6-2 Solving Systems Using Substitution

Common Core State Standards

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

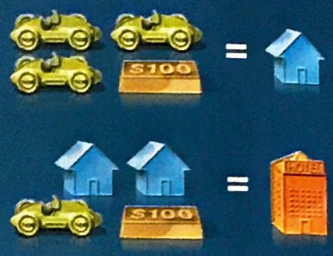
MP 1, MP 2, MP 3, MP 4

Objective To solve systems of equations using substitution

How can you get started? One house equals 3 cars plus $100. Two houses equal . . .

Getting Ready!

A board game allows players to trade game pieces of equal value. The diagram shows two fair trades. The hotel is worth $2400. How much is a car worth? Explain your reasoning.



MATHEMATICAL PRACTICES

You can model fair trades with a linear system. You can solve linear systems by solving one of the equations for one of the variables. Then substitute the expression for the variable into the other equation. This is called the **substitution method.**

Lesson Vocabulary
- substitution method

Essential Understanding Systems of equations can be solved in more than one way. When a system has at least one equation that can be solved quickly for a variable, the system can be solved efficiently using substitution.

Problem 1 Using Substitution

What is the solution of the system? Use substitution.

$$y = 3x$$
$$x + y = -32$$

Plan

How can you get started?
If one equation is already solved for one variable, use it for the substitution. If both equations are solved for a variable, you can use either one.

Step 1 Because $y = 3x$, you can substitute $3x$ for y in $x + y = -32$.

$x + y = -32$ Write the second equation.

$x + 3x = -32$ Substitute $3x$ for y.

$4x = -32$ Simplify.

$x = -8$ Divide each side by 4.

Step 2 Substitute -8 for x in either equation and solve for y.

$y = 3x$ Write either equation.

$y = 3(-8) = -24$ Substitute -8 for x and solve.

The solution is $(-8, -24)$. Check by substituting $(-8, -24)$ into each equation.

Check

$y = 3x$

$-24 \stackrel{?}{=} 3(-8)$

$-24 = -24$ ✔

$x + y = -32$

$-8 + (-24) \stackrel{?}{=} -32$

$-32 = -32$ ✔

Got It? **1.** What is the solution of the system? Use substitution. Check your answer.

$y = 2x + 7$

$y = x - 1$

To use substitution to solve a system of equations, one of the equations must be solved for a variable.

Problem 2 Solving for a Variable and Using Substitution

What is the solution of the system? Use substitution. $3y + 4x = 14$, $-2x + y = -3$

Know

Neither equation is solved for one of the variables.

Need

The solution of the system

Plan

Solve one of the equations for one of the variables. Then use the substitution method to find the solution of the system.

Think

Which variable should you solve for?
If one equation has a variable with a coefficient of 1 or −1, solve for that variable. It is generally easier to solve for a variable with a coefficient of 1 or −1.

Step 1 Solve one of the equations for one of the variables.

$-2x + y = -3$ Write the second equation.

$-2x + y + 2x = -3 + 2x$ Add $2x$ to each side.

$y = 2x - 3$ Simplify.

Step 2 Substitute $2x - 3$ for y in the other equation and solve for x.

$3y + 4x = 14$ Write the first equation.

$3(2x - 3) + 4x = 14$ Substitute $2x - 3$ for y. Use parentheses.

$6x - 9 + 4x = 14$ Distributive Property

$10x = 23$ Add 9 to each side. Simplify.

$x = 2.3$ Divide each side by 10.

Step 3 Substitute 2.3 for x in either equation and solve for y.

$-2x + y = -3$ Write either equation.

$-2(2.3) + y = -3$ Substitute 2.3 for x.

$-4.6 + y = -3$ Simplify.

$y = 1.6$ Add 4.6 to each side.

The solution is (2.3, 1.6).

Got It? **2. a.** What is the solution of the system? Use substitution.

$6y + 5x = 8$

$x + 3y = -7$

b. Reasoning In your first step in part (a), which variable did you solve for? Which equation did you use to solve for the variable?

Problem 3 Using Systems of Equations

GRIDDED RESPONSE

Snack Bar A snack bar sells two sizes of snack packs. A large snack pack is \$5, and a small snack pack is \$3. In one day, the snack bar sold 60 snack packs for a total of \$220. How many small snack packs did the snack bar sell?

