8-2 Multiplying and Factoring

Common Core State Standards

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

MP 1, MP 2, MP 3, MP 4

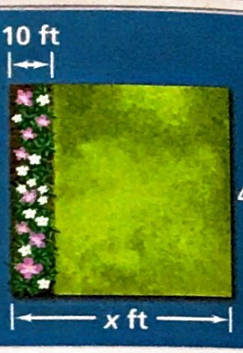
Objectives To multiply a monomial by a polynomial
To factor a monomial from a polynomial

Sketch a diagram. A diagram can help you understand all the parts of this problem.

SOLVE IT!

Getting Ready!

You set aside part of a rectangular plot of land for a garden and seed the rest of the plot with grass, as shown. Grass seed costs \$.03 per square foot. Write an expression for the total cost of the seed. Suppose you buy \$50 worth of seed. How wide can the section of grass be? Explain your reasoning.

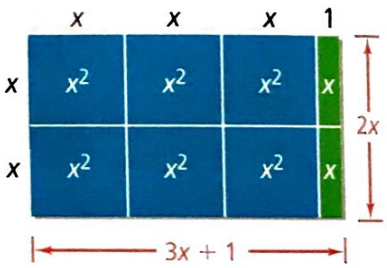


MATHEMATICAL PRACTICES **Essential Understanding** You can use the Distributive Property to multiply a monomial by a polynomial.

For example, consider the product $2x(3x + 1)$.

$$\begin{aligned} 2x(3x + 1) &= 2x(3x) + 2x(1) \\ &= 6x^2 + 2x \end{aligned}$$

You can show why the multiplication makes sense using the area model at the right.



Problem 1 Multiplying a Monomial and a Trinomial

Multiple Choice What is a simpler form of $-x^3(9x^4 - 2x^3 + 7)$?

(A) $-9x^{12} + 2x^9 - 7x^3$

(B) $9x^7 - 2x^6 + 7x^3$

(C) $-9x^7 - 2x^3 + 7$

(D) $-9x^7 + 2x^6 - 7x^3$

Plan

What should I keep in mind when multiplying?
Remember to distribute $-x^3$ to *all* of the terms. Also remember to add the exponents instead of multiplying them.

$$-x^3(9x^4 - 2x^3 + 7) = -x^3(9x^4) - x^3(-2x^3) - x^3(7)$$ Use the Distributive Property.

$$= -9x^{3+4} + 2x^{3+3} - 7x^3$$ Multiply coefficients and add exponents.

$$= -9x^7 + 2x^6 - 7x^3$$ Simplify.

The correct answer is D.

Got It? 1. What is a simpler form of $5n(3n^3 - n^2 + 8)$?

Essential Understanding Factoring a polynomial reverses the multiplication process. When factoring a monomial from a polynomial, the first step is to find the greatest common factor (GCF) of the polynomial's terms.

Problem 2 Finding the Greatest Common Factor

What is the GCF of the terms of $5x^3 + 25x^2 + 45x$?

List the prime factors of each term. Identify the factors common to all terms.

$5x^3 = 5 \cdot x \cdot x \cdot x$

$25x^2 = 5 \cdot 5 \cdot x \cdot x$

$45x = 3 \cdot 3 \cdot 5 \cdot x$

Remember to list only the prime factors of the variables.

The GCF is $5 \cdot x$, or $5x$.

Think

Why use the factors 5 and x to form the GCF, but not 3?
Both 5 and x are factors of *every* term of the polynomial, but 3 is only a factor of the last term.

Got It? **2.** What is the GCF of the terms of $3x^4 - 9x^2 - 12x$?

Once you find the GCF of a polynomial's terms, you can factor it out of the polynomial.

Problem 3 Factoring Out a Monomial

What is the factored form of $4x^5 - 24x^3 + 8x$?

Think

To factor the polynomial, first factor each term.

Find the GCF of the three terms.

Factor out the GCF from each term. Then factor it out of the polynomial.

Write

$4x^5 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x$

$24x^3 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$

$8x = 2 \cdot 2 \cdot 2 \cdot x$

The GCF is $2 \cdot 2 \cdot x$, or $4x$.

