6-3 Solving Systems Using Elimination

Common Core State Standards

A-REI.C.5 Prove that, given . . . two equations . . . replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **Also A-REI.C.6**

MP 1, MP 2, MP 3, MP 4, MP 6

Objective To solve systems by adding or subtracting to eliminate a variable

Hmm . . . Can the methods from earlier lessons be used to solve this?

SOLVE IT!

Getting Ready!

A cafeteria sells fresh fruit by weight. All apples weigh the same, and all oranges weigh the same. What is the weight of an apple? What is the weight of an orange? How do you know?

MATHEMATICAL PRACTICES

By the Addition and Subtraction Properties of Equality, if $a = b$ and $c = d$, then $a + c = b + d$ and $a - c = b - d$. For example, $5 + 1 = 6$ and $3 + 4 = 7$, so $(5 + 1) + (3 + 4) = 6 + 7$. In the **elimination method**, you use these properties to add or subtract equations in order to eliminate a variable in a system.

Lesson Vocabulary
- elimination method

Essential Understanding There is more than one way to solve a system of equations. Some systems are written in a way that makes eliminating a variable a good method to use.

Problem 1 Solving a System by Adding Equations

What is the solution of the system? Use elimination. $2x + 5y = 17$, $6x - 5y = -9$

Plan

Which variable should you eliminate? You can eliminate either variable. Since the coefficients of y are opposites, you can add the equations to eliminate y in one step.

Step 1 Eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y.

$$\begin{array}{rl} 2x + 5y = 17 & \\ 6x - 5y = -9 & \\ \hline 8x + 0 = 8 & \text{Add the two equations.} \\ x = 1 & \text{Solve for } x. \end{array}$$

Step 2 Substitute 1 for x to solve for the eliminated variable.

$$\begin{array}{rl} 2x + 5y = 17 & \text{You can use the first equation.} \\ 2(1) + 5y = 17 & \text{Substitute 1 for } x. \\ 2 + 5y = 17 & \text{Simplify.} \\ y = 3 & \text{Solve for } y. \end{array}$$

Since $x = 1$ and $y = 3$, the solution is $(1, 3)$.

Got It? 1. What is the solution of each system? Use elimination.

a. $5x - 6y = -32$
$3x + 6y = 48$

b. $-3x - 3y = 9$
$3x - 4y = 5$

Problem 2 Solving a System by Subtracting Equations

Multiple Choice The theater club sells a total of 101 tickets to its first play. A student ticket costs $1. An adult ticket costs $2.50. Total ticket sales are $164. How many student tickets were sold?

(A) 25 (B) 42 (C) 59 (D) 76

Define Let a = the number of adult tickets sold.
Let s = the number of student tickets sold.

Relate total number of tickets | total ticket sales

Write $a + s = 101$ | $2.5a + s = 164$

Think
How is this problem similar to Problem 1?
In each problem, you are looking for coefficients of one variable that are either the same or opposites. Here, the coefficients of s are the same, so eliminate s.

Step 1 Eliminate one variable. Since the difference of the coefficients of s is 0, eliminate s.

$$\begin{array}{rl} a + s = 101 & \\ \underline{2.5a + s = 164} & \\ -1.5a + 0 = -63 & \text{Subtract the equations.} \\ a = 42 & \text{Solve for } a. \end{array}$$

Step 2 Solve for the eliminated variable. Use either equation.

$$\begin{array}{rl} a + s = 101 & \text{You can use the first equation.} \\ 42 + s = 101 & \text{Substitute 42 for } a. \\ s = 59 & \text{Solve for } s. \end{array}$$

There were 59 student tickets sold. The correct answer is C.

Check 42 is close to 40 and 59 is close to 60. The total number of tickets is about $40 + 60 = 100$, which is close to 101. The total sales are about $\$2.50(40) + \$60 = \$160$, which is close to $164. The solution is reasonable.

Got It? 2. Washing 2 cars and 3 trucks takes 130 min. Washing 2 cars and 5 trucks takes 190 min. How long does it take to wash each type of vehicle?

