

6-6

Systems of Linear Inequalities

Common Core State Standards

A-REI.D.12 Graph . . . the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

MP 1, MP 2, MP 3, MP 4, MP 7

Objectives To solve systems of linear inequalities by graphing
To model real-world situations using systems of linear inequalities



See how many combinations you can find that satisfy this situation.



Getting Ready!

You want to buy at least 6 new ring tones from a Web site, but you cannot spend more than \$15. How many premium ring tones and how many top-10 ring tones can you buy? Explain how you found your answer.

RING TONES

PREMIUM \$1.50 BUY NOW

TOP 10 \$3.00 BUY NOW

MATHEMATICAL PRACTICES

A **system of linear inequalities** is made up of two or more linear inequalities. A **solution of a system of linear inequalities** is an ordered pair that makes *all* the inequalities in the system true. The graph of a system of linear inequalities is the set of points that represent all of the solutions of the system.

Essential Understanding You can graph the solutions of a system of linear inequalities in the coordinate plane. The graph of the system is the region where the graphs of the individual inequalities overlap.



Lesson Vocabulary

- system of linear inequalities
- solution of a system of linear inequalities



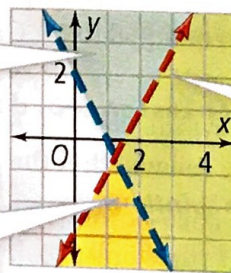
Problem 1 Graphing a System of Inequalities

What is the graph of the system? $y < 2x - 3$
 $2x + y > 2$

Graph $y < 2x - 3$ and $2x + y > 2$.

The blue region represents solutions of $2x + y > 2$.

The yellow region represents solutions of $y < 2x - 3$.



The green region represents solutions of *both* inequalities.

The system's solutions lie in the green region where the graphs overlap.

Think

Have you seen a problem like this before?

Yes. The solution of a system of equations is shown by the intersection of two lines. The solutions of a system of inequalities are shown by the intersection of two shaded areas.

Check $(3, 0)$ is in the green region. See if $(3, 0)$ satisfies both inequalities.

$$y \stackrel{?}{\geq} 2x - 3$$

← Write both inequalities. →

$$2x + y \stackrel{?}{\geq} 2$$

$$0 \stackrel{?}{\geq} 2(3) - 3$$

← Substitute $(3, 0)$ for (x, y) . →

$$2(3) + 0 \stackrel{?}{\geq} 2$$

$$0 < 3 \checkmark$$

← Simplify. The solution checks. →

$$6 > 2 \checkmark$$



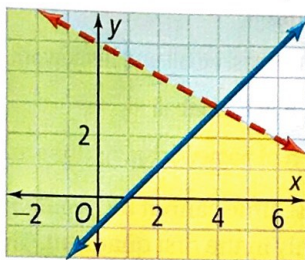
Got It? 1. What is the graph of the system? $y \geq -x + 5$
 $-3x + y \leq -4$

You can combine your knowledge of linear equations with your knowledge of inequalities to describe a graph using a system of inequalities.



Problem 2 Writing a System of Inequalities From a Graph

What system of inequalities is represented by the graph below?



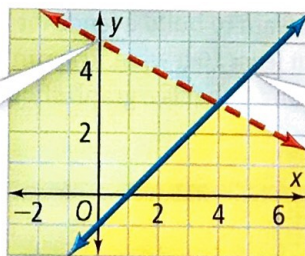
Think

Have you seen a problem like this one before?

Yes. You wrote an inequality from a graph in Lesson 6-5. Now you'll write two inequalities.

To write a system that is represented by the graph, write an inequality that represents the yellow region and an inequality that represents the blue region.

The red boundary line is $y = -\frac{1}{2}x + 5$. The region does not include the line, only points below. The inequality is $y < -\frac{1}{2}x + 5$.

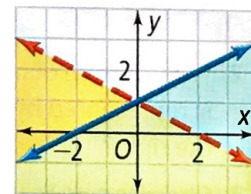


The blue boundary line is $y = x - 1$. The region includes the boundary line and points above. The inequality is $y \geq x - 1$.

The graph shows the intersection of the system $y < -\frac{1}{2}x + 5$ and $y \geq x - 1$.



Got It? 2. a. What system of inequalities is represented by the graph?
 b. **Reasoning** In part (a), is the point where the boundary lines intersect a solution of the system? Explain.



You can model many real-world situations by writing and graphing systems of linear inequalities. Some real-world situations involve three or more restrictions, so you must write a system of at least three inequalities.



Problem 3 Using a System of Inequalities

Time Management You are planning what to do after school. You can spend at most 6 h daily playing basketball and doing homework. You want to spend less than 2 h playing basketball. You must spend at least $1\frac{1}{2}$ h on homework. What is a graph showing how you can spend your time?

