# 8-1 Adding and Subtracting Polynomials

**Common Core State Standards**

**A-APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**MP 1, MP 2, MP 3, MP 4, MP 6**

**Objective** To classify, add, and subtract polynomials

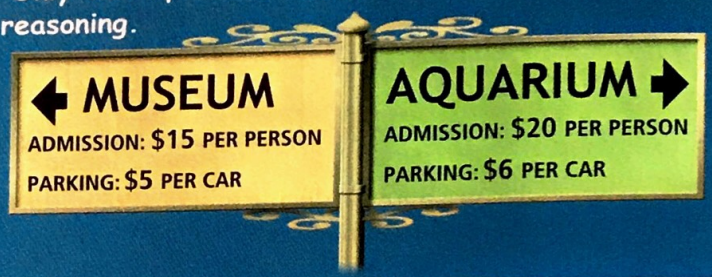
Try breaking this problem into simpler parts. How much do they spend on admission? On parking?

MATHEMATICAL PRACTICES

SOLVE IT!

**Getting Ready!**

The costs of admission and parking for a museum and an aquarium are given. What is the total cost of m people in n cars visiting only the museum? Only the aquarium? Both the museum and the aquarium? Explain your reasoning.




In some cases, you can model a situation with an expression composed of *monomials*. A **monomial** is a real number, a variable, or a product of a real number and one or more variables with whole-number exponents. Here are some examples of monomials.

$$18 \qquad z \qquad -4x^2 \qquad 2.5xy^3 \qquad \frac{a}{3}$$

**Lesson Vocabulary**
- monomial
- degree of a monomial
- polynomial
- standard form of a polynomial
- degree of a polynomial
- binomial
- trinomial

**Essential Understanding** You can use monomials to form larger expressions called *polynomials*. Polynomials can be added and subtracted.

The **degree of a monomial** is the sum of the exponents of its variables. The degree of a nonzero constant is 0. Zero has no degree.

## Problem 1 Finding the Degree of a Monomial

**What is the degree of each monomial?**

**A** $5x$ Degree: 1 $5x = 5x^1$. The exponent is 1.

**B** $6x^3y^2$ Degree: 5 The exponents are 3 and 2. Their sum is 5.

**C** $4$ Degree: 0 $4 = 4x^0$. The degree of a nonzero constant is 0.

**Think**

**Why is the degree of a nonzero constant 0?** You can write a nonzero constant $c$ as $cx^0$. The exponent is 0, so the degree is 0 also.

**Got It?** **1.** What is the degree of each monomial?

**a.** $8xy$ **b.** $-7y^4z$ **c.** 11

You can add or subtract monomials by adding or subtracting like terms.

## Problem 2 Adding and Subtracting Monomials

**What is the sum or difference?**

**A** $3x^2 + 5x^2 = 8x^2$ Combine like terms.

**B** $4x^3y - x^3y = 3x^3y$ Combine like terms.

**Think**
**Will the sum of two monomials always be a monomial?**
No. The monomials must be like terms.

**Got It?** **2.** What is the sum $-6x^4 + 11x^4$? What is the difference $2x^2y^4 - 7x^2y^4$?

A **polynomial** is a monomial or a sum of monomials. The following polynomial is the sum of the monomials $3x^4$, $5x^2$, $-7x$, and 1.

$$3x^4 + 5x^2 - 7x + 1$$

Degree of each monomial: 4, 2, 1, 0

The polynomial shown above is in *standard form*. **Standard form of a polynomial** means that the degrees of its monomial terms decrease from left to right. The **degree of a polynomial** in one variable is the same as the degree of the monomial with the greatest exponent. The degree of $3x^4 + 5x^2 - 7x + 1$ is 4.

You can name a polynomial based on its degree or the number of monomials it contains.

| Polynomial | Degree | Name Using Degree | Number of Terms | Name Using Number of Terms |
|---|---|---|---|---|
| 6 | 0 | Constant | 1 | Monomial |
| $5x + 9$ | 1 | Linear | 2 | Binomial |
| $4x^2 + 7x + 3$ | 2 | Quadratic | 3 | Trinomial |
| $2x^3$ | 3 | Cubic | 1 | Monomial |
| $8x^4 - 2x^3 + 3x$ | 4 | Fourth degree | 3 | Trinomial |

## Problem 3 Classifying Polynomials

**Write each polynomial in standard form. What is the name of the polynomial based on its degree and number of terms?**

**Think**
**Why do you need to combine like terms in part (B)?**
To name a polynomial correctly based on its number of terms, you must first combine all like terms.

**A** $3x + 4x^2$

$4x^2 + 3x$ Place terms in order.

This is a quadratic binomial.

**B** $4x - 1 + 5x^3 + 7x$

$5x^3 + 4x + 7x - 1$ Place terms in order.

$5x^3 + 11x - 1$ Combine like terms.

This is a cubic trinomial.

**Got It?** **3. a.** Write $2x - 3 + 8x^2$ in standard form. What is the name of the polynomial based on its degree and number of terms?

**b. Reasoning** How does writing a polynomial in standard form help you name the polynomial?

You can add polynomials by adding like terms.

