7-8 Geometric Sequences

Common Core State Standards

F-BF.A.2 Write . . . geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **Also F-BF.A.1a, F-LE.A.2**

MP 1, MP 2, MP 3, MP 4, MP 6

Objective To write and use recursive formulas for geometric sequences

SOLVE IT! Getting Ready!

Imagine working part time at a clothing store. Each week a coat doesn't sell, its price is marked down 20%. What will the sale price for the coat shown at the right be after three weeks?

MATHEMATICAL PRACTICES In the Solve It, the sales prices form a *geometric sequence.*

Essential Understanding In a **geometric sequence**, the ratio of any term to its preceding term is a constant value.

Lesson Vocabulary
- geometric sequence

take note

Key Concept Geometric Sequence

A geometric sequence with a *starting value* a and a *common ratio* r is a sequence of the form $a, ar, ar^2, ar^3, \ldots$

A *recursive definition* for the sequence has two parts:

$a_1 = a$ Initial condition

$a_n = a_{n-1} \cdot r$, for $n \geq 2$ Recursive formula

An *explicit definition* for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Every geometric sequence has a starting value and a common ratio. The starting value and common ratio define a unique geometric sequence.

Problem 1 Identifying Geometric Sequences

Think

How do you find the common ratio between two adjacent terms? Divide each term by the previous term.

Which of the following are geometric sequences?

A 20 200 2,000 20,000 200,000, . . .

× 10 × 10 × 10 × 10

There is a common ratio, $r = 10$. So, the sequence is geometric.

B 2 4 6 8 10, . . .

× 2 × 1.5 × 1.33 × 1.25

There is no common ratio. So, the sequence is not geometric.

C 5 −5 5 −5 5, . . .

× −1 × −1 × −1 × −1

There is a common ratio, $r = -1$. So, the sequence is geometric.

Got It? **1.** Which of the following are geometric sequences? If the sequence is not geometric, is it arithmetic?

a. 3, 6, 12, 24, 48, . . .

b. 3, 6, 9, 12, 15, . . .

c. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

d. 4, 7, 11, 16, 22, . . .

Any geometric sequence can be written with both an explicit and a recursive formula. The recursive formula is useful for finding the next term in the sequence. The explicit formula is more convenient when finding the nth term.

Problem 2 Finding Recursive and Explicit Formulas

Plan

What do you need to know in order to write recursive and explicit formulas for a geometric sequence? You need the common ratio and the starting value.

Find the recursive and explicit formulas for the sequence 7, 21, 63, 189, . . .

The starting value a_1 is 7. The common ratio r is $\frac{21}{7} = 3$.

$a_1 = a; a_n = a_{n-1} \cdot r$	Use the formula.	$a_n = a_1 \cdot r^{n-1}$
$a_1 = 7; a_n = a_{n-1} \cdot r$	Substitute the starting value for a_1.	$a_n = 7 \cdot r^{n-1}$
$a_1 = 7; a_n = a_{n-1} \cdot 3$	Substitute the common ratio for r.	$a_n = 7 \cdot 3^{n-1}$

The recursive formula is $a_1 = 7; a_n = a_{n-1} \cdot 3$.

The explicit formula is $a_n = 7 \cdot 3^{n-1}$.

Got It? **2.** Find the recursive and explicit formulas for each of the following.

a. 2, 4, 8, 16, . . .

b. 40, 20, 10, 5, . . .

Problem 3 Using Sequences

Two managers at a clothing store created sequences to show the original price and the marked-down prices of an item. Write a recursive formula and an explicit formula for each sequence. What will the price of the item be after the 6th markdown?

Think

Which term number represents the 6th markdown?

The 1st term represents the original price, the 2nd term represents the 1st markdown, the 3rd term represents the 2nd markdown, and so on. Therefore, the 7th term represents the 6th markdown.

First Sequence		Second Sequence
\$60, \$51, \$43.35, \$36.85, ...		\$60, \$52, \$44, \$36, ...
common ratio = 0.85		common difference = −8
$a_1 = 60$		$a_1 = 60$
$a_n = a_{n-1} \cdot 0.85$	Recursive formula	$a_n = a_{n-1} - 8$
$a_n = 60 \cdot 0.85^{n-1}$	Explicit formula	$a_n = -8(n-1) + 60$
$a_7 = 60 \cdot 0.85^{7-1}$	Substitute 7 for *n*.	$a_7 = -8(7-1) + 60$
$a_7 \approx \$22.63$	Simplify.	$a_7 = \$12$

The price continuing the first sequence is \$22.63 after the 6th markdown.

The price continuing the second sequence is \$12 after the 6th markdown.

Got It? **3.** Write a recursive formula and an explicit formula for each sequence. Find the 8th term of each sequence.

a. 14, 84, 504, 3024, ... **b.** 648, 324, 162, 81, ...