$4x^5 - 24x^3 + 8x = 4x(x^4) + 4x(-6x^2) + 4x(2)$

$= 4x(x^4 - 6x^2 + 2)$

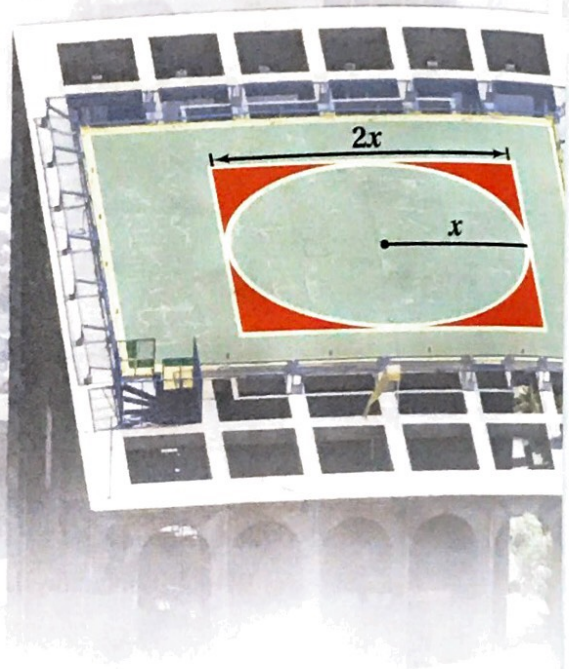
The factored form of the polynomial is $4x(x^4 - 6x^2 + 2)$.

Got It? **3. a.** What is the factored form of $9x^6 + 15x^4 + 12x^2$?

b. Reasoning What is $-6x^4 - 18x^3 - 12x^2$ written as the product of a polynomial with positive coefficients and a monomial?

Problem 4 Factoring a Polynomial Model

Helipads A helicopter landing pad, or helipad, is sometimes marked with a circle inside a square so that it is visible from the air. What is the area of the shaded region of the helipad at the right? Write your answer in factored form.



Plan

How can you find the shaded region's area? The shaded region is the entire square except for the circular portion. So, subtract the area of the circle from the area of the square.

Step 1 Find the area of the shaded region.

$A_1 = s^2$ Area of a square

$= (2x)^2$ Substitute $2x$ for s.

$= 4x^2$ Simplify.

$A_2 = \pi r^2$ Area of a circle

$= \pi x^2$ Substitute x for r.

The area of the shaded region is $A_1 - A_2$, or $4x^2 - \pi x^2$.

Step 2 Factor the expression.

First find the GCF.

$4x^2 = 2 \cdot 2 \cdot x \cdot x$

$\pi x^2 = \pi \cdot x \cdot x$

The GCF is $x \cdot x$, or x^2.

Step 3 Factor out the GCF.

$4x^2 - \pi x^2 = x^2(4) + x^2(-\pi)$

$= x^2(4 - \pi)$

The factored form of the area of the shaded region is $x^2(4 - \pi)$.

Got It? **4.** In Problem 4, suppose the side length of the square is $6x$ and the radius of the circle is $3x$. What is the factored form of the area of the shaded region?

Lesson Check

Do you know HOW?

1. What is a simpler form of $6x(2x^3 + 7x)$?

2. What is the GCF of the terms in $4a^4 + 6a^2$?

Factor each polynomial.

3. $6m^2 - 15m$

4. $4x^3 + 8x^2 + 12x$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Match each pair of monomials with its GCF.

5. $14n^2, 35n^4$ **A.** 1

6. $21n^3, 18n^2$ **B.** $7n^2$

7. $7n^2, 9$ **C.** $3n^2$

8. Reasoning Write a binomial with $9x^2$ as the GCF of its terms.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Simplify each product.

See Problem 1.

9. $7x(x + 4)$
10. $(b + 11)2b$
11. $3m^2(10 + m)$
12. $-w^2(w - 15)$
13. $4x(2x^3 - 7x^2 + x)$
14. $-8y^3(7y^2 - 4y - 1)$

Find the GCF of the terms of each polynomial.

See Problem 2.

15. $12x + 20$
16. $8w^2 - 18w$
17. $45b + 27$
18. $a^3 + 6a^2 - 11a$
19. $4x^3 + 12x - 28$
20. $14z^4 - 42z^3 + 21z^2$

Factor each polynomial.

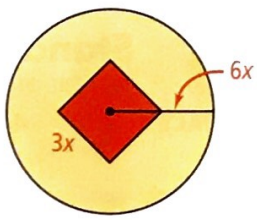
See Problem 3.

21. $9x - 6$
22. $t^2 + 8t$
23. $14n^3 - 35n^2 + 28$
24. $5k^3 + 20k^2 - 15$
25. $14x^3 - 2x^2 + 8x$
26. $g^4 + 24g^3 + 12g^2 + 4g$

27. **Art** A circular mirror is surrounded by a square metal frame. The radius of the mirror is $5x$. The side length of the metal frame is $15x$. What is the area of the metal frame? Write your answer in factored form.