In Problems 1 and 2, a variable is eliminated because the sum or difference of its coefficients is zero. From the Multiplication Property of Equality, you know that you can multiply each side of an equation to get a new equation that is equivalent to the original. That is, $a + b = c$ is equivalent to $d(a + b) = dc$, or $da + db = dc$. Since this is true, you can eliminate a variable by adding or subtracting, if you first multiply an equation by an appropriate number. You can prove that the results are the same simply by substituting the values for the variables in the original equations to show that the equations are true.

Problem 3 Solving a System by Multiplying One Equation

What is the solution of the system? Use elimination. $-2x + 15y = -32$, $7x - 5y = 17$

Know
A system of equations that can't quickly be solved by graphing or substitution

Need
The solution of the system

Plan
Multiply one or both equations by a constant so that the coefficients of one variable are the same or opposites. Then eliminate the variable.

Step 1 To eliminate one variable, you can multiply $7x - 5y = 17$ by 3 and then add.

$-2x + 15y = -32$
$7x - 5y = 17$ Multiply by 3. $21x - 15y = 51$

$-2x + 15y = -32$
$19x + 0 = 19$ Add the equations.

$x = 1$ Solve for x.

Step 2 Solve for the eliminated variable. Use either of the original equations.

$7x - 5y = 17$ You can use the second equation.
$7(1) - 5y = 17$ Substitute 1 for x.
$y = -2$ Solve for y.

The solution is $(1, -2)$.

Think
How can you show that this is a solution to the original system?
Substitute (–1, 2) in the original system.

Got It? **3. a.** How can you use the Multiplication Property of Equality to change an equation in this system in order to solve it using elimination? $-5x - 2y = 6$, $3x + 6y = 6$

b. Write and solve a revised system.

c. Show that the solution of the revised system is a solution of the original system.

Problem 4 Solving a System by Multiplying Both Equations

What is the solution of the system? Use elimination. $3x + 2y = 1$, $4x + 3y = -2$

Step 1 Multiply each equation so you can eliminate one variable.

$3x + 2y = 1$ Multiply by 3. $9x + 6y = 3$
$4x + 3y = -2$ Multiply by 2. $8x + 6y = -4$

$x + 0 = 7$ Subtract the equations.

Step 2 Solve for the eliminated variable. Use either of the original equations.

$3x + 2y = 1$ You can use the first equation.
$3(7) + 2y = 1$ Substitute 7 for x.
$2y = -20$ Subtract 21 from each side. Simplify.
$y = -10$ Solve for y.

The solution is $(7, -10)$.

Plan
How can you get started?
Find the LCM of the coefficients of the variable that you want to eliminate. Multiply to make the coefficients equal to the LCM.

Got It? **4. a.** How can you use the Multiplication Property of Equality to change the equations in this system in order to solve it using elimination?

$$4x + 3y = -19$$
$$3x - 2y = -10$$

b. Write and solve a revised system.

c. Show that the solution of the revised system is a solution of the original system.

Recall that if you get a false statement as you solve a system, then the system has no solution. If you get an identity, then the system has infinitely many solutions.

Problem 5 Finding the Number of Solutions

How many solutions does the system have?

$$2x + 6y = 18$$
$$x + 3y = 9$$

Think

Could you have solved this problem another way?
Yes. For example, you could have multiplied the second equation by 2 and subtracted.

Multiply the second equation by -2.

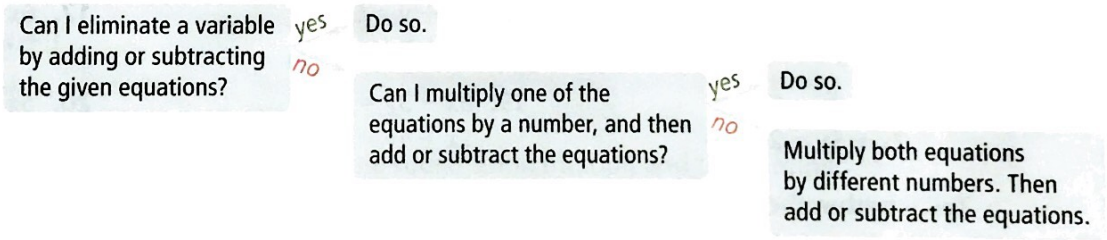
$$\begin{array}{l} 2x + 6y = 18 \\ x + 3y = 9 \end{array} \quad \text{Multiply by } -2. \quad \begin{array}{l} 2x + 6y = 18 \\ -2x - 6y = -18 \\ \hline 0 = 0 \end{array} \quad \text{Add the equations.}$$

Because $0 = 0$ is an identity, there are infinitely many solutions.

