7-4 Division Properties of Exponents

Common Core State Standards

N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

MP 1, MP 2, MP 3, MP 4, MP 7

Objectives To divide powers with the same base
To raise a quotient to a power

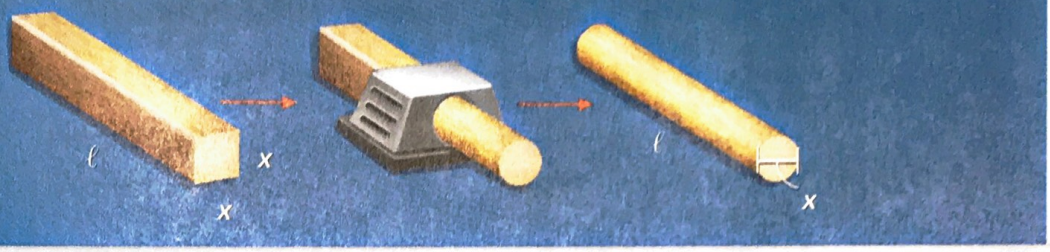
Solve a simpler problem first. Use a value for x to understand all of the relationships in this problem.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

A machine makes wooden dowels by removing material from a block of wood as shown in the diagram. What percent of the wood does the machine remove from the original piece of wood to form the dowel? Explain how you found your answer. (Hint: What is the volume of the dowel?)



In the Solve It, the expression for the volume of the dowel involves a quotient raised to a power.

Essential Understanding You can use properties of exponents to divide powers with the same base.

You can use repeated multiplication to simplify quotients of powers with the same base. Expand the numerator and the denominator. Then divide out the common factors.

$$\frac{4^5}{4^3} = \frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}\cdot 4\cdot 4}{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}} = 4^2$$

This example suggests the following property of exponents.

take note

Property Dividing Powers With the Same Base

Words To divide powers with the same base, subtract the exponents.

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m and n are rational numbers

Examples $\frac{2^6}{2^2} = 2^{6-2} = 2^4$ $\quad \frac{x^4}{x^7} = x^{4-7} = x^{-3} = \frac{1}{x^3}$ $\quad \frac{s^{\frac{3}{4}}}{s^{\frac{1}{2}}} = s^{\frac{3}{4}-\frac{1}{2}} = s^{\frac{3}{4}-\frac{2}{4}} = s^{\frac{1}{4}}$

Problem 1 Dividing Algebraic Expressions

What is the simplified form of each expression?

Think

How are the properties for dividing powers and multiplying powers similar?
For both properties, the bases of the powers must be the same. Dividing a power is the same as multiplying by a negative exponent.

A $\dfrac{x^{\frac{5}{2}}}{x^2}$

$\dfrac{x^{\frac{5}{2}}}{x^2} = x^{\frac{5}{2}-2}$ Subtract exponents when dividing powers with the same base.

$= x^{\frac{1}{2}}$ Simplify.

B $\dfrac{m^2n^4}{m^5n^3}$

$\dfrac{m^2n^4}{m^5n^3} = m^{2-5}n^{4-3}$ Subtract exponents when dividing powers with the same base.

$= m^{-3}n^1$ Simplify the exponents.

$= \dfrac{n}{m^3}$ Rewrite using positive exponents.

Got It? **1.** What is the simplified form of each expression?

a. $\dfrac{y^{\frac{3}{4}}}{y^{\frac{1}{2}}}$ b. $\dfrac{d^{\frac{7}{2}}}{d^3}$ c. $\dfrac{k^6j^2}{kj^5}$ d. $\dfrac{a^{-3}b^7}{a^5b^2}$ e. $\dfrac{x^4y^{-1}z^8}{x^4y^{-5}z}$

You can use the property of dividing powers with the same base to divide numbers in scientific notation.

Problem 2 Dividing Numbers in Scientific Notation

Demographics **Population density describes the number of people per unit area. During one year, the population of Angola was 1.21×10^7 people. The area of Angola is 4.81×10^5 mi^2. What was the population density of Angola that year?**

Know
- The population
- The area

Need
The population density

Plan
Write the ratio of population to area.

