7-6 Exponential Functions

Common Core State Standards

F-IF.C.7e Graph exponential . . . functions, showing intercepts and end behavior . . . **Also F-IF.B.4, F-IF.B.5, F-IF.C.9, F-LE.A.2**

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To evaluate and graph exponential functions

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

Your soccer team wants to practice a drill for a certain amount of time each day. Which plan will give your team more total practice time over 4 days? Over 8 days? Explain your reasoning.

The two plans in the Solve It have different patterns of growth. You can model each type of growth with a different type of function.

Lesson Vocabulary
- exponential function

Essential Understanding Some functions model an initial amount that is repeatedly multiplied by the same positive number. In the rules for these functions, the independent variable is an exponent.

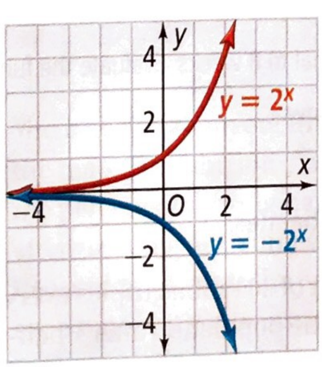
take note

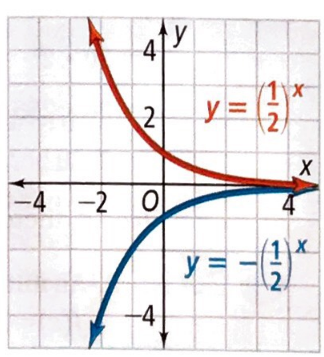
Key Concept Exponential Function

Definition

An **exponential function** is a function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$, and x is a real number.

Examples





Suppose all the x-values in a table have a common difference. If all the y-values have a common difference, then the table represents a linear function. If all of the y-values have a common ratio, then the table represents an exponential function.

Think

How can you identify a constant ratio between y-values?
When you multiply each y-value by the same constant and get the next y-value, there is a constant ratio between the values.

Problem 1 Identifying Linear and Exponential Functions

Does the table or rule represent a linear or an exponential function? Explain.

A

x	0	1	2	3
y	−1	−3	−9	−27

The difference between each x-value is 1.

+1 +1 +1

x	0	1	2	3
y	−1	−3	−9	−27

×3 ×3 ×3

The ratio between each y-value is 3.

The table represents an exponential function. There is a common difference between x-values and a common ratio between y-values.

B $y = 3x$

The rule represents a linear function. The independent variable x is not an exponent.

Got It? **1.** Does the table or rule represent a linear or an exponential function? Explain.

a.

x	1	2	3	4
y	−1	1	3	5

b. $y = 3 \cdot 6^x$

Think

Why is the function $30 \cdot 2^x$ not $2 \cdot 30^x$?
In an exponential function, a is the starting value and b is the common ratio.

Problem 2 Evaluating an Exponential Function — GRIDDED RESPONSE

Population Growth **Suppose 30 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles each week. The function $f(x) = 30 \cdot 2^x$ gives the population after x weeks. How many beetles will there be after 56 days?**

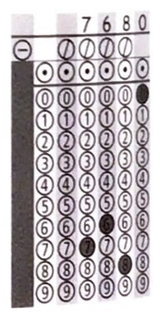
$f(x) = 30 \cdot 2^x$

$= 30 \cdot 2^8$ 56 days is equal to 8 weeks. Evaluate the function for $x = 8$.

$= 30 \cdot 256$ Simplify the power.

$= 7680$ Simplify.

After 56 days, there will be 7680 beetles.



Got It? **2.** An initial population of 20 rabbits triples every half year. The function $f(x) = 20 \cdot 3^x$ gives the population after x half-year periods. How many rabbits will there be after 3 yr?

Problem 3 Graphing an Exponential Function

Think

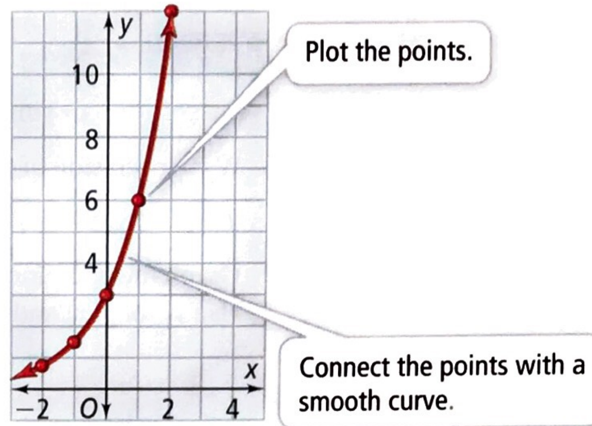
What are the domain and range of the function?