Step 1 Write the system of equations. Let x = the number of large \$5 snack packs, and let y = the number of small \$3 snack packs.

$x + y = 60$ Represent the total number of snack packs.

$5x + 3y = 220$ Represent the amount earned from 60 snack packs.

Step 2 $x + y = 60$ Use the first equation to solve for y.

$y = 60 - x$ Subtract x from each side.

Step 3 $5x + 3(60 - x) = 220$ Substitute $60 - x$ for y in the second equation.

$5x + 180 - 3x = 220$ Distributive Property

$2x = 40$ Simplify.

$x = 20$ Divide each side by 2.

Step 4 $20 + y = 60$ Substitute 20 for x in the first equation.

$y = 40$ Subtract 20 from each side.

The system's solution is (20, 40). The snack bar sold 40 small snack packs.

Think

What does the solution represent in the real world?
Check what the assigned variables represent. Here, (20, 40) represents 20 large snack packs and 40 small snack packs.

Got It? **3.** You pay \$22 to rent 6 video games. The store charges \$4 for new games and \$2 for older games. How many new games did you rent?

If you get an identity, like $2 = 2$, when you solve a system of equations, then the system has infinitely many solutions. If you get a false statement, like $8 = 2$, then the system has no solution.

Problem 4 Systems With Infinitely Many Solutions or No Solution

How many solutions does each system have?

A $x = -2y + 4$
$3.5x + 7y = 14$

Substitute $-2y + 4$ for x in $3.5x + 7y = 14$.

$3.5x + 7y = 14$

$3.5(-2y + 4) + 7y = 14$

$-7y + 14 + 7y = 14$

$14 = 14$ ✔

The system has infinitely many solutions.

B $y = 3x - 11$
$y - 3x = -13$

Substitute $3x - 11$ for y in $y - 3x = -13$.

$y - 3x = -13$

$(3x - 11) - 3x = -13$

$-11 = -13$ ✘

The system has no solution.

Think

How many solutions can a system of linear equations have?
A system can have exactly one solution, infinitely many solutions, or no solution.

Got It? **4.** How many solutions does the system have? $6y + 5x = 8$
$2.5x + 3y = 4$

Lesson Check

Do you know HOW?

Solve each system using substitution. Check your solution.

1. $4y = x$
   $3x - y = 70$

2. $-2x + 5y = 19$
   $3x - 4 = y$

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

3. $y = 2x + 1$
   $4x - 2y = 6$

4. $-x + \frac{1}{2}y = 13$
   $x + 15 = \frac{1}{2}y$

5. **Talent Show** In a talent show of singing and comedy acts, singing acts are 5 min long and comedy acts are 3 min long. The show has 12 acts and lasts 50 min. How many singing acts and how many comedy acts are in the show?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** When is the substitution method a better method than graphing for solving a system of linear equations?

For each system, tell which equation you would first use to solve for a variable in the first step of the substitution method. Explain your choice.

7. $-2x + y = -1$
   $4x + 2y = 12$

8. $2.5x - 7y = 7.5$
   $6x - y = 1$

Tell whether each statement is *true* or *false*. Explain.

9. When solving a system using substitution, if you obtain an identity, then the system has no solution.

10. You cannot use substitution to solve a system that does not have a variable with a coefficient of 1 or -1.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice **Solve each system using substitution. Check your answer.** See Problems 1 and 2.

11. $x + y = 8$
    $y = 3x$

12. $2x + 2y = 38$
    $y = x + 3$

13. $x + 3 = y$
    $3x + 4y = 7$

14. $y = 8 - x$
    $7 = 2 - y$

15. $y = -2x + 6$
    $3y - x + 3 = 0$

16. $3x + 2y = 23$
    $\frac{1}{2}x - 4 = y$

17. $y - 2x = 3$
    $3x - 2y = 5$

18. $4x = 3y - 2$
    $18 = 3x + y$

19. $2 = 2y - x$
    $23 = 5y - 4x$

20. $4y + 3 = 3y + x$
    $2x + 4y = 18$

21. $7x - 2y = 1$
    $2y = x - 1$

22. $4y - x = 5 + 2y$
    $3x + 7y = 24$

23. **Theater Tickets** Adult tickets to a play cost \$22. Tickets for children cost \$15. Tickets for a group of 11 people cost a total of \$228. Write and solve a system of equations to find how many children and how many adults were in the group. See Problem 3.