$\dfrac{1.21 \times 10^7}{4.81 \times 10^5} = \dfrac{1.21}{4.81} \times 10^{7-5}$ Subtract exponents when dividing powers with the same base.

$= \dfrac{1.21}{4.81} \times 10^2$ Simplify the exponent.

$\approx 0.252 \times 10^2$ Divide. Round to the nearest thousandth.

$= 25.2$ Write in standard notation.

The population density of Angola was about 25.2 people per square mile.

Got It? **2.** During one year, Honduras had a population of 7.33×10^6 people. The area of Honduras is 4.33×10^4 mi^2. What was the population density of Honduras that year?

You can use repeated multiplication to simplify a quotient raised to a power.

$$\left(\frac{x}{y}\right)^4 = \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} = \frac{x \cdot x \cdot x \cdot x}{y \cdot y \cdot y \cdot y} = \frac{x^4}{y^4}$$

This suggests another property of exponents.

Take Note

Property Raising a Quotient to a Power

Words To raise a quotient to a power, raise the numerator and the denominator to the power and simplify.

Algebra $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $a \neq 0$, $b \neq 0$, and n is an rational number.

Examples $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$ $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$ $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Problem 3 Raising a Quotient to a Power

Multiple Choice What is the simplified form of $\left(\frac{e^{\frac{4}{7}}}{5}\right)^3$?

(A) $\frac{e^{\frac{12}{7}}}{15}$ (B) $\frac{e^3}{15}$ (C) $\frac{e^{\frac{12}{7}}}{125}$ (D) $\frac{e^3}{125}$

Think

How can you check your answer?
Substitute the same number for the variable in the original expression and the simplified expression. The expressions should be equal.

$\left(\frac{e^{\frac{4}{7}}}{5}\right)^3 = \frac{\left(e^{\frac{4}{7}}\right)^3}{5^3}$ Raise the numerator and the denominator to the third power.

$= \frac{e^{\frac{4}{7} \cdot 3}}{5^3}$ Multiply the exponents in the numerator.

$= \frac{e^{\frac{12}{7}}}{125}$ Simplify.

The correct answer is C.

Got It? 3. a. What is the simplified form of $\left(\frac{4}{x^3}\right)^2$?

b. **Reasoning** Describe two different ways to simplify the expression $\left(\frac{a^3}{a^5}\right)^4$. Which method do you prefer? Explain.

You can write an expression of the form $\left(\frac{a}{b}\right)^{-n}$ using positive exponents.

$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n}$ Use the definition of negative exponent.

$= \frac{1}{\left(\frac{a^n}{b^n}\right)}$ Raise the quotient to a power.

$= 1 \cdot \frac{b^n}{a^n}$ Multiply by the reciprocal of $\frac{a^n}{b^n}$ which is $\frac{b^n}{a^n}$.

$= \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$ Simplify. Write the quotient using one exponent.

So, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ for all nonzero numbers a and b and positive integers n.

Problem 4 Simplifying an Exponential Expression

What is the simplified form of $\left(\frac{2x^6}{y^4}\right)^{-3}$?

Plan

How do you write an expression in simplified form?
Use the properties of exponents to write each variable with a single positive exponent.

$\left(\frac{2x^6}{y^4}\right)^{-1} = \left(\frac{y^4}{2x^6}\right)^{1}$ Rewrite using the reciprocal of $\frac{2x^6}{y^4}$.

$= \frac{(y^4)^3}{(2x^6)^3}$ Raise the numerator and denominator to the third power.

$= \frac{y^{12}}{8x^{18}}$ Simplify.

Got It? 4. What is the simplified form of $\left(\frac{a}{5b}\right)^{-2}$?

Lesson Check

Do you know HOW?

Simplify each expression.

1. $\frac{y^3}{y^{10}}$
2. $\left(\frac{x^4}{3}\right)^3$
3. $\left(\frac{m}{n}\right)^{-3}$
4. $\left(\frac{3x^2}{5y^4}\right)^{-4}$

5. A large cube is made up of many small cubes. The volume of the large cube is 7.506×10^5 mm^3. The volume of each small cube is 2.78×10^4 mm^3. How many small cubes make up the large cube?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** How is the property for raising a quotient to a power similar to the property for raising a product to a power?