Know

- At most 6 h playing basketball and doing homework
- Less than 2 h playing basketball
- At least $1\frac{1}{2}$ h doing homework

Need

To find different ways you can spend your time

Plan

Write and graph an inequality for each restriction. Find the region where all three restrictions are met.

Let x = the number of hours playing basketball.

Let y = the number of hours doing homework.

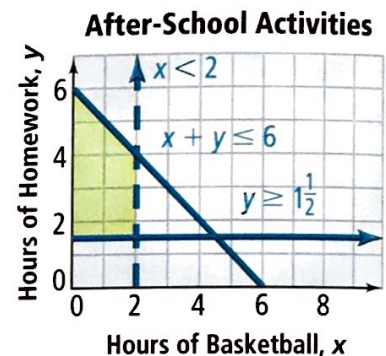
Write a system of inequalities.

$$x + y \leq 6 \quad \text{At most 6 h of basketball and homework}$$

$$x < 2 \quad \text{Less than 2 h of basketball}$$

$$y \geq 1\frac{1}{2} \quad \text{At least } 1\frac{1}{2} \text{ h of homework}$$

Graph the system. Because time cannot be negative, the graph makes sense only in the first quadrant. The solutions of the system are all of the points in the shaded region, including the points on the solid boundary lines.



Got It? 3. You want to build a fence for a rectangular dog run. You want the run to be at least 10 ft wide. The run can be at most 50 ft long. You have 126 ft of fencing. What is a graph showing the possible dimensions of the dog run?

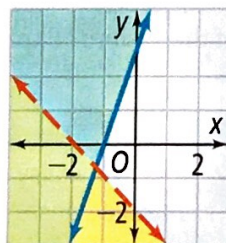


Lesson Check

Do you know HOW?

1. What is the graph of the system? $y > 3x - 2$
 $2y - x \leq 6$

2. What system of inequalities is represented by the graph at the right?



3. Cherries cost \$4/lb. Grapes cost \$2.50/lb. You can spend no more than \$15 on fruit, and you need at least 4 lb in all. What is a graph showing the amount of each fruit you can buy?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** How can you determine whether an ordered pair is a solution of a system of linear inequalities?
5. **Reasoning** Suppose you are graphing a system of two linear inequalities, and the boundary lines for the inequalities are parallel. Does that mean that the system has no solution? Explain.
6. **Writing** How is finding the solution of a system of inequalities different from finding the solution of a system of equations? How is it the same? Explain.

A Practice

Determine whether the ordered pair is a solution of the given system.

◀ See Problem 1.

7. $(2, 12)$;
 $y > 2x + 4$
 $y < 3x + 7$

8. $(8, 2)$;
 $3x - 2y \leq 17$
 $0.3x + 4y > 9$

9. $(-3, 17)$;
 $y > -5x + 2$
 $y \geq -3x + 7$

Solve each system of inequalities by graphing.

10. $y < 2x + 4$
 $-3x - 2y \geq 6$

11. $y < 2x + 4$
 $2x - y \leq 4$

12. $y > 2x + 4$
 $2x - y \leq 4$

13. $y > \frac{1}{4}x$
 $y \leq -x + 4$

14. $y < 2x - 3$
 $y > 5$

15. $y \leq -\frac{1}{3}x + 7$
 $y \geq -x + 1$

16. $x + 2y \leq 10$
 $x + 2y \geq 9$

17. $y \geq -x + 5$
 $y \leq 3x - 4$

18. $y \leq 0.75x - 2$
 $y > 0.75x - 3$

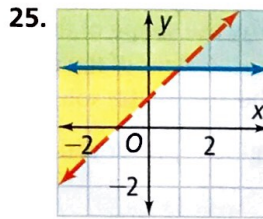
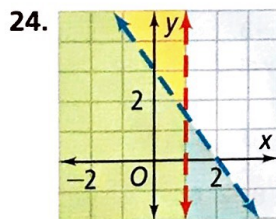
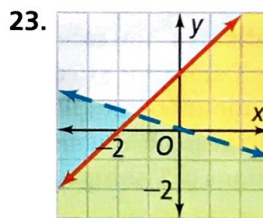
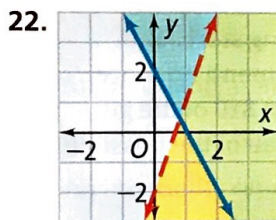
19. $8x + 4y \geq 10$
 $3x - 6y > 12$

20. $2x - \frac{1}{4}y < 1$
 $4x + 8y > 4$

21. $6x - 5y < 15$
 $x + 2y \geq 7$

Write a system of inequalities for each graph.

◀ See Problem 2.