You can also represent a sequence by using function notation. This allows you to plot the sequence using the points (n, a_n), where n is the term number and a_n is the term.

Problem 4 Writing Geometric Sequences as Functions

A geometric sequence has an initial value of 6 and a common ratio of 2. Write a function to represent the sequence. Graph the function.

Think

What is the domain of the sequence? What is the domain of the function?

The domain of the sequence is positive integers ≥ 1. The domain of the function is all real numbers.

$a_n = a_1 \cdot r^{n-1}$ Explicit formula

$f(x) = 6 \cdot 2^{x-1}$ Substitute $f(x)$ for a_n, 6 for a_1, and 2 for r.

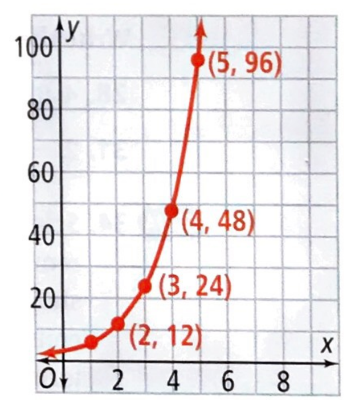
The function $f(x) = 6 \cdot 2^{x-1}$ represents the geometric sequence.

Determine the first few terms of the sequence by using the function.

$f(2) = 6 \cdot 2^{2-1} = 12$ $f(4) = 6 \cdot 2^{4-1} = 48$

$f(3) = 6 \cdot 2^{3-1} = 24$ $f(5) = 6 \cdot 2^{5-1} = 96$

Plot the points (1, 6), (2, 12), (3, 24), (4, 48), and (5, 96).



Got It? **4.** A geometric sequence has an initial value of 2 and a common ratio of 3. Write a function to represent the sequence. Graph the function.

Lesson Check

Do you know HOW?

Are the following geometric sequences? If so, find the common ratio.

1. 3, 9, 27, 81, . . .
2. 200, 50, 12.5, 3.125, . . .
3. 10, 8, 6, 4, . . .

Write the explicit and recursive formulas for each geometric sequence.

4. 5, 20, 80, 320, . . .
5. 4, −8, 16, −32, . . .
6. 162, 108, 72, 48, . . .
7. 3, 6, 12, 24, . . .

Do you UNDERSTAND? MATHEMATICAL PRACTICES

8. **Error Analysis** A friend says that the recursive formula for the geometric sequence 1, −1, 1, −1, 1, . . . is $a_n = 1 \cdot (-1)^{n-1}$. Explain your friend's error and give the correct recursive formula for the sequence.

9. **Critical Thinking** Describe the similarities and differences between arithmetic and geometric sequences.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Determine whether the sequence is a geometric sequence. Explain.

See Problem 1.

10. 2, 8, 32, 128, . . .
11. 5, 10, 15, 20, . . .
12. 162, 54, 18, 6, . . .
13. 256, 192, 144, 108, . . .
14. 6, −12, 24, −48, . . .
15. 10, 20, 40, 80, . . .

Find the common ratio for each geometric sequence.

16. 3, 6, 12, 24, . . .
17. 81, 27, 9, 3, . . .
18. 128, 96, 72, 54, . . .
19. 5, 20, 80, 320, . . .
20. 7, −7, 7, −7, . . .
21. 2, −6, 18, −54, . . .

Write the explicit formula for each geometric sequence.

See Problem 2.

22. 2, 6, 18, 54, . . .
23. 3, 6, 12, 24, . . .
24. 200, 40, 8, $1\frac{3}{5}$, . . .
25. 3, −12, 48, −192, . . .
26. 8, −8, 8, −8, . . .
27. 686, 98, 14, 2, . . .

Write the recursive formula for each geometric sequence.

28. 4, 8, 16, 32, . . .
29. 1, 5, 25, 125, . . .
30. 100, 50, 25, 12.5, . . .
31. 2, −8, 32, −64, . . .
32. $-\frac{1}{36}, \frac{1}{12}, -\frac{1}{4}, \frac{3}{4}, \ldots$
33. 192, 128, $85\frac{1}{3}$, $56\frac{8}{9}$, . . .

STEM 34. **Science** When a radioactive substance decays, measurements of the amount remaining over constant intervals of time form a geometric sequence. The table shows the amount of Fl-18, remaining after different constant intervals. Write the explicit and recursive formulas for the geometric sequence formed by the amount of Fl-18 remaining.

See Problem 3.

Fluorine-18 Remaining

Time (min)	0	110	220	330
Fl-18 (picograms)	260	130	65	32.5

35. A store manager plans to offer discounts on some sweaters according to this sequence: \$48, \$36, \$27, \$20.25, . . . Write the explicit and recursive formulas for the sequence.