## Problem 4 Adding Polynomials

**Travel** **A researcher studied the number of overnight stays in U.S. National Park Service campgrounds and in the backcountry of the national park system over a 5-yr period. The researcher modeled the results, in thousands, with the following polynomials.**

**Campgrounds:** $-7.1x^2 - 180x + 5800$

**Backcountry:** $21x^2 - 140x + 1900$

**In each polynomial, $x = 0$ corresponds to the first year in the 5-yr period. What polynomial models the total number of overnight stays in both campgrounds and backcountry?**

**Know**
- Overnight stays in campgrounds: $-7.1x^2 - 180x + 5800$
- Overnight stays in backcountry: $21x^2 - 140x + 1900$

**Need**
A polynomial for the total number of overnight stays in campgrounds and backcountry

**Plan**
The word *both* implies addition, so add the two polynomials to find a polynomial that represents the total.

**Method 1** Add vertically.
Line up like terms. Then add the coefficients.

$$\begin{array}{r} -7.1x^2 - 180x + 5800 \\ +\ 21x^2 - 140x + 1900 \\ \hline 13.9x^2 - 320x + 7700 \end{array}$$

**Method 2** Add horizontally.
Group like terms. Then add the coefficients.

$$\begin{aligned} &(-7.1x^2 - 180x + 5800) + (21x^2 - 140x + 1900) \\ &= (-7.1x^2 + 21x^2) + (-180x - 140x) + (5800 + 1900) \\ &= 13.9x^2 - 320x + 7700 \end{aligned}$$

A polynomial that models the number of stays (in thousands) in campgrounds and backcountry over the 5-yr period is $13.9x^2 - 320x + 7700$.

**Got It?** **4.** A nutritionist studied the U.S. consumption of carrots and celery and of broccoli over a 6-yr period. The nutritionist modeled the results, in millions of pounds, with the following polynomials.

Carrots and celery: $-12x^3 + 106x^2 - 241x + 4477$

Broccoli: $14x^2 - 14x + 1545$

In each polynomial, $x = 0$ corresponds to the first year in the 6-yr period. What polynomial models the total number of pounds, in millions, of carrots, celery, and broccoli consumed in the United States during the 6-yr period?

Recall that subtraction means to add the opposite. So when you subtract a polynomial, change each of the terms to its opposite. Then add the coefficients.

## Problem 5 Subtracting Polynomials

**What is a simpler form of $(x^3 - 3x^2 + 5x) - (7x^3 + 5x^2 - 12)$?**

**Think**
**Is the sum or difference of two polynomials always a polynomial?**
Yes. The set of polynomials is *closed* under addition and subtraction, which means that adding or subtracting polynomials always gives you another polynomial.

**Method 1** Subtract vertically.

$$\begin{array}{rl} & x^3 - 3x^2 + 5x \\ \underline{-\,(7x^3 + 5x^2} & \underline{\qquad - 12)} \\ & x^3 - 3x^2 + 5x \\ & -7x^3 - 5x^2 \qquad + 12 \\ \hline & -6x^3 - 8x^2 + 5x + 12 \end{array}$$

Line up like terms.

Then add the opposite of each term in the polynomial being subtracted.

**Method 2** Subtract horizontally.

$$(x^3 - 3x^2 + 5x) - (7x^3 + 5x^2 - 12)$$

$= x^3 - 3x^2 + 5x - 7x^3 - 5x^2 + 12$ — Write the opposite of each term in the polynomial being subtracted.

$= (x^3 - 7x^3) + (-3x^2 - 5x^2) + 5x + 12$ — Group like terms.

$= -6x^3 - 8x^2 + 5x + 12$ — Simplify.

**Got It?** **5.** What is a simpler form of $(-4m^3 - m + 9) - (4m^2 + m - 12)$?

## Lesson Check

### Do you know HOW?

Find the degree of each monomial.

1. $-7x^4$
2. $8y^2z^3$

Simplify each sum or difference.

3. $(5r^3 + 8) + (6r^3 + 3)$
4. $(x^2 - 2) - (3x + 5)$

### Do you UNDERSTAND? MATHEMATICAL PRACTICES

**Vocabulary** Name each polynomial based on its degree and number of terms.

5. $5x^2 + 2x + 1$
6. $3z - 2$

7. **Compare and Contrast** How are the processes of adding monomials and adding polynomials alike? How are the processes different?

## Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

**A Practice** Find the degree of each monomial. See Problem 1.

8. $3x$
9. $8a^3$
10. $20$
11. $2b^8c^2$
12. $-7y^3z$
13. $-3$
14. $12w^4$
15. $0$

**Simplify.**

See Problem 2.

**16.** $12p^2 + 8p^2$ **17.** $2m^3n^3 + 9m^3n^3$ **18.** $8w^2x + w^2x$ **19.** $3t^4 + 11t^4$

**20.** $x^3 - 9x^3$ **21.** $30v^4w^3 - 12v^4w^3$ **22.** $7x^2 - 2x^2$ **23.** $5bc^4 - 13bc^4$

**Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.**

See Problem 3.