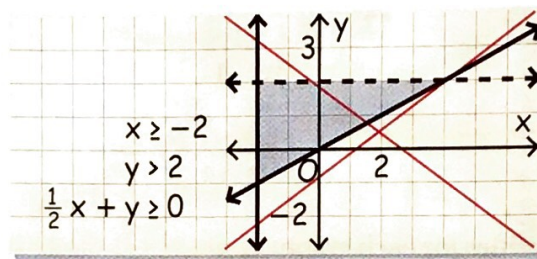
26. **Earnings** Suppose you have a job mowing lawns that pays \$12 per hour. You also have a job at a clothing store that pays \$10 per hour. You need to earn at least \$350 per week, but you can work no more than 35 h per week. You must work a minimum of 10 h per week at the clothing store. What is a graph showing how many hours per week you can work at each job?

◀ See Problem 3.

27. **Driving** Two friends agree to split the driving on a road trip from Philadelphia, Pennsylvania, to Denver, Colorado. One friend drives at an average speed of 60 mi/h. The other friend drives at an average speed of 55 mi/h. They want to drive at least 500 mi per day. They plan to spend no more than 10 h driving each day. The friend who drives slower wants to drive fewer hours. What is a graph showing how they can split the driving each day?

B Apply

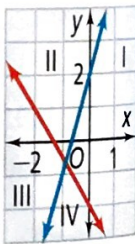
28. **Think About a Plan** You are fencing in a rectangular area for a garden. You have only 150 ft of fence. You want the length of the garden to be at least 40 ft. You want the width of the garden to be at least 5 ft. What is a graph showing the possible dimensions your garden could have?
- What variables will you use? What will they represent?
 - How many inequalities do you need to write?
29. a. Graph the system $y > 3x + 3$ and $y \leq 3x - 5$.
 b. **Writing** Will the boundary lines $y = 3x + 3$ and $y = 3x - 5$ ever intersect? How do you know?
 c. Do the shaded regions in the graph from part (a) overlap?
 d. Does the system of inequalities have any solutions? Explain.
30. **Error Analysis** A student graphs the system as shown below. Describe and correct the student's error.



31. **Gift Certificates** You received a \$100 gift certificate to a clothing store. The store sells T-shirts for \$15 and dress shirts for \$22. You want to spend no more than the amount of the gift certificate. You want to leave at most \$10 of the gift certificate unspent. You need at least one dress shirt. What are all of the possible combinations of T-shirts and dress shirts you could buy?
32. a. **Geometry** Graph the system of linear inequalities.
 b. Describe the shape of the solution region.
 c. Find the vertices of the solution region.
 d. Find the area of the solution region.
33. Which region represents the solution of the system?

- (A) I (C) III
 (B) II (D) IV

$$\begin{aligned} x &\geq 2 \\ y &\geq -3 \\ x + y &\leq 4 \\ y &\leq -\frac{3}{2}x - 2 \\ 3y - 9x &\geq 6 \end{aligned}$$



34. **Open-Ended** Write a system of linear inequalities with the given characteristic.

34. All solutions are in Quadrant III.

35. There are no solutions.

C Challenge

36. **Business** A jeweler plans to produce a ring made of silver and gold. The price of gold is about \$25 per gram. The price of silver is approximately \$.40 per gram. She considers the following in deciding how much gold and silver to use in the ring.
- The total mass must be more than 10 g but less than 20 g.
 - The ring must contain at least 2 g of gold.
 - The total cost of the gold and silver must be less than \$90.
- a. Write and graph the inequalities that describe this situation.
 b. For one solution, find the mass of the ring and the cost of the gold and silver.

37. Solve $|y| \geq x$. (Hint: Write two inequalities and then graph them.)

38. **Student Art** A teacher wants to post a row of student artwork on a wall that is 20 ft long. Some pieces are 8.5 in. wide. Other pieces are 11 in. wide. She is going to leave 3 in. of space to the left of each art piece. She wants to post at least 16 pieces of art. Write and graph a system of inequalities that describes how many pieces of each size she can post.

Apply What You've Learned



MP 2

Look back at the information on page 363 about Miguel's exercise program at the gym. Choose from the following inequalities to complete the sentences below. In each inequality, x represents the number of minutes Miguel spends on the stair machine and y represents the number of minutes Miguel spends on the rowing machine.

$$x + y \leq 60$$

$$2y \leq x$$

$$x - y \geq 30$$

$$x + y \geq 30$$

$$2y \geq x$$

$$y \geq 2x$$

$$x + 2y \geq 30$$

$$x + y \geq 60$$

- An inequality representing the first condition given by Miguel's trainer is ? .
- An inequality representing the second condition given by Miguel's trainer is ? .
- An inequality representing the third condition given by Miguel's trainer is ? .