7. a. **Reasoning** Ross simplifies $\frac{a^3}{a^7}$ as shown at the right. Explain why Ross's method works.

$$\frac{a^3}{a^7} = \frac{1}{a^{7-3}} = \frac{1}{a^4}$$

b. **Open-Ended** Write a quotient of powers and use Ross's method to simplify it.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Copy and complete each equation. See Problem 1.

8. $\frac{5^9}{5^2} = 5^{\blacksquare}$
9. $\frac{2^{\blacksquare}}{2^2} = 2^{\blacksquare}$
10. $\frac{3^2}{3^{\blacksquare}} = 3^{\blacksquare}$
11. $\frac{5^2 5^{\blacksquare}}{5^{\blacksquare}5^2} = 5^{\blacksquare}$

Simplify each expression.

12. $\frac{3^0}{3^6}$
13. $\frac{9^{\frac{1}{3}}}{9^{\frac{1}{3}}}$
14. $\frac{d^{14}}{d^{17}}$
15. $\frac{n^{-1}}{n^{-4}}$
16. $\frac{5s^{-7}}{10s^{-9}}$
17. $\frac{x^{11}y^3}{x^{11}y}$
18. $\frac{c^{\frac{4}{3}}d^{-5}}{c^{\frac{1}{3}}d^{-1}}$
19. $\frac{10m^6n^3}{5m^2n^7}$
20. $\frac{m^{\frac{4}{3}}n^2}{m^{-1}n^3}$
21. $\frac{3^2m^5t^6}{3^5m^7t^{-5}}$
22. $\frac{x^5y^{-\frac{9}{2}}z^3}{xy^{-4}z^3}$
23. $\frac{12a^{-1}b^6c^{-3}}{4a^5b^{-1}c^5}$

Simplify each quotient. Write each answer in scientific notation. See Problem 2.

24. $\frac{5.2 \times 10^{13}}{1.3 \times 10^{7}}$ **25.** $\frac{3.6 \times 10^{-10}}{9 \times 10^{-6}}$ **26.** $\frac{6.5 \times 10^{4}}{5 \times 10^{6}}$

27. $\frac{8.4 \times 10^{-5}}{2 \times 10^{-8}}$ **28.** $\frac{4.65 \times 10^{-4}}{3.1 \times 10^{2}}$ **29.** $\frac{3.5 \times 10^{6}}{5 \times 10^{8}}$

30. Computers The average time it takes a computer to execute one instruction is measured in picoseconds. There are 3.6×10^{15} picoseconds per hour. What fraction of a second is a picosecond?

31. Wildlife Data from a deer count in a forested area show that an estimated 3.16×10^{3} deer inhabit 7.228×10^{4} acres of land. What is the density of the deer population?

STEM **32. Astronomy** The sun's mass is 1.998×10^{30} kg. Saturn's mass is 5.69×10^{26} kg. How many times as great as the mass of Saturn is the mass of the sun?

Simplify each expression. See Problems 3 and 4.

33. $\left(\frac{3}{8}\right)^{2}$ **34.** $\left(\frac{1}{a}\right)^{3}$ **35.** $\left(\frac{3x}{y}\right)^{4}$ **36.** $\left(\frac{2x}{3y}\right)^{5}$

37. $\left(\frac{6}{5^{2}}\right)^{3}$ **38.** $\left(\frac{2^{2}}{2^{3}}\right)^{5}$ **39.** $\left(\frac{8}{n^{5}}\right)^{6}$ **40.** $\left(\frac{2p}{9}\right)^{3}$

41. $\left(\frac{2}{5}\right)^{-1}$ **42.** $\left(\frac{5}{4}\right)^{-4}$ **43.** $\left(-\frac{7x^{\frac{3}{2}}}{5y^{4}}\right)^{-2}$ **44.** $\left(-\frac{2x^{\frac{1}{6}}}{3y^{4}}\right)^{-3}$

45. $\left(\frac{3x^{\frac{1}{2}}}{15}\right)^{2}$ **46.** $\left(\frac{6n^{2}}{3n}\right)^{-3}$ **47.** $\left(\frac{b^{\frac{4}{5}}}{b^{7}}\right)^{-5}$ **48.** $\left(\frac{3}{5c^{2}}\right)^{0}$

B Apply

Explain why each expression is *not* in simplest form.

