

Applications of Linear Systems

Common Core State Standards

A-REI.C.6 Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. **Also N-Q.A.3, A-CED.A.3**
MP 1, MP 2, MP 3, MP 4

Objective To choose the best method for solving a system of linear equations



Use what you've learned about systems to solve this problem.



Getting Ready!

The taller candle burns at a rate of 1.15 in. per hour. The shorter candle burns at a rate of 0.75 in. per hour. After how many hours will they be the same height? Explain your reasoning.



Essential Understanding You can solve systems of linear equations using a graph, the substitution method, or the elimination method. The best method to use depends on the forms of the given equations and how precise the solution should be.

take note

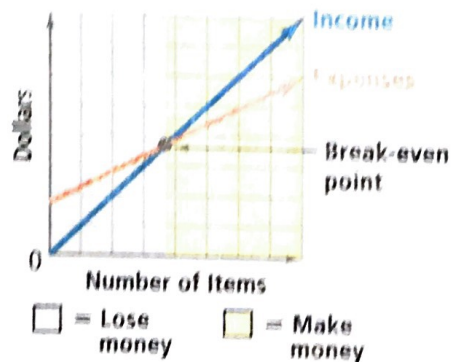
Concept Summary Choosing a Method for Solving Linear Systems

Method	When to Use
Graphing	When you want a visual display of the equations, or when you want to estimate a solution
Substitution	When one equation is already solved for one of the variables, or when it is easy to solve for one of the variables
Elimination	When the coefficients of one variable are the same or opposites, or when it is not convenient to use graphing or substitution

Systems of equations are useful for modeling problems involving mixtures, rates, and break-even points.

The break-even point for a business is the point at which income equals expenses. The graph shows the break-even point for one business.

Notice that the values of y on the red line represent dollars spent on expenses. The values of y on the blue line represent dollars received as income. So y is used to represent both expenses and income.



Problem 1 Finding a Break-Even Point

Business A fashion designer makes and sells hats. The material for each hat costs \$5.50. The hats sell for \$12.50 each. The designer spends \$1400 on advertising. How many hats must the designer sell to break even?

Think

What equations should you write?

The break-even point is when income equals expenses, so write one equation for income and one equation for expenses.

Step 1 Write a system of equations. Let x = the number of hats sold, and let y = the number of dollars of expense or income.

$$\text{Expense: } y = 5.5x + 1400 \qquad \text{Income: } y = 12.5x$$

Step 2 Choose a method. Use substitution since both equations are solved for y .

$$\begin{aligned} y &= 5.5x + 1400 && \text{Start with one equation.} \\ 12.5x &= 5.5x + 1400 && \text{Substitute } 12.5x \text{ for } y. \\ 7x &= 1400 && \text{Subtract } 5.5x \text{ from each side.} \\ x &= 200 && \text{Divide each side by 7.} \end{aligned}$$

Since x is the number of hats, the designer must sell 200 hats to break even.



Got It? 1. A puzzle expert wrote a new sudoku puzzle book. His initial costs are \$864. Binding and packaging each book costs \$.80. The price of the book is \$2. How many copies must be sold to break even?

In real-world situations, you need to consider the constraints described in the problem in order to write equations. Once you solve an equation, you need to consider the viability of the solution. For example, a solution that has a negative number of hours is not a viable solution.

Problem 2 Identifying Constraints and Viable Solutions

Zoo The local zoo is filling two water tanks for the elephant exhibit. One water tank contains 50 gal of water and is filled at a constant rate of 10 gal/h. The second water tank contains 29 gal of water and is filled at a constant rate of 3 gal/h. When will the two tanks have the same amount of water? Explain.