Any value substituted for x results in a positive y-value. The domain is all real numbers. The range is all positive real numbers.

What is the graph of $y = 3 \cdot 2^x$?

Make a table of x- and y-values.

x	$y = 3 \cdot 2^x$	(x, y)
-2	$3 \cdot 2^{-2} = \frac{3}{2^2} = \frac{3}{4}$	$\left(-2, \frac{3}{4}\right)$
-1	$3 \cdot 2^{-1} = \frac{3}{2^1} = 1\frac{1}{2}$	$\left(-1, 1\frac{1}{2}\right)$
0	$3 \cdot 2^0 = 3 \cdot 1 = 3$	$(0, 3)$
1	$3 \cdot 2^1 = 3 \cdot 2 = 6$	$(1, 6)$
2	$3 \cdot 2^2 = 3 \cdot 4 = 12$	$(2, 12)$



Got It? 3. What is the graph of each function?

a. $y = 0.5 \cdot 3^x$

b. $y = -0.5 \cdot 3^x$

Problem 4 Graphing an Exponential Model

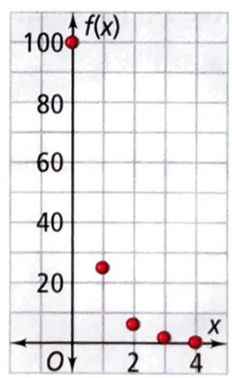
Think

Should you connect the points of the graph?

No. The number of times you zoom in must be a nonnegative integer.

Maps **Computer mapping software allows you to zoom in on an area to view it in more detail. The function $f(x) = 100 \cdot 0.25^x$ models the percent of the original area the map shows after zooming in x times. Graph the function.**

x	$f(x) = 100 \cdot 0.25^x$	$(x, f(x))$
0	$100 \cdot 0.25^0 = 100$	$(0, 100)$
1	$100 \cdot 0.25^1 = 25$	$(1, 25)$
2	$100 \cdot 0.25^2 = 6.25$	$(2, 6.25)$
3	$100 \cdot 0.25^3 \approx 1.56$	$(3, 1.56)$
4	$100 \cdot 0.25^4 \approx 0.39$	$(4, 0.39)$



Got It? 4. a. You can also zoom out to view a larger area on the map. The function $f(x) = 100 \cdot 4^x$ models the percent of the original area the map shows after zooming out x times. Graph the function.

b. Reasoning What is the percent change in area each time you zoom out in part (a)?

In the Concept Byte after Lesson 6-1, you solved one-variable linear equations using graphs and a graphing calculator. In the next example, you will write each side of the equation as a function and graph the functions. The x-value where the functions intersect is a solution.

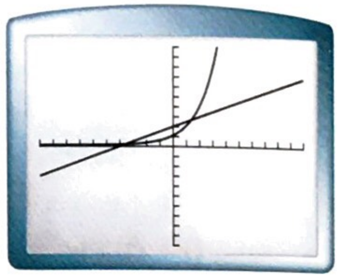
Problem 5 Solving One-Variable Equations

What is the solution or solutions of $2^x = 0.5x + 2$?

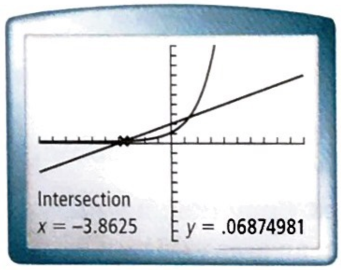
Step 1 Write each side of the equation as a function equation.

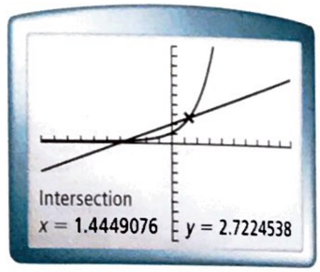
$f(x) = 2^x$ and $g(x) = 0.5x + 2$

Step 2 Graph the equations using a graphing calculator. Use y_1 for $f(x)$ and y_2 for $g(x)$.