49. $5^{3}m^{3}$ **50.** $x^{5}y^{-2}$ **51.** $(2c)^{4}$ **52.** $x^{0}y$ **53.** $\frac{d^{7}}{d}$

54. Think About a Plan During one year, about 163 million adults over 18 years old in the United States spent a total of about 93 billion hours online at home. On average, how many hours per day did each adult spend online at home?

- How do you write each number in scientific notation?
- How do you convert the units to hours per day?

55. Television During one year, people in the United States older than 18 years old watched a total of 342 billion hours of television. The population of the United States older than 18 years old was about 209 million people.

a. On average, how many hours of television did each person older than 18 years old watch that year? Round to the nearest hour.

b. On average, how many hours per week did each person older than 18 years old watch that year? Round to the nearest hour.

Which property or properties of exponents would you use to simplify each expression?

56. 2^{-3} **57.** $\frac{2^{2}}{2^{5}}$ **58.** $\frac{1}{2^{-4}2^{7}}$ **59.** $\frac{(2^{\frac{2}{3}})^{3}}{2^{15}}$

Simplify each expression.

60. $\frac{3n^2(5^0)}{2n^3}$

61. $\left(\frac{2m^4}{m^2}\right)^{-4}$

62. $\frac{3x^3}{(3x)^3}$

63. $\frac{(2a^6)(4a)}{8a^3}$

64. $\left(\frac{9t^{\frac{2}{3}}}{36t}\right)^3$

65. $\left(\frac{a^4a}{a^2}\right)^{-3}$

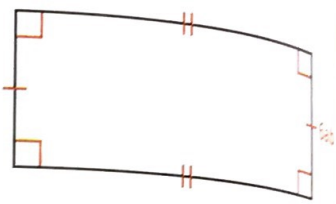
66. $\left(\frac{2x^2}{5x^3}\right)^{-2}$

67. $\frac{4x^{-2}y^4}{8x^3(y^{-2})^3}$

68. a. **Open-Ended** Write three numbers greater than 1000 in scientific notation.
    b. Divide each number by 2.
    c. **Reasoning** Is the exponent of the power of 10 divided by 2 when you divide a number in scientific notation by 2? Explain.

69. Simplify the expression $\left(\frac{3}{x^2}\right)^{-3}$ in three different ways. Justify each step.

70. **Geometry** The area of the rectangle is $72a^3b^4$. What is the length of the rectangle?



(A) $\frac{a^3b^4}{12}$ (C) $\frac{12}{a^3b^4}$

(B) $12a^2b^3$ (D) $12a^3b^4$

Simplify each expression.

71. $\left(\frac{3xy^5}{x^4y}\right)^{-2}$

72. $\frac{m^4n^3p^{-3}}{m^{-2}n^7p^{-8}}$

73. $\frac{\left(\frac{1}{4}\right)^{-2}}{\left(\frac{1}{6}\right)^{-3}}$

74. $\frac{0.2^3 \cdot 0.2^4}{0.2^7}$

75. $\left(\frac{a^{-1}b^3c}{a^2b^4}\right)^6$

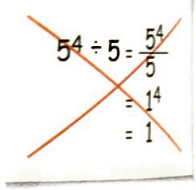
76. $\left(\frac{(-4)^2}{(-3)^{-3}}\right)^2$

77. $\left(\frac{(4x)^2y}{xy^4}\right)^{-2}$

78. $\frac{(6a^3)(8b^4)}{(2a^4)(36b^{-1})}$

STEM 79. **Physics** The wavelength of a radio wave is defined as speed divided by frequency. An FM radio station has a frequency of 9×10^7 waves per second. The speed of the waves is about 3×10^8 meters per second. What is the wavelength of the station?