Step 1 Write a system of equations. Let x = the number of hours the tanks are filling and let y = the number of gallons in the tank.

$$\text{Tank 1: } y = 10x + 50 \qquad \text{Tank 2: } y = 3x + 29$$

Step 2 The system is easy to solve using substitution. Substitute $10x + 50$ for y in the second equation and solve for x .

$$\begin{aligned} y &= 3x + 29 && \text{Write the second equation.} \\ 10x + 50 &= 3x + 29 && \text{Substitute } 10x + 50 \text{ for } y. \\ 7x + 50 &= 29 && \text{Subtract } 3x \text{ from each side. Then simplify.} \\ 7x &= -21 && \text{Subtract 50 from each side. Then simplify.} \\ x &= -3 && \text{Divide each side by 7.} \end{aligned}$$

Step 3 Substitute -3 for x in either equation and solve for y .

$$\begin{aligned} y &= 10(-3) + 50 && \text{Substitute } -3 \text{ for } x \text{ in the first equation.} \\ y &= 20 && \text{Simplify.} \end{aligned}$$

The solution to the system is $(-3, 20)$. The solution $(-3, 20)$ is not a viable solution because it is not possible to have time be -3 hours. So, the tanks never have the same amount of water.

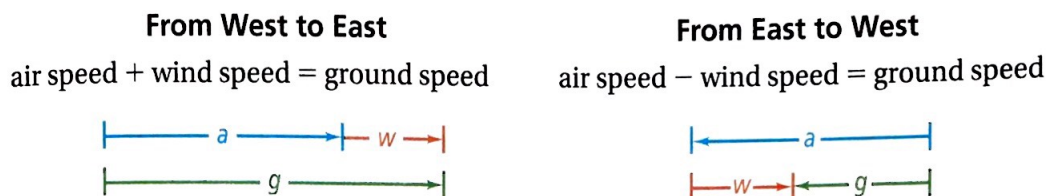
Think

What are the constraints of the system?

If x represents time, then $x \geq 0$. If y represents the number of gallons, then $y \geq 0$.

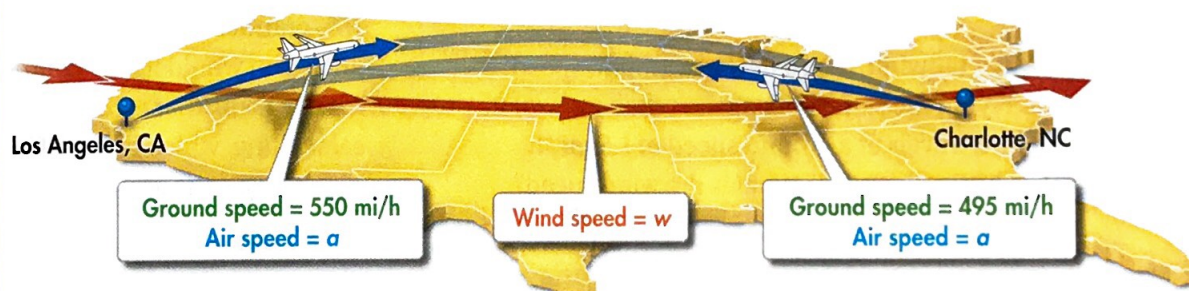
- Got It?** 2. The zoo has two other water tanks that are leaking. One tank contains 10 gal of water and is leaking at a constant rate of 2 gal/h. The second tank contains 6 gal of water and is leaking at a constant rate of 4 gal/h. When will the tanks have the same amount of water? Explain.

When a plane travels from west to east across the United States, the steady west-to-east winds act as tailwinds. This increases the plane's speed relative to the ground. When a plane travels from east to west, the winds act as headwinds. This decreases the plane's speed relative to the ground.



Problem 3 Solving a Wind or Current Problem

Travel A traveler flies from Charlotte, North Carolina, to Los Angeles, California. At the same time, another traveler flies from Los Angeles to Charlotte. The air speed of each plane is the same. The ground speeds are shown below. What is the air speed? What is the wind speed?