Got It? **5.** How many solutions does the system have?

$$-2x + 5y = 7$$
$$-2x + 5y = 12$$

The flowchart below can help you decide which steps to take when solving a system of equations using elimination.



Lesson Check

Do you know HOW?

Solve each system using elimination.

1. $3x - 2y = 0$
   $4x + 2y = 14$
2. $3p + q = 7$
   $2p - 2q = -6$
3. $3x - 2y = 1$
   $8x + 3y = 2$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** If you add two equations in two variables and the sum is an equation in one variable, what method are you using to solve the system? Explain.
5. **Reasoning** Explain how the Addition Property of Equality allows you to add equations.
6. **Writing** Explain how you would solve a system of equations using elimination.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

See Problems 1 and 2

Solve each system using elimination.

7. $3x + 3y = 27$
   $x - 3y = -11$

8. $-x + 5y = 13$
   $x - y = 15$

9. $2x + 4y = 22$
   $2x - 2y = -8$

10. $4x - 7y = 3$
    $x - 7y = -15$

11. $5x - y = 0$
    $3x + y = 24$

12. $6x + 5y = 39$
    $3x + 5y = 27$

13. **Talent Show** Your school's talent show will feature 12 solo acts and 2 ensemble acts. The show will last 90 min. The 6 solo performers judged best will give a repeat performance at a second 60-min show, which will also feature the 2 ensemble acts. Each solo act lasts x minutes, and each ensemble act lasts y minutes.
    a. Write a system of equations to model the situation.
    b. Solve the system from part (a). How long is each solo act? How long is each ensemble act?

14. **Furniture** A carpenter is designing a drop-leaf table with two drop leaves of equal size. The lengths of the table when one leaf is folded up and when both leaves are folded up are shown. How long is the table when no leaves are folded up?

See Problems 3 and 4

Solve each system using elimination.

15. $2x + 3y = 9$
    $x + 5y = 8$

16. $3x + y = 5$
    $2x - 2y = -2$

17. $6x + 4y = 42$
    $-3x + 3y = -6$

18. $3x + 2y = 17$
    $2x + 5y = 26$

19. $6x - 3y = 15$
    $7x + 4y = 10$

20. $5x - 9y = -43$
    $3x + 8y = 68$

See Problem 5

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

21. $9x + 8y = 15$
    $9x + 8y = 30$

22. $3x + 4y = 24$
    $6x + 8y = 24$

23. $5x - 3y = 10$
    $10x + 6y = 20$

24. $2x - 5y = 17$
    $6x - 15y = 51$

25. $4x - 7y = 15$
    $-8x + 14y = -30$

26. $4x - 8y = 15$
    $-5x + 10y = -30$

B Apply

27. **Think About a Plan** A photo studio offers portraits in 8×10 and wallet-sized formats. One customer bought two 8×10 portraits and four wallet-sized portraits and paid \$52. Another customer bought three 8×10 portraits and two wallet-sized portraits and paid \$50. What is the cost of an 8×10 portrait? What is the cost of a wallet-sized portrait?
    - Can you eliminate a variable simply by adding or subtracting?
    - If not, how many of the equations do you need to multiply by a constant?

28. **Reasoning** A toy store worker packed two boxes of identical dolls and plush toys for shipping in boxes that weigh 1 oz when empty. One box held 3 dolls and 4 plush toys. The worker marked the weight as 12 oz. The other box held 2 dolls and 3 plush toys. The worker marked the weight as 10 oz. Explain why the worker must have made a mistake.

29. **Error Analysis** A student solved a system of equations by elimination. Describe and correct the error made in the part of the solution shown.