**24.** $5y - 2y^2$ **25.** $-2q + 7$ **26.** $x^2 + 4 - 3x$

**27.** $6x^2 - 13x^2 - 4x + 4$ **28.** $c + 8c^3 - 3c^7$ **29.** $3z^4 - 5z - 2z^2$

**Simplify.**

See Problem 4.

**30.** $\begin{array}{r} 4w - 5 \\ +\ 9w + 2 \\ \hline \end{array}$ **31.** $\begin{array}{r} 6x^2 + 7 \\ +\ 3x^2 + 1 \\ \hline \end{array}$ **32.** $\begin{array}{r} 2k^2 - k + 3 \\ +\ 5k^2 + 3k - 7 \\ \hline \end{array}$

**33.** $(5x^2 + 3) + (15x^2 + 2)$ **34.** $(2g^4 - 3g + 9) + (-g^3 + 12g)$

**35. Education** The number of students at East High School and the number of students at Central High School over a 10-year period can be modeled by the following polynomials.

East High School: $-11x^2 + 133x + 1200$
Central High School: $-7x^2 + 95x + 1100$

In each polynomial, $x = 0$ corresponds to the first year in the 10-year period. What polynomial models the total number of students at both high schools?

**Simplify.**

See Problem 5.

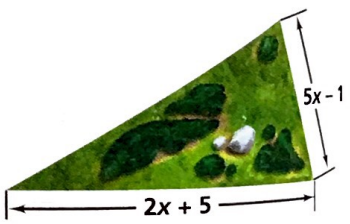
**36.** $\begin{array}{r} 5n - 2 \\ -(3n + 8) \\ \hline \end{array}$ **37.** $\begin{array}{r} 6x^3 + 17 \\ -(4x^3 + \ 9) \\ \hline \end{array}$ **38.** $\begin{array}{r} 2c^2 + \ 7c - 1 \\ -(c^2 - 10c + 4) \\ \hline \end{array}$

**39.** $(14h^4 + 3h^3) - (9h^4 + 2h^3)$ **40.** $(-6w^4 + w^2) - (-2w^3 + 4w^2 - w)$

## B Apply

**41. Think About a Plan** The perimeter of a triangular park is $16x + 3$. What is the missing length?

- What is the sum of the two given side lengths?
- What operation should you use to find the remaining side length?




**42. Geometry** The perimeter of a trapezoid is $39a - 7$. Three sides have the following lengths: $9a$, $5a + 1$, and $17a - 6$. What is the length of the fourth side?

**43. Error Analysis** Describe and correct the error in finding the difference of the polynomials.

$$(4x^2 - x + 3) - (3x^2 - 5x - 6) = 4x^2 - x + 3 - 3x^2 - 5x - 6$$
$$= 4x^2 - 3x^2 - x - 5x + 3 - 6$$
$$= x^2 - 6x - 3$$

**Simplify. Write each answer in standard form.**

**44.** $(5x^2 - 3x + 7x) + (9x^2 + 2x^2 + 7x)$

**45.** $(y^3 - 4y^2 - 2) - (6y^3 + 4 - 6y^2)$

**46.** $(-9r^3 + 2r - 1) - (-5r^2 + r + 8)$

**47.** $(3z^3 - 4z + 7z^2) + (8z^2 - 6z - 5)$

**48. a.** Is the sum of two polynomials always a polynomial? Explain.

**b.** Is the difference of two polynomials always a polynomial? Explain.

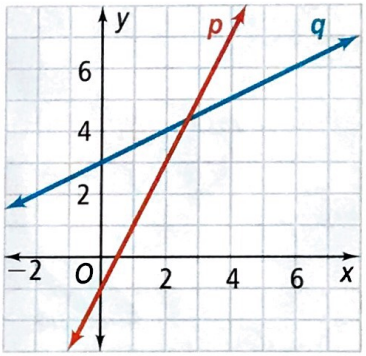
**Challenge**

**49. a.** Write the equations for line $p$ and line $q$. Use slope-intercept form.

**b.** Use your equations from part (a) to write a function for the vertical distance $D(x)$ between points on lines $p$ and $q$ with the same $x$-value.

**c.** For what value of $x$ does $D(x)$ equal zero?

**d. Reasoning** How does the $x$-value in part (c) relate to the graph?




**Simplify each expression.**

**50.** $(ab^2 + ba^3) + (4a^3b - ab^2 - 5ab)$

**51.** $(9pq^6 - 11p^4q) - (-5pq^6 + p^4q^4)$

## Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at Kelly's and Roberto's original plots on page 485. Choose from the following words, numbers, and expressions to complete the sentences below.

| | | | | |
|---|---|---|---|---|
| monomial | binomial | trinomial | 1 | 2 |
| $x^2$ | $x + 2$ | $x^2 + 2$ | $x + 6$ | $2x + 6$ |

Two polynomials that represent the length and width of Kelly's plot are **a.** ? and **b.** ? . Each of these polynomials is an example of a **c.** ? . The polynomial that represents the area of Roberto's flower bed is **d.** ? . This polynomial is an example of a **e.** ? . The degree of this polynomial is **f.** ? .