Use the ground speed with the tailwind and with the headwind to write the system. Let a = the air speed of the planes. Let w = the wind speed.

air speed	+	wind speed	=	ground speed	air speed	-	wind speed	=	ground speed
a		w		with tailwind	a		w		with headwind
a	+	w	=	550	a	-	w	=	495

Choose a method to solve the system. Use elimination.

$$\begin{array}{r} a + w = 550 \\ a - w = 495 \\ \hline 2a + 0 = 1045 \end{array}$$

Add the equations.

$$a = 522.5 \quad \text{Solve for } a.$$

Substitute 522.5 for a in either equation and solve for w .

$$522.5 + w = 550 \quad \text{Substitute 522.5 for } a \text{ in the first equation.}$$

$$w = 27.5 \quad \text{Solve for } w.$$

The air speed is 522.5 mi/h. The wind speed is 27.5 mi/h.

Think

How are the speeds related?

The air speed is a plane's speed with no wind. Add wind speed and air speed to get the ground speed with a tailwind. Subtract wind speed from air speed to find the ground speed with a headwind.



Got It?

3. a. You row upstream at a speed of 2 mi/h. You travel the same distance downstream at a speed of 5 mi/h. What would be your rowing speed in still water? What is the speed of the current?
- b. **Reasoning** Suppose your rowing speed in still water is 3 mi/h and the speed of the current is 4 mi/h. What happens when you try to row upstream?



Lesson Check

Do you know HOW?

- Newsletters** Printing a newsletter costs \$1.50 per copy plus \$450 in printer's fees. The copies are sold for \$3 each. How many copies of the newsletter must be sold to break even?
- Jewelry** A metal alloy is a metal made by blending 2 or more types of metal. A jeweler has supplies of two metal alloys. One alloy is 30% gold and the other is 10% gold. How much of each alloy should the jeweler combine to create 4 kg of an alloy containing 15% gold?
- Flying** With a tailwind, a bird flew at a ground speed of 3 mi/h. Flying the same path against the same wind, the bird travels at a ground speed of 1.5 mi/h. What is the bird's air speed? What is the wind speed?

Do you UNDERSTAND?



- Vocabulary** What is the relationship between income and expenses before a break-even point is reached? What is the relationship between income and expenses after a break-even point is reached?
- Reasoning** Which method would you use to solve the following system? Explain.

$$3x + 2y = 9$$

$$-2x + 3y = 5$$
- Reasoning** One brand of cranberry-apple drink is 15% cranberry juice. Another brand is 40% cranberry juice. You would like to combine the brands to make a drink that is 25% cranberry juice. Without calculating, which brand of juice will you need more of to make your drink? Explain.



Practice and Problem-Solving Exercises



A Practice

- Business** A bicycle store costs \$2400 per month to operate. The store pays an average of \$60 per bike. The average selling price of each bicycle is \$120. How many bicycles must the store sell each month to break even?
- Theater** Producing a musical costs \$88,000 plus \$5900 per performance. One sold-out performance earns \$7500 in revenue. If every performance sells out, how many performances are needed to break even?
- Investment** You split \$1500 between two savings accounts. Account A pays annual 5% interest and Account B pays 4% annual interest. After one year, you have earned a total of \$69.50 in interest. How much money did you invest in each account? Explain.

See Problem 1.

See Problem 2.



- Biology** A group of scientists studied the effect of a chemical on various strains of bacteria. Strain A started with 6000 cells and decreased at a constant rate of 2000 cells per hour after the chemical was applied. Strain B started with 2000 cells and decreased at a constant rate of 1000 cells per hour after the chemical was applied. When will the strains have the same number of cells? Explain.

11. **Airports** A traveler is walking on a moving walkway in an airport. The traveler must walk back on the walkway to get a bag he forgot. The traveler's groundspeed is 2 ft/s against the walkway and 6 ft/s with the walkway. What is the traveler's speed off the walkway? What is the speed of the moving walkway?
12. **Kayaking** A kayaker paddles upstream from camp to photograph a waterfall and returns. The kayaker's speed while traveling upstream and downstream is shown below. What is the kayaker's speed in still water? What is the speed of the current?



13. **Money** You have a jar of pennies and quarters. You want to choose 15 coins that are worth exactly \$4.35.
- Write and solve a system of equations that models the situation.
 - Is your solution reasonable in terms of the original problem? Explain.

Apply

Solve each system. Explain why you chose the method you used.

