

6-5

Linear Inequalities

Objectives To graph linear inequalities in two variables
To use linear inequalities when modeling real-world situations

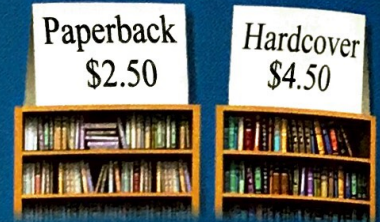


One of these and one of those . . . no, wait. Three of these . . .



Getting Ready!

You are buying paperback and hardcover books at a book sale. You can spend at most \$20. What are the possible combinations of paperback and hardcover books that you can buy? Explain.



Lesson Vocabulary

- linear inequality
- solution of an inequality

A **linear inequality** in two variables, such as $y > x - 3$, can be formed by replacing the equal sign in a linear equation with an inequality symbol. A **solution of an inequality** in two variables is an ordered pair that makes the inequality true.

Essential Understanding A linear inequality in two variables has an infinite number of solutions. These solutions can be represented in the coordinate plane as the set of all points on one side of a boundary line.

Think

Have you tested solutions before?

Yes. You have tested whether ordered pairs are solutions of equations. Now you will test ordered pairs to see whether they satisfy an inequality.



Problem 1 Identifying Solutions of a Linear Inequality

Is the ordered pair a solution of $y > x - 3$?

A (1, 2)

$$y > x - 3$$

← Write the inequality. →

$$2 \stackrel{?}{>} 1 - 3$$

← Substitute. →

$$2 > -2 \checkmark$$

← Simplify. →

(1, 2) is a solution.

B (-3, -7)

$$y > x - 3$$

$$-7 \stackrel{?}{>} -3 - 3$$

$$-7 > -6 \times$$

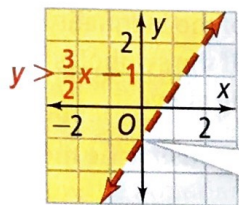
(-3, -7) is *not* a solution.



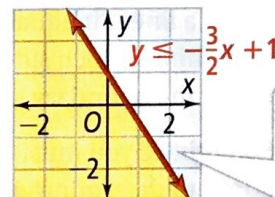
Got It? 1. a. Is (3, 6) a solution of $y \leq \frac{2}{3}x + 4$?

b. **Reasoning** Suppose an ordered pair is not a solution of $y > x + 10$. Must it be a solution of $y < x + 10$? Explain.

The graph of a linear inequality in two variables consists of all points in the coordinate plane that represent solutions. The graph is a region called a *half-plane* that is bounded by a line. All points on one side of the boundary line are solutions, while all points on the other side are not solutions.



Each point on a *dashed* line is not a solution. A dashed line is used for inequalities with $>$ or $<$.



Each point on a *solid* line is a solution. A solid line is used for inequalities with \geq or \leq .

Think

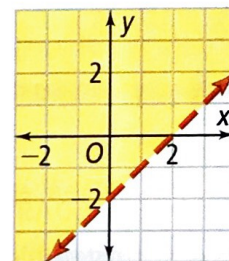
Why does $y = x - 2$ represent the **boundary line**?

For any value of x , the corresponding value of y is the boundary between values of y that are greater than $x - 2$ and values of y that are less than $x - 2$.

Problem 2 Graphing an Inequality in Two Variables

What is the graph of $y > x - 2$?

First, graph the boundary line $y = x - 2$. Since the inequality symbol is $>$, the points on the boundary line are *not* solutions. Use a dashed line to indicate that the points are not included in the solution.



To determine which side of the boundary line to shade, test a point that is not on the line. For example, test the point $(0, 0)$.

$$y > x - 2$$

$$0 \stackrel{?}{>} 0 - 2 \quad \text{Substitute } (0, 0) \text{ for } (x, y).$$

$$0 > -2 \quad \checkmark \quad (0, 0) \text{ is a solution.}$$

Because the point $(0, 0)$ is a solution of the inequality, so are all the points on the same side of the boundary line as $(0, 0)$. Shade the area above the boundary line.