24. **Transportation** A school is planning a field trip for 142 people. The trip will use six drivers and two types of vehicles: buses and vans. A bus can seat 51 passengers. A van can seat 10 passengers. Write and solve a system of equations to find how many buses and how many vans will be needed.

25. **Geometry** The measure of one acute angle in a right triangle is four times the measure of the other acute angle. Write and solve a system of equations to find the measures of the acute angles.

Tell whether the system has *one solution, infinitely many solutions,* or *no solution.*

See Problem 4.

26. $y = \frac{1}{2}x + 3$
$2y - x = 6$

27. $6y = -5x + 24$
$2.5x + 3y = 12$

28. $x = -7y + 34$
$x + 7y = 32$

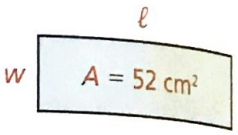
29. $5 = \frac{1}{2}x + 3y$
$10 - x = 6y$

30. $17 = 11y + 12x$
$12x + 11y = 14$

31. $1.5x + 2y = 11$
$3x + 6y = 22$

B Apply

32. Geometry The rectangle shown has a perimeter of 34 cm and the given area. Its length is 5 more than twice its width. Write and solve a system of equations to find the dimensions of the rectangle.



33. Writing What would your first step be in solving the system below? Explain.

$$1.2x + y = 2$$
$$1.4y = 2.8x + 1$$

34. Coins You have \$3.70 in dimes and quarters. You have 5 more quarters than dimes. How many of each type of coin do you have?

35. Error Analysis Describe and correct the error at the right in finding the solution of the following system:

$$7x + 5y = 14$$
$$x + 8y = 21$$

Step 1 $x + 8y = 21$
$x = 21 - 8y$
Step 2 $x + 8y = 21$
$(21 - 8y) + 8y = 21$
$21 = 21$
The system has infinitely many solutions.

36. Art An artist is going to sell two sizes of prints at an art fair. The artist will charge \$20 for a small print and \$45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs \$510. How many of each size print must the artist sell in order to break even at the fair?

37. Think About a Plan At a certain high school, 350 students are taking an algebra course. The ratio of boys to girls taking algebra is 33 : 37. How many more girls are taking algebra than boys?

- How can you write a system of equations to model the situation?
- Which equation will you solve for a variable in the first step of solving the system? Why?
- How can you interpret the solution in the context of the problem?

38. a. Compare and Contrast Using a graph, how can you tell when a system of linear equations has no solution?

b. Using substitution, how can you tell when a system of linear equations has no solution?

c. How can you tell by looking at a table of values if two lines will intersect in one point, no points, or an infinite number of points?

39. Fireworks A pyrotechnician plans for two fireworks to explode together at the same height in the air. They travel at speeds shown at the right. Firework B is launched 0.25 s before Firework A. How many seconds after Firework B launches will both fireworks explode?

40. Writing Let a be any real number. Will the system at the right *always*, *sometimes*, or *never* have a solution? Explain.

$$y = ax$$
$$y = ax + 4$$

41. Reasoning Explain how you can use substitution to show that the system at the right has no solution.

$$y + x = x$$
$$\frac{3x}{2y} = 4$$

Challenge

42. Agriculture A farmer grows corn, tomatoes, and sunflowers on a 320-acre farm. This year, the farmer wants to plant twice as many acres of tomatoes as acres of sunflowers. The farmer also wants to plant 40 more acres of corn than of tomatoes. How many acres of each crop should the farmer plant?

43. Track and Field Michelle and Pam are running a 200-m race. Michelle runs at an average of 7.5 m/s. Pam averages 7.8 m/s, but she starts 1 s after Michelle.

a. How long will it take Pam to catch up to Michelle?

b. Will Pam overtake Michelle before the finish line? Explain.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1, MP 4

Look back at the information on page 363 about the amounts of time Ashley uses the stair machine and rowing machine at the gym.

a. Define two variables to represent the amounts of time Ashley can spend on the two exercise machines.

b. Write a system of two equations that describes the relationships between the amounts of time Ashley spends on the two machines.

c. Solve the system of equations.

d. Interpret the solution of the system.