Step 3 Use the **CALC** feature. Chose **INTERSECT** to find the points where the lines intersect.





The solutions of $2^x = 0.5x + 2$ are about -3.86 and 1.45.

Think

How can you check that the *x*-value is a solution?

Substitute for x in the original equation. Make sure you use the same x-value for each instance of x.

Got It? 5. What is the solution or solutions of each equation?

a. $0.3^x = 5$ **b.** $1.25^x = -2x$ **c.** $-(2^x) = \frac{3}{4}x - 4$

Lesson Check

Do you know HOW?

Evaluate each function for the given value.

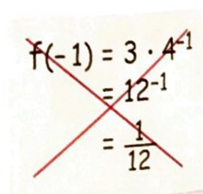
1. $f(x) = 6 \cdot 2^x$ for $x = 3$
2. $g(w) = 45 \cdot 3^w$ for $w = -2$

Graph each function.

3. $y = 3^x$
4. $f(x) = 4\left(\frac{1}{2}\right)^x$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Describe the differences between a linear function and an exponential function.
6. **Reasoning** Is $y = (-2)^x$ an exponential function? Justify your answer.
7. **Error Analysis** A student evaluated the function $f(x) = 3 \cdot 4^x$ for $x = -1$ as shown at the right. Describe and correct the student's mistake.



Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Determine whether each table or rule represents a linear or an exponential function. Explain why or why not.

See Problem 1.

8.

x	1	2	3	4
y	2	8	32	128

9.

x	0	1	2	3
y	6	9	12	15

10. $y = 4 \cdot 5^x$

11. $y = 12 \cdot x$

12. $y = -5 \cdot 0.25^x$

13. $y = 7x + 3$

Evaluate each function for the given value.

See Problem 2.

14. $f(x) = 6^x$ for $x = 2$

15. $g(t) = 2 \cdot 0.4^t$ for $t = -2$

16. $y = 20 \cdot 0.5^x$ for $x = 3$

17. $h(w) = -0.5 \cdot 4^w$ for $w = 18$

18. **Finance** An investment of \$5000 doubles in value every decade. The function $f(x) = 5000 \cdot 2^x$, where x is the number of decades, models the growth of the value of the investment. How much is the investment worth after 30 yr?

19. **Wildlife Management** A population of 75 foxes in a wildlife preserve quadruples in size every 15 yr. The function $y = 75 \cdot 4^x$, where x is the number of 15-yr periods, models the population growth. How many foxes will there be after 45 yr?

Graph each exponential function.

See Problem 3.

20. $y = 4^x$

21. $y = -4^x$

22. $y = \left(\frac{1}{3}\right)^x$

23. $y = -\left(\frac{1}{3}\right)^x$

24. $y = 10 \cdot \left(\frac{3}{2}\right)^x$

25. $y = 0.1 \cdot 2^x$

26. $y = \frac{1}{4} \cdot 2^x$

27. $y = 1.25^x$

28. **Admissions** A new museum had 7500 visitors this year. The museum curators expect the number of visitors to grow by 5% each year. The function $y = 7500 \cdot 1.05^x$ models the predicted number of visitors each year after x years. Graph the function.

See Problem 4.

29. **Environment** A solid waste disposal plan proposes to reduce the amount of garbage each person throws out by 2% each year. This year, each person threw out an average of 1500 lb of garbage. The function $y = 1500 \cdot 0.98^x$ models the average amount of garbage each person will throw out each year after x years. Graph the function.

What is the solution or solutions of each equation?

See Problem 5.

30. $4^x = \frac{3}{2}x + 5$

31. $x + 3 = 3^x$

B Apply

Evaluate each function over the domain $\{-2, -1, 0, 1, 2, 3\}$. As the values of the domain increase, do the values of the range *increase* or *decrease*?