See Problem 4.

28. **Design** A circular table is painted yellow with a red square in the middle. The radius of the tabletop is $6x$. The side length of the red square is $3x$. What is the area of the yellow part of the tabletop? Write your answer in factored form.



B Apply

Simplify. Write in standard form.

29. $-2x(5x^2 - 4x + 13)$
30. $-5y^2(-3y^3 + 8y)$
31. $10a(-6a^2 + 2a - 7)$
32. $p(p + 2) - 3p(p - 5)$
33. $t^2(t + 1) - t(2t^2 - 1)$
34. $3c(4c^2 - 5) - c(9c)$

35. **Think About a Plan** A rectangular wooden frame has side lengths $5x$ and $7x + 1$. The rectangular opening for a picture has side lengths $3x$ and $5x$. What is the area of the wooden part of the frame? Write your answer in factored form.
- How can drawing a diagram help you solve the problem?
- How can you express the area of the wooden part of the frame as a difference of areas?

36. **Error Analysis** Describe and correct the error made in multiplying.

$$-3x(2x - 5) = -3x(2x) - 3x(5)$$
$$= -6x^2 - 15x$$

Factor each polynomial.

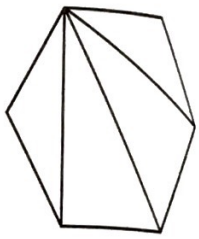
37. $17xy^4 + 51x^2y^3$
38. $9m^4n^5 - 27m^2n^3$
39. $31a^6b^3 + 63a^5$

40. **a.** Factor $n^2 + n$.
b. Writing Suppose n is an integer. Is $n^2 + n$ *always*, *sometimes*, or *never* an even integer? Justify your answer.

41. Reasoning The GCF of two numbers p and q is 7. What is the GCF of p^2 and q^2? Justify your answer.

Challenge

42. a. Geometry How many sides does the polygon have? How many of its diagonals come from one vertex?

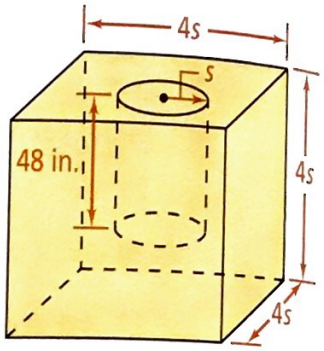


b. A polygon has n sides. How many diagonals will it have from one vertex?

c. The number of diagonals from all the vertices is $\frac{n}{2}(n-3)$. Write this polynomial in standard form.

d. A polygon has 8 sides. How many diagonals does it have?

STEM **43. Manufacturing** The diagram shows a cube of metal with a cylinder cut out of it. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.



a. Write a formula for the volume of the cube in terms of s.

b. Write a formula for the volume of the cylinder in terms of s.

c. Write a formula in terms of s for the volume V of the metal left after the cylinder has been removed.

d. Factor your formula from part (c).

e. Find V in cubic inches for $s = 15$ in. Use $\pi = 3.14$.

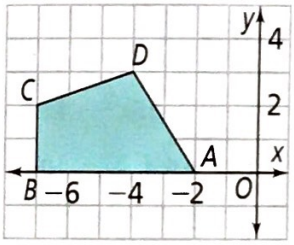
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

44. Simplify the product $4x(5x^2 + 3x + 7)$. What is the coefficient of the x^2-term?

45. What is the slope of the line that passes through $\overline{CD}$?



46. What is the solution of the equation $7x - 11 = 3$?

47. Simplify the product $8x^3(2x^2)$. What is the exponent?

48. The expression $9x^3 - 15x$ can be factored as $ax(3x^2 - 5)$. What is the value of a?

Mixed Review

Simplify each sum or difference. See Lesson 8-1.

49. $(5x^2 + 4x - 2) + (3x^2 + 7)$

50. $(4x^4 - 3x^2 - 1) + (3x^4 + 6x^2)$

51. $(3x^3 - 2x) - (8x^3 + 4x)$

52. $(7x^4 + 3x^3 - 5x + 1) - (x^3 + 8x^2 - 5x - 3)$

Solve each inequality for y. Then graph the inequality. See Lesson 6-5.

53. $4x - 5y \geq 10$

54. $7x - 2y \leq 8$

55. $-3y - x > 9$

Get Ready! To prepare for Lesson 8-3, do Exercises 56–58.

Use the Distributive Property to simplify each expression. See Lesson 1-7.

56. $8(x - 5)$

57. $-3(w + 4)$

58. $0.25(6c + 16)$