14. $4x + 5y = 3$
 $3x - 2y = 8$

15. $2x + 7y = -20$
 $y = 3x + 7$

16. $5x + 2y = 17$
 $x - 2y = 8$

17. **Reasoning** Find A and B so that the system below has the solution $(2, 3)$.

$$\begin{aligned} Ax - 2By &= 6 \\ 3Ax - By &= -12 \end{aligned}$$

18. **Think About a Plan** A tugboat can pull a boat 24 mi downstream in 2 h. Going upstream, the tugboat can pull the same boat 16 mi in 2 h. What is the speed of the tugboat in still water? What is the speed of the current?
- How can you use the formula $d = rt$ to help you solve the problem?
 - How are the tugboat's speeds when traveling upstream and downstream related to its speed in still water and the speed of the current?

19. **Open-Ended** Without solving, decide which method you would use to solve each system: *graphing*, *substitution*, or *elimination*. Explain.

19. $y = 3x - 1$
 $y = 4x$

20. $3m - 4n = 1$
 $3m - 2n = -1$

21. $4s - 3t = 8$
 $t = -2s - 1$

22. **Business** A perfume maker has stocks of two perfumes on hand. Perfume A sells for \$15 per ounce. Perfume B sells for \$35 per ounce. How much of each should be combined to make a 3-oz bottle of perfume that can be sold for \$63?

23. **Chemistry** In a chemistry lab, you have two vinegars. One is 5% acetic acid, and one is 6.5% acetic acid. You want to make 200 mL of a vinegar with 6% acetic acid. How many milliliters of each vinegar do you need to mix together?

24. **Boating** A boat is traveling in a river with a current that has a speed of 1.5 km/h. In one hour, the boat can travel twice the distance downstream that it can travel upstream. What is the boat's speed in still water?

© 25. **Reasoning** A student claims that the best way to solve the system at the right is by substitution. Do you agree? Explain.

26. **Entertainment** A contestant on a quiz show gets 150 points for every correct answer and loses 250 points for each incorrect answer. After answering 20 questions, the contestant has 200 points. How many questions has the contestant answered correctly? Incorrectly?

Challenge

27. **Number Theory** You can represent the value of any two-digit number with the expression $10a + b$, where a is the tens' place digit and b is the ones' place digit. For example, if a is 5 and b is 7, then the value of the number is $10(5) + 7$, or 57. What two-digit number is described below?

- The ones' place digit is one more than twice the tens' place digit.
- The value of the number is two more than five times the ones' place digit.

28. **Mixed Nuts** You want to sell 1-lb jars of mixed peanuts and cashews for \$5. You pay \$3 per pound for peanuts and \$6 per pound for cashews. You plan to combine 4 parts peanuts and 1 part cashews to make your mix. You have spent \$70 on materials to get started. How many jars must you sell to break even?

$$y - 3x = 4$$
$$y - 6x = 12$$

Standardized Test Prep

SAT/ACT

29. Last year, one fourth of the students in your class played an instrument. This year, 6 students joined the class. Four of the new students play an instrument. Now, one third of the students play an instrument. How many students are in your class now?

- (A) 18 (B) 24 (C) 30 (D) 48

30. Which answer choice shows $2x - y = z$ correctly solved for y ?

- (F) $y = 2x + z$ (G) $y = 2x - z$ (H) $y = -2x + z$ (I) $y = -2x - z$

Short Response

31. What is an equation of a line passing through the points (3, 1) and (4, 3) written in slope-intercept form?

Mixed Review

Solve each system using elimination.

32. $x + 3y = 11$
 $2x + 3y = 4$

33. $2x + 4y = -12$
 $-6x + 5y = 2$

34. $5x + 8y = 40$
 $3x - 10y = -13$

Get Ready! To prepare for Lesson 6-5, do Exercises 35–37.

Solve each inequality. Check your solution.

35. $3a + 5 > 20$

36. $2d - 3 \geq 4d + 2$

37. $3(q + 4) \leq -2q - 8$

See Lesson 6-3

See Lesson 3-4