphs of the equations? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each system by graphing. Check your solution. See Problem 1.

10. $y = 2x$
    $y = -2x + 8$

11. $y = \frac{1}{2}x + 7$
    $y = \frac{3}{2}x + 3$

12. $y = \frac{1}{3}x + 1$
    $y = -3x + 11$

13. $y = x - 4$
    $y = -x$

14. $y = -x + 3$
    $y = x + 1$

15. $4x - y = -1$
    $-x + y = x - 5$

16. $y = -\frac{1}{2}x + 2$
    $y = \frac{1}{2}x + 6$

17. $2x - y = -5$
    $-2x - y = -1$

18. $x = -3$
    $y = 5$

19. **Student Statistics** The number of right-handed students in a mathematics class is nine times the number of left-handed students. The total number of students in the class is 30. How many right-handed students are in the class? How many left-handed students are in the class? See Problem 2.

20. **Plants** A plant nursery is growing a tree that is 3 ft tall and grows at an average rate of 1 ft per year. Another tree at the nursery is 4 ft tall and grows at an average rate of 0.5 ft per year. After how many years will the trees be the same height?

21. **Fitness** At a local fitness center, members pay a \$20 membership fee and \$3 for each aerobics class. Nonmembers pay \$5 for each aerobics class. For what number of aerobics classes will the cost for members and nonmembers be the same?

Solve each system by graphing. Tell whether the system has *one solution, infinitely many solutions,* or *no solution.*

See Problem 3.

22. $y = x + 3$
$y = x - 1$

23. $y = 2x - 1$
$3y = 6x - 5$

24. $3x + y = 2$
$4y = 12 - 12x$

25. $2x - 2y = 5$
$y = x - 4$

26. $y = 2x - 2$
$2y = 4x - 4$

27. $y - x = 5$
$3y = 3x + 15$

28. $2x + 2y = 4$
$12 - 3x = 3y$

29. $2y = x - 2$
$3y = \frac{3}{2}x - 3$

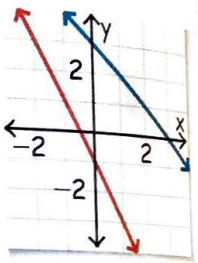
30. $3x - y = 2$
$4y = -x + 5$

B Apply

31. Think About a Plan You are looking for an after-school job. One job pays \$9 per hour. Another pays \$12 per hour, but you must buy a uniform that costs \$39. After how many hours of work would your net earnings from either job be the same?

- What equations can you write to model the situation?
- How will graphing the equations help you solve the problem?

32. Error Analysis A student graphs the system $y = -x + 3$ and $y = -2x - 1$ as shown at the right. The student concludes there is no solution. Describe and correct the student's error.



33. Reasoning Suppose you graph a system of linear equations and the intersection point appears to be (3, 7). Can you be sure that the ordered pair (3, 7) is the solution? What must you do to be sure?

34. Cell Phone Plans A cell phone provider offers a plan that costs \$40 per month plus \$.20 per text message sent or received. A comparable plan costs \$60 per month but offers unlimited text messaging.

a. How many text messages would you have to send or receive in order for the plans to cost the same each month?

b. If you send or receive an average of 50 text messages each month, which plan would you choose? Why?

Without graphing, decide whether each system has *one solution, infinitely many solutions,* or *no solution.* Justify your answer.

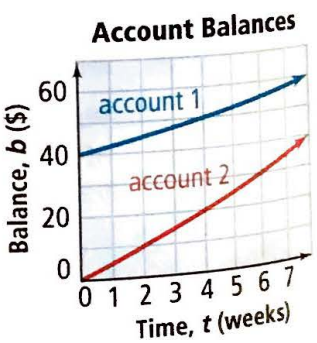
35. $y = x - 4$
$y = x - 3$

36. $x - y = -\frac{1}{2}$
$2x - 2y = -1$

37. $y = 5x - 1$
$10x = 2y + 2$

38. $3x + 2y = 1$
$4y = 6x + 2$

39. Banking The graph at the right shows the balances in two bank accounts over time. Use the graph to write a system of equations giving the amount in each account over time. Let t = the time in weeks and let b = the balance in dollars. If the accounts continue to grow as shown, when will they have the same balance?



40. Open-Ended One equation in a system is $y = \frac{1}{2}x - 2$.

a. Write a second equation so that the system has one solution.

b. Write a second equation so that the system has no solution.

c. Write a second equation so that the system has infinitely many solutions.

Challenge

41. Reasoning Consider the system at the right.

$$y = gx + 3$$
$$y = hx + 7$$

a. If $g \geq h$, will the system *always, sometimes,* or *never* have exactly one solution? Explain your reasoning.

b. If $g \leq h$, will the system *always, sometimes,* or *never* have infinitely many solutions? Explain your reasoning.

42. Hiking Two hikers are walking along a marked trail. The first hiker starts at a point 6 mi from the beginning of the trail and walks at a speed of 4 mi/h. At the same time, the second hiker starts 1 mi from the beginning and walks at a speed of 3 mi/h.

a. What is a system of equations that models the situation?

b. Graph the two equations and find the intersection point.

c. Is the intersection point meaningful in this situation? Explain.

Standardized Test Prep

SAT/ACT

43. Which ordered pair is the solution of the system?

$$2x + 3y = -17$$
$$3x + 2y = -8$$

Ⓐ $(2, -7)$ Ⓑ $(-4, 2)$ Ⓒ $(-2, -1)$ Ⓓ $\left(-\frac{4}{3}, -2\right)$

44. Which expression is equivalent to $5(m - 12) + 8$?

Ⓕ $5m - 68$ Ⓖ $5m - 20$ Ⓗ $5m - 4$ Ⓘ $5m - 52$

Extended Response

45. The costs for parking in two different parking garages are given in the table at the right.

a. What is a system of equations that models the situation?

b. How many hours of parking would cost the same parking in either garage?

c. If you needed to park a car for 3 h, which garage would you choose? Why?

Garage Parking Fees

Garage	Flat Fee	Hourly Fee
A	$5	$2.50
B	$20	$0

Mixed Review

Graph each function by translating the graph of $y = |x|$. See Lesson 5-8.

46. $y = |x| - 2$ **47.** $y = |x| - 1$ **48.** $y = |x + 3|$ **49.** $y = |x + 2|$

Find the slope of a line that is parallel to the graph of the equation. See Lesson 5-6.

50. $y = x + 3$ **51.** $y = -\frac{1}{2}x - 4$ **52.** $3y + 2x = 7$ **53.** $3x = 5y + 10$

Get Ready! To prepare for Lesson 6-2, do Exercises 54–57.

Solve each equation for y. See Lesson 2-5.

54. $4x + 2y = 38$ **55.** $\frac{1}{2}x + \frac{1}{3}y = 5$ **56.** $\frac{3}{2}y = \frac{4}{5}x$ **57.** $1.5x - 4.5y = 21$