Got It? 2. What is the graph of $y \leq \frac{1}{2}x + 1$?

An inequality in one variable can be graphed on a number line or in the coordinate plane. The boundary line will be a horizontal or vertical line.

Problem 3 Graphing a Linear Inequality in One Variable

What is the graph of each inequality in the coordinate plane?

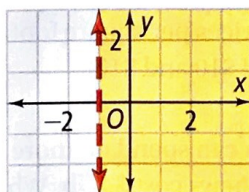
A $x > -1$

Graph $x = -1$ using a dashed line. Use $(0, 0)$ as a test point.

$$x > -1$$

$$0 > -1 \quad \checkmark$$

Shade on the side of the line that contains $(0, 0)$.



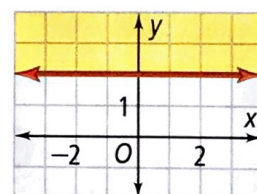
B $y \geq 2$

Graph $y = 2$ using a solid line. Use $(0, 0)$ as a test point.

$$y \geq 2$$

$$0 \geq 2 \quad \times$$

Shade on the side of the line that does *not* contain $(0, 0)$.



Think

Have you graphed inequalities like these before?

Yes. In Lesson 3-1, you graphed inequalities in one variable on a number line. Here you graph them in the coordinate plane.

- Got It?** 3. What is the graph of each inequality?
 a. $x < -5$

b. $y \leq 2$

When a linear inequality is solved for y , the direction of the inequality symbol determines which side of the boundary line to shade. If the symbol is $<$ or \leq , shade below the boundary line. If the symbol is $>$ or \geq , shade above it.

Sometimes you must first solve an inequality for y before using the method described above to determine where to shade.

Problem 4 Rewriting to Graph an Inequality

Interior Design An interior decorator is going to remodel a kitchen. The wall above the stove and the counter is going to be redone as shown. The owners can spend \$420 or less. Write a linear inequality and graph the solutions. What are three possible prices for the wallpaper and tiles?



Let x = the cost per square foot of the paper.

Let y = the cost per square foot of the tiles.

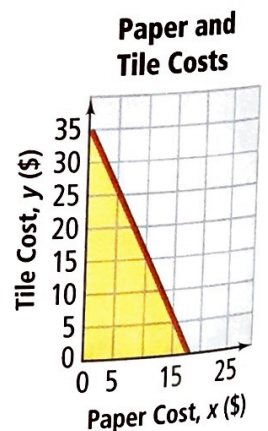
Write an inequality and solve it for y .

$$24x + 12y \leq 420 \quad \text{Total cost is \$420 or less.}$$

$$12y \leq -24x + 420 \quad \text{Subtract } 24x \text{ from each side.}$$

$$y \leq -2x + 35 \quad \text{Divide each side by 12.}$$

Graph $y \leq -2x + 35$. The inequality symbol is \leq , so the boundary line is solid and you shade below it. The graph only makes sense in the first quadrant. Three possible prices per square foot for wallpaper and tile are \$5 and \$25, \$5 and \$15, and \$10 and \$10.



Think

Which inequality symbol should you use?

You must read the problem statement carefully. Here "\$420 or less" means that the solution includes, but cannot exceed, \$420, so use \leq .

- Got It?** 4. For a party, you can spend no more than \$12 on nuts. Peanuts cost \$2/lb. Cashews cost \$4/lb. What are three possible combinations of peanuts and cashews you can buy?

Think

Can you eliminate choices?

Yes. The boundary line is solid and the region below it is shaded, so you know the inequality symbol must be \leq . You can eliminate choices C and D.



Problem 5 Writing an Inequality From a Graph

Multiple Choice Which inequality represents the graph at the right?