36. A geometric sequence has an initial value of 18 and a common ratio of $\frac{1}{2}$. Write a function to represent this sequence. Graph the function.

See Problem 4.

37. Write and graph the function that represents the sequence in the table.

x	1	2	3	4
$f(x)$	8	16	32	64

B Apply

Determine if each sequence is a geometric sequence. If it is, find the common ratio and write the explicit and recursive formulas.

38. 5, 10, 20, 40, . . .

39. 20, 15, 10, 5, . . .

40. 3, −9, 27, −81, . . .

41. 98, 14, 2, $\frac{2}{7}$, . . .

42. −3, −1, 1, 3, . . .

43. 200, −100, 50, −25, . . .

Identify each sequence as *arithmetic*, *geometric*, or *neither*.

44. 1.5, 4.5, 13.5, 40.5, . . .

45. 42, 38, 34, 30, . . .

46. 4, 9, 16, 25, . . .

47. −4, 1, 6, 11, . . .

48. 1, 2, 3, 5, . . .

49. 2, 8, 32, 128, . . .

50. **Think about a Plan** Suppose you are rehearsing for a concert. You plan to rehearse the piece you will perform four times the first day and then to double the number of times you rehearse the piece each day until the concert. What are two formulas you can write to describe the sequence of how many times you will rehearse the piece each day?
- How can you write a sequence of numbers to represent this situation?
- Is the sequence arithmetic, geometric, or neither?
- How can you write explicit and recursive formulas for this sequence?

51. **Open-Ended** Write a geometric sequence. Then write the explicit and recursive formulas for your sequence.

STEM 52. **Science** A certain culture of yeast increases by 50% every three hours. A scientist places 9 grams of the yeast in a culture dish. Write the explicit and recursive formulas for the geometric sequence formed by the growth of the yeast.

C Challenge

53. The differences between consecutive terms in a geometric sequence form a new geometric sequence. For instance, when you take the differences between the consecutive terms of the geometric sequence 5, 15, 45, 135, . . . you get 15 − 5, 45 − 15, 135 − 45, . . . The new geometric sequence is 10, 30, 90, . . . Compare the two sequences. How are they similar, and how do they differ?

Standardized Test Prep

54. Which of the following is the explicit formula for the geometric sequence 15, 3, 0.6, 0.12, . . .?

Ⓐ $a_n = 15 \cdot 0.2^{n-1}$

Ⓑ $a_1 = 0.12; a_n = 5 \cdot a^{n-1}$

Ⓒ $a_1 = 15; a_n = 0.2 \cdot a^{n-1}$

Ⓓ $a_n = 0.2 \cdot 15^{n-1}$

55. Which of the following is the recursive formula for the geometric sequence 2, 12, 72, 432, . . .?

Ⓕ $a_n = 2 \cdot 6^{n-1}$

Ⓖ $a_1 = 2; a_n = 6 \cdot a^{n-1}$

Ⓗ $a_1 = 6; a_n = 2 \cdot a^{n-1}$

Ⓘ $a_n = 6 \cdot 2^{n-1}$

56. Which of the following is the explicit formula for the geometric sequence 12, 18, 27, 40.5, . . .?

Ⓐ $a_n = 15 \cdot 1.2^{n-1}$

Ⓑ $a_1 = 12; a_n = 0.5 \cdot a^{n-1}$

Ⓒ $a_1 = 12; a_n = 1.5 \cdot a^{n-1}$

Ⓓ $a_n = 12 \cdot 1.5^{n-1}$

57. Which of the following is both an arithmetic sequence and a geometric sequence?

Ⓕ 1, −1, 1, −1, . . .

Ⓖ 16, 24, 36, 54, . . .

Ⓗ 1, 4, 9, 16, . . .

Ⓘ 5, 5, 5, 5, . . .

Short Response

58. Write the explicit and recursive formulas for the geometric sequence 27, 36, 48, 64, . . .

Mixed Review

Write the recursive and explicit formulas for each arithmetic sequence. See Lesson 4-7.

59. 0, 9, 18, 27, . . .

60. 5, 3, 1, −1, . . .

61. −7, −3, 1, 5, . . .

Solve each equation. See Lesson 2-3.

62. $3x + 7 = 2x - 1$

63. $\frac{y}{2} + 5 = -y + 2$

64. $5(a - 3) = 20 + a$

Solve the following problems. See Lesson 2-10.

65. A price changes from \$40 to \$70. What is the percent change?

66. The value 25 is increased by 50%. What is the new value?

Get Ready! **To prepare for Lesson 8-1, do Exercises 67–69.**

Simplify the following expressions. See Lesson 1-8.

67. $(2x + 3y) + 2y$

68. $(4a + 5b) - 3b$

69. $(-6c + 5d) + 2c$