32. $f(x) = 5^x$

33. $y = 2.5^x$

34. $h(x) = 0.1^x$

35. $f(x) = 5 \cdot 4^x$

36. $y = 0.5^x$

37. $y = 8^x$

38. $g(x) = 4 \cdot 10^x$

39. $y = 100 \cdot 0.3^x$

40. Compare the rule and the function table below. Which function has the greater value when $x = 12$? Explain.

Function 1

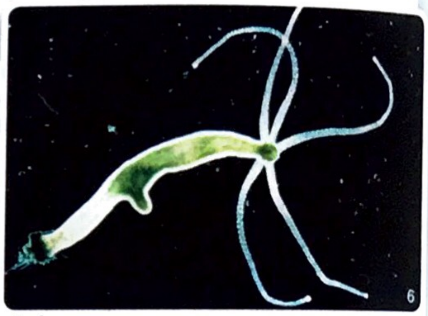
$y = 4^x$

Function 2

x	1	2	3	4
y	5	25	125	625

41. You have just read a journal article about a population of fungi that doubles every 3 weeks. The beginning population was 10. The function $y = 10 \cdot 2^{\frac{n}{3}}$ represents the population after n weeks.
   a. You have a population of 15 of the same fungi. Assuming the journal articles gives the correct rate of increase, write the function that represents the population of fungi after n weeks.
   b. Suppose you find another article that states that the fungi population triples every 4 weeks. If there are currently 15 fungi in your population, write the function that represents the population after n weeks.

42. **Think About a Plan** Hydra are small freshwater animals. They can double in number every two days in a laboratory tank. Suppose one tank has an initial population of 60 hydra. When will there be more than 5000 hydra?
   - How can a table help you identify a pattern?
   - What function models the situation?


Hydra

43. a. Graph $y = 2^x$, $y = 4^x$, and $y = 0.25^x$ on the same axes.
   b. What point is on all three graphs?
   c. Does the graph of an exponential function intersect the x-axis? Explain.
   d. **Reasoning** How does the graph of $y = b^x$ change as the base b increases or decreases?

Which function has the greater value for the given value of x?

44. $y = 4^x$ or $y = x^4$ for $x = 2$

45. $f(x) = 10 \cdot 2^x$ or $f(x) = 200 \cdot x^2$ for $x = 7$

46. $y = 3^x$ or $y = x^3$ for $x = 5$

47. $f(x) = 2^x$ or $f(x) = 100x^2$ for $x = 10$

48. **Computers** A computer valued at \$1500 loses 20% of its value each year.
   a. Write a function rule that models the value of the computer.
   b. Find the value of the computer after 3 yr.
   c. In how many years will the value of the computer be less than \$500?

49. a. Graph the functions $y = x^2$ and $y = 2^x$ on the same axes.
   b. What do you notice about the graphs for the values of x between 1 and 3?
   c. **Reasoning** How do you think the graph of $y = 8^x$ would compare to the graphs of $y = x^2$ and $y = 2^x$?

50. **Writing** Find the range of the function $f(x) = 500 \cdot 1^x$ using the domain $\{1, 2, 3, 4, 5\}$. Explain why the definition of *exponential function* states that $b \neq 1$.

Challenge **Solve each equation.**

51. $2^x = 64$

52. $3^x = \frac{1}{27}$

53. $3 \cdot 2^x = 24$

54. $5 \cdot 2^x - 152 = 8$

55. Suppose (0, 4) and (2, 36) are on the graph of an exponential function.
   a. Use (0, 4) in the general form of an exponential function, $y = a \cdot b^x$, to find the value of the constant a.
   b. Use your answer from part (a) and (2, 36) to find the value of the constant b.
   c. Write a rule for the function.
   d. Evaluate the function for $x = -2$ and $x = 4$.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 7

Look back at the information on page 417 about the two CDs that Emilio is considering for the investment of his prize money, and at your work in the Apply What You've Learned in Lesson 7-3.

Let r be the annual interest rate, expressed as a decimal, of an account for which interest is compounded n times per year, and let P represent the starting principal (the initial deposit).

a. Write an expression in terms of r and n for the interest rate used to calculate the interest for each compounding period.

b. Write an expression in terms of P, r, and n for the value of the account after the first compounding period.

c. Explain how you can use the Distributive Property to rewrite your expression from part (b).

d. Using a process similar to the one you used to complete the table on page 438, you can show that the formula $A = P\left(1 + \frac{r}{n}\right)^x$ gives the value of the account after x compounding periods. Use this formula to confirm your result in part (b) of the Apply What You've Learned on page 438.

e. Suppose Emilio chooses the First Bank CD. Use the formula given in part (d) to find the value of the CD after one year.

f. Is the formula given in part (d) an exponential function? Explain.