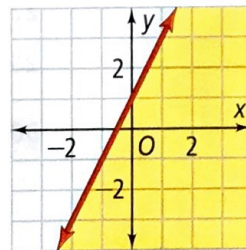
(A) $y \leq 2x + 1$

(C) $y \geq 2x + 1$

(B) $y \leq x + 1$

(D) $y < 2x + 1$

The slope of the line is 2 and the y -intercept is 1, so the equation of the boundary line is $y = 2x + 1$. The boundary line is solid, so the inequality symbol is either \leq or \geq . The symbol must be \leq , because the region below the boundary line is shaded. The inequality is $y \leq 2x + 1$.



The correct answer is A.



Got It! 5. You are writing an inequality from a graph. The boundary line is dashed and has slope $\frac{1}{3}$ and y -intercept -2 . The area above the line is shaded. What inequality should you write?



Lesson Check

Do you know HOW?

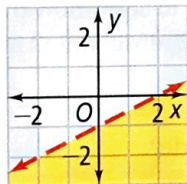
1. Is $(-1, 4)$ a solution of the inequality $y < 2x + 5$?

Graph each linear inequality.

2. $y \leq -2x + 3$

3. $x < -1$

4. What is an inequality that represents the graph at the right?



Do you UNDERSTAND?



MATHEMATICAL PRACTICES

5. **Vocabulary** How is a linear inequality in two variables like a linear equation in two variables? How are they different?
6. **Writing** To graph the inequality $y < \frac{3}{2}x + 3$, do you shade above or below the boundary line? Explain.
7. **Reasoning** Write an inequality that describes the region of the coordinate plane *not* included in the graph of $y < 5x + 1$.



Practice and Problem-Solving Exercises



MATHEMATICAL PRACTICES

A Practice

Determine whether the ordered pair is a solution of the linear inequality.

See Problem 1.

8. $y \leq -2x + 1$; $(2, 2)$

9. $x < 2$; $(-1, 0)$

10. $y \geq 3x - 2$; $(0, 0)$

11. $y > x - 1$; $(0, 1)$

12. $y \geq -\frac{2}{5}x + 4$; $(0, 0)$

13. $3y > 5x - 12$; $(-6, 1)$

Graph each linear inequality.

See Problem 2.

14. $y \leq x - 1$

15. $y \geq 3x - 2$

16. $y < -4x - 1$

17. $y > 2x - 6$

18. $y < 5x - 5$

19. $y \leq \frac{1}{2}x - 3$

20. $y > -3x$

21. $y \geq -x$

Graph each inequality in the coordinate plane.

◀ See Problems 3 and 4.

22. $x \leq 4$

23. $y \geq -1$

24. $x > -2$

25. $y < -4$

26. $-2x + y \geq 3$

27. $x + 3y < 15$

28. $4x - y > 2$

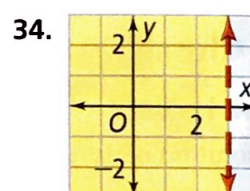
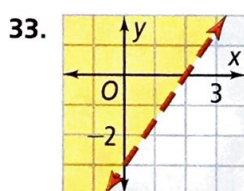
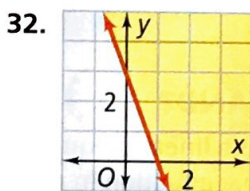
29. $-x + 0.25y \leq -1.75$

30. **Carpentry** You budget \$200 for wooden planks for outdoor furniture. Cedar costs \$2.50 per foot and pine costs \$1.75 per foot. Let x = the number of feet of cedar and let y = the number of feet of pine. What is an inequality that shows how much of each type of wood can be bought? Graph the inequality. What are three possible amounts of each type of wood that can be bought within your budget?

31. **Business** A fish market charges \$9 per pound for cod and \$12 per pound for flounder. Let x = the number of pounds of cod. Let y = the number of pounds of flounder. What is an inequality that shows how much of each type of fish the store must sell today to reach a daily quota of at least \$120? Graph the inequality. What are three possible amounts of each fish that would satisfy the quota?

Write a linear inequality that represents each graph.

◀ See Problem 5.