80. a. **Error Analysis** What mistake did the student make in simplifying the expression at the right?
    b. What is the correct simplified form of the expression?



81. **Writing** Suppose $\frac{a^x}{a^y} = a^3$ and $\frac{a^x}{a^{3y}} = a^{-5}$. Find the values of x and y. Explain how you found your answer.

82. a. **Finance** In 2000, the United States government owed about \$5.63 trillion to its creditors. The population of the United States was 282.4 million people. How much did the government owe per person in 2000? Round to the nearest dollar.
    b. In 2005, the debt had grown to \$7.91 trillion, with a population of 296.9 million. How much did the government owe per person? Round to the nearest dollar.
    c. What was the percent increase in the average amount owed per person from 2000 to 2005?

Write each expression with only one exponent. You may need to use parentheses.

83. $\frac{m^7}{n^7}$

84. $\frac{10^7 \cdot 10^0}{10^{-3}}$

85. $\frac{27x^3}{8y^3}$

86. $\frac{4m^2}{169m^4}$

87. **a.** Use the property for dividing powers with the same base to write $\frac{a^0}{a^n}$ as a power of a.

b. Use the definition of a zero exponent to simplify $\frac{a^0}{a^n}$.

c. **Reasoning** Explain how your results from parts (a) and (b) justify the definition of a negative exponent.

Challenge **Simplify each expression.**

88. $n^{x+2} \div n^x$ **89.** $n^{5x} \div n^x$ **90.** $\left(\frac{x^n}{x^{n-2}}\right)^3$ **91.** $\frac{\left(\frac{m^4}{m^5}\right)}{m^2}$

92. **Reasoning** Use the division property of exponents to show why 0^0 is undefined.

STEM **93.** **Astronomy** The density of an object is the ratio of its mass to its volume. Neptune has a mass of 1.02×10^{26} kg. The radius of Neptune is 2.48×10^4 km. What is the density of Neptune in grams per cubic meter? (*Hint:* $V = \frac{4}{3}\pi r^3$)

Standardized Test Prep

SAT/ACT

94. Which expression is equivalent to $\frac{(2x)^5}{x^3}$?

Ⓐ $2x^2$ Ⓑ $32x^2$ Ⓒ $2x^8$ Ⓓ $32x^{-2}$

95. Which equation is an equation of the line that contains the point $(8, -3)$ and is perpendicular to the line $y = -4x + 5$?

Ⓕ $y = -\frac{1}{4}x - 1$ Ⓖ $y = \frac{1}{4}x + \frac{35}{4}$ Ⓗ $y = \frac{1}{4}x - 5$ Ⓘ $y = 4x - 35$

96. What is the solution of the system of equations $y = -3x + 5$ and $y = -4x - 1$?

Ⓐ $(23, 6)$ Ⓑ $(6, 23)$ Ⓒ $(-6, 23)$ Ⓓ $(-6, -23)$

Short Response

97. You have 8 bags of grass seed. Each bag covers 1200 ft^2 of ground. The function $A(b) = 1200b$ represents the area $A(b)$, in square feet, that b bags cover. What domain and range are reasonable for the function? Explain.

Mixed Review

Simplify each expression. See Lesson 7-3.

98. $(2m^{\frac{2}{3}})^3$ **99.** $2(3s^{-2})^{-3}$ **100.** $(4^3c^2)^{\frac{1}{6}}$ **101.** $(-3)^2(r^{\frac{1}{4}})^2$ **102.** $(7^0n^{-3})^2(n^7)^3$

Solve each system by graphing. See Lesson 6-1.

103. $y = 3x$, $y = -2x$ **104.** $y = 2x + 1$, $y = x - 3$ **105.** $y = 5$, $x = 3$ **106.** $y = 7$, $y = 8$

Get Ready! **To prepare for Lesson 7-5, do Exercises 107–110.**

Graph each function. See Lesson 4-4.

107. $y = 4x$ **108.** $y = 5x$ **109.** $y = -3x$ **110.** $y = 1.5x$