$5x + 4y = 2 \quad \times 3 \rightarrow \quad 15x + 12y = 6$

$3x + 3y = -3 \quad \times 4 \rightarrow \quad 12x + 12y = -3$

$3x + 0 = 9$

$x = 3$

30. **Nutrition** Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

31. **Open-Ended** Write a system of equations that can be solved efficiently by elimination. Explain what you would do to eliminate one of the variables. Then solve the system.

Solve each system using any method. Explain why you chose the method you used.

32. $y = 2.5x$
$2y + 3x = 32$

33. $2x + y = 4$
$6x + 7y = 12$

34. $3x + 2y = 5$
$4x + 5y = 16$

35. $y = \frac{2}{3}x + 1$
$2x + 3y = 27$

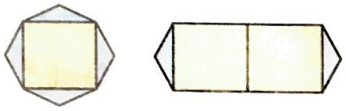
36. $x + y = 1.5$
$2x + y = 1$

37. $\frac{1}{3}x + \frac{1}{2}y = 0$
$\frac{1}{2}x + \frac{1}{5}y = \frac{11}{5}$

38. **Compare and Contrast** What do the substitution method and the elimination method have in common? Explain. Give an example of a system that you would prefer to solve using one method instead of the other. Justify your choice.

39. **Vacations** A hotel offers two activity packages. One costs \$192 and includes 3 h of horseback riding and 2 h of parasailing. The second costs \$213 and includes 2 h of horseback riding and 3 h of parasailing. What is the cost for 1 h of each activity?

40. **Geometry** Each of the squares in the figures shown at the right has the same area, and each of the triangles has the same area. The total area of Figure A is 141 cm^2. The total area of Figure B is 192 cm^2. What is the area of each square and each triangle?



A B

Challenge

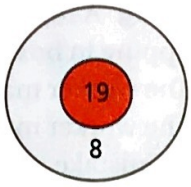
Solve each system using elimination.

41. $\frac{2}{x} - \frac{3}{y} = -5$
$\frac{4}{x} + \frac{6}{y} = 14$

42. $2x = 5(2 - y)$
$y = 3(-x + 5)$

43. $2x - 3y + z = 0$
$2x + y + z = 12$
$y - z = 4$

44. Reasoning Use the dartboard at the right. Can you score exactly 100 points with seven darts that all land on the board? Explain.



Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

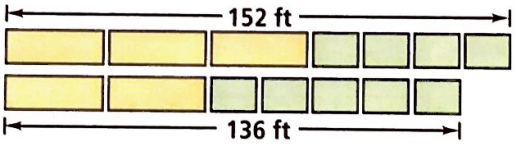
45. What is the value of the y-coordinate of the solution of the given system?

$$4x + 3y = 33$$
$$3x + 2y = 23$$

46. What is the y-intercept of $2x + 5y = 15$?

47. You buy a toothbrush for \$2.83 and a tube of toothpaste for \$2.37. There is a 5% sales tax. Including the tax, what is the total cost in dollars of your purchases?

48. Three fire trucks and 4 ambulances can fit into a parking lane 152 ft long. Two fire trucks and 5 ambulances can fit into a lane 136 ft long. How many feet long must a parking lane be for 1 fire truck and 5 ambulances? Assume there is 1 ft of space between each vehicle.



49. You are competing in a mountain bike race. Your average speed is 10 mi/h. If the racecourse is 65 mi long, how many minutes will it take you to finish the race?

Mixed Review

Solve each system using substitution.

See Lesson 6-2.

50. $y = \frac{1}{2}x$
$2y + 3x = 28$

51. $x - 7 = y$
$2x - y = 41$

52. $x + 2y = -1$
$3x - 5y = 30$

Solve each inequality.

See Lesson 3-4.

53. $4 - 2a < 3a - 1$

54. $3(2x - 1) \geq 5x + 4$

55. $2.7 + 2b > 3.4 - 1.5b$

Get Ready! **To prepare for Lesson 6-4, do Exercise 56.**

56. Two trains run on two sets of parallel tracks. The first train leaves a city $\frac{1}{2}$ h before the second train. The first train travels at 55 mi/h. The second train travels at 65 mi/h. How long does it take for the second train to pass the first train?

See Lesson 2-4.