B Apply

35. **Think About a Plan** A truck that can carry no more than 6400 lb is being used to transport refrigerators and upright pianos. Each refrigerator weighs 250 lb and each piano weighs 475 lb. Write and graph an inequality to show how many refrigerators and how many pianos the truck could carry. Will 12 refrigerators and 8 pianos overload the truck? Explain.

- What inequality symbol should you use?
- Which side of the boundary line should you shade?

36. **Employment** A student with two summer jobs earns \$10 per hour at a cafe and \$8 per hour at a market. The student would like to earn at least \$800 per month.

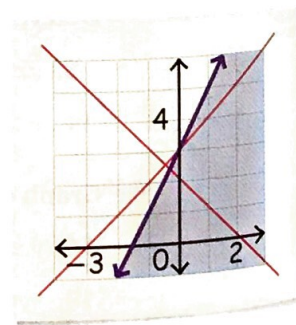
- Write and graph an inequality to represent the situation.
- The student works at the market for 60 h per month and can work at most 90 h per month. Can the student earn at least \$800 each month? Explain how you can use your graph to determine this.

37. **Error Analysis** A student graphed $y \geq 2x + 3$ as shown at the right. Describe and correct the student's error.

38. **Writing** When graphing an inequality, can you always use $(0, 0)$ as a test point to determine where to shade? If not, how would you choose a test point?

C Challenge

39. **Music Store** A music store sells used CDs for \$5 each and buys used CDs for \$1.50 each. You go to the store with \$20 and some CDs to sell. You want to have at least \$10 left when you leave the store. Write and graph an inequality to show how many CDs you could buy and sell.



40. **Groceries** At your grocery store, milk normally costs \$3.60 per gallon. Ground beef costs \$3 per pound. Today there are specials: Milk is discounted \$.50 per gallon, and ground beef is 20% off. You want to spend no more than \$20. Write and graph a linear inequality to show how many gallons of milk and how many pounds of ground beef you can buy today.

© 41. **Reasoning** You are graphing a linear inequality of the form $y > mx + b$. The point $(1, 2)$ is not a solution, but $(3, 2)$ is. Is the slope of the boundary line *positive*, *negative*, *zero*, or *undefined*? Explain.

Standardized Test Prep

SAT/ACT

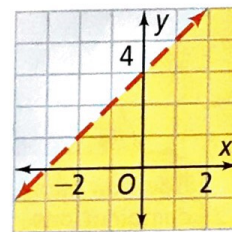
42. What is the equation of the graph shown?

(A) $y + x \geq -3$

(C) $x - y > -3$

(B) $y - x \geq 3$

(D) $y > -x + 3$



43. You secure pictures to your scrapbook using 3 stickers. You started with 24 stickers. There are now 2 pictures in your scrapbook. You write the equation $3(x + 2) = 24$ to find the number x of additional pictures you can put in your scrapbook. How many more pictures can you add?

(F) 4

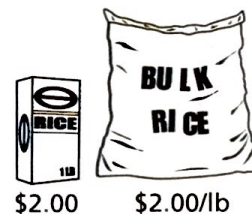
(H) 8

(G) 6

(I) 12

Short Response

44. At Market A, 1-lb packages of rice are sold for the price shown. At Market B, rice is sold in bulk for the price shown. For each market, write a function describing the cost of buying rice in terms of the weight. How are the domains of the two functions different?



Mixed Review

45. **Small Business** An electrician spends \$12,000 on initial costs to start a new business. He estimates his expenses at \$25 per day. He expects to earn \$150 per day. If his estimates are correct, after how many working days will he break even?

◀ See Lesson 6-4.

46. What compound inequality represents the phrase "all real numbers that are greater than 2 and less than or equal to 7"? Graph the solutions.

◀ See Lesson 3-6.

Get Ready! To prepare for Lesson 6-6, do Exercises 47-49.

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

◀ See Lesson 6-1.

47. $y = \frac{3}{2}x$
 $-2x + y = 3$

48. $3x + y = 6$
 $2x - y = 4$

49. $x + y = 11$
 $x + y = 16$