8-3 Multiplying Binomials

Common Core State Standards

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

MP 1, MP 2, MP 3, MP 4, MP 7, MP 8

Objective To multiply two binomials or a binomial by a trinomial

You've got a diagram. How can you use it to create expressions for the width and for the length?

SOLVE IT!

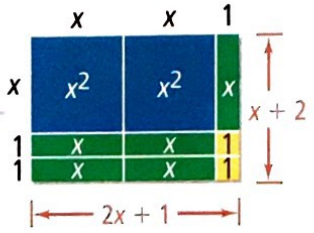
Getting Ready!

A park has a rectangular dog run with length 30 ft and width 20 ft. The parks department wants to expand each end of each side of the dog run by the same amount x. What will be the total area of the expanded dog run? Justify your reasoning.

MATHEMATICAL PRACTICES **Essential Understanding** There are several ways to find the product of two binomials, including models, algebra, and tables.

One way to find the product of two binomials is to use an area model, as shown below.

This model shows that $(2x + 1)(x + 2)$ can be written in standard form as $2x^2 + 5x + 2$.



You can also use the Distributive Property to find the product of two binomials.

Problem 1 Using the Distributive Property

What is a simpler form of $(2x + 4)(3x - 7)$?

$(2x + 4)(3x - 7) = 2x(3x - 7) + 4(3x - 7)$ Distribute the second factor, $3x - 7$.

$= 6x^2 - 14x + 4(3x - 7)$ Distribute $2x$.

$= 6x^2 - 14x + 12x - 28$ Distribute 4.

$= 6x^2 - 2x - 28$ Combine like terms.

Think

Is the product of two polynomials always a polynomial?

Yes. The set of polynomials is *closed* under multiplication, which means that multiplying polynomials always gives you another polynomial.

Got It? 1. What is a simpler form of $(x - 6)(4x + 3)$?

When you use the Distributive Property to multiply binomials, notice that you multiply each term of the first binomial by each term of the second binomial. A table can help you organize your work.

Problem 2 Using a Table

What is a simpler form of $(x - 3)(4x - 5)$?

Know
Binomial factors

Need
Product of binomials written in standard form

Plan
Use a table.

Make a table of products.

Think
Is this the only table you can make?
No. You can write the terms of $x - 3$ in a row and the terms of $4x - 5$ in a column.

	$4x$	-5
x	$4x^2$	$-5x$
-3	$-12x$	15

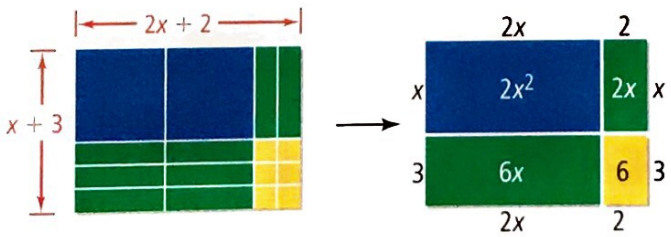
When labeling the rows and columns, think of $x - 3$ as $x + (-3)$. Think of $4x - 5$ as $4x + (-5)$.

The product is $4x^2 - 5x - 12x + 15$, or $4x^2 - 17x + 15$.

Got It? **2.** What is a simpler form of $(3x + 1)(x + 4)$? Use a table.

There is a shortcut you can use to multiply two binomials. Consider the product of $2x + 2$ and $x + 3$. The large rectangle below models this product. You can divide the large rectangle into four smaller rectangles.

The area of the large rectangle is the sum of the areas of the four smaller rectangles.



$$\begin{aligned}(2x + 2)(x + 3) &= (2x)(x) + (2x)(3) + (2)(x) + (2)(3)\\ &= 2x^2 + 6x + 2x + 6\\ &= 2x^2 + 8x + 6\end{aligned}$$

The area of each rectangle is the product of one term of $2x + 2$ and one term of $x + 3$.

This model illustrates another way to find the product of two binomials. You find the sum of the products of the First terms, the Outer terms, the Inner terms, and the Last terms of the binomials. The acronym FOIL may help you remember this method.

Problem 3 Using FOIL

What is a simpler form of $(5x - 3)(2x + 1)$?

Plan

How can a diagram help you multiply two binomials?
Draw arrows from each term of the first binomial to each term of the second binomial. This will help you organize the products of the terms.

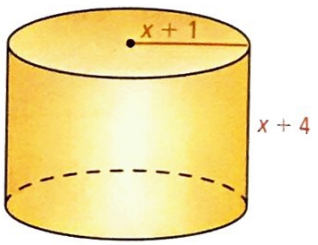
$$(5x - 3)(2x + 1) = \underset{\text{First}}{(5x)(2x)} + \underset{\text{Outer}}{(5x)(1)} + \underset{\text{Inner}}{(-3)(2x)} + \underset{\text{Last}}{(-3)(1)}$$
$$= 10x^2 + 5x - 6x - 3$$
$$= 10x^2 - x - 3$$

The product is $10x^2 - x - 3$.

Got It? **3.** What is a simpler form of each product? Use the FOIL method.
a. $(3x - 4)(x + 2)$ **b.** $(n - 6)(4n - 7)$ **c.** $(2p^2 + 3)(2p - 5)$

Problem 4 Applying Multiplication of Binomials

Multiple Choice **A cylinder has the dimensions shown in the diagram. Which polynomial in standard form best describes the total surface area of the cylinder?**



(A) $2\pi x^2 + 4\pi x + 2\pi$ (C) $4\pi x^2 + 14\pi x + 10\pi$
(B) $2\pi x^2 + 10\pi x + 8\pi$ (D) $2\pi x^2 + 2\pi x + 10\pi$

The total surface area (S.A.) of a cylinder is given by the formula S.A. $= 2\pi r^2 + 2\pi rh$, where r is the radius of the cylinder and h is the height.

Think

How can you simplify $(x + 1)^2$?
Write the expression as $(x + 1)(x + 1)$ and multiply the binomials. You do not "distribute" the exponent to each term: $(x + 1)^2 \neq x^2 + 1^2$.

S.A. $= 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$= 2\pi(x + 1)^2 + 2\pi(x + 1)(x + 4)$	Substitute $x + 1$ for r and $x + 4$ for h.
$= 2\pi(x + 1)(x + 1) + 2\pi(x + 1)(x + 4)$	Write $(x + 1)^2$ as $(x + 1)(x + 1)$.
$= 2\pi(x^2 + x + x + 1) + 2\pi(x^2 + 4x + x + 4)$	Multiply binomials.
$= 2\pi(x^2 + 2x + 1) + 2\pi(x^2 + 5x + 4)$	Combine like terms.
$= 2\pi(x^2 + 2x + 1 + x^2 + 5x + 4)$	Factor out 2π.
$= 2\pi(2x^2 + 7x + 5)$	Combine like terms.
$= 4\pi x^2 + 14\pi x + 10\pi$	Write in standard form.

The correct answer is C.

Got It? **4.** What is the total surface area of a cylinder with radius $x + 2$ and height $x + 4$? Write your answer as a polynomial in standard form.

You can use the FOIL method when you multiply two binomials, but it is not helpful when multiplying a trinomial and a binomial. In this case, you can use a vertical method to distribute each term.

Problem 5 Multiplying a Trinomial and a Binomial

What is a simpler form of $(3x^2 + x - 5)(2x - 7)$?

Plan

How should you align the polynomials? Write the polynomials so that like terms are vertically aligned.

Multiply by arranging the polynomials vertically as shown.

$$\begin{array}{rl} & 3x^2 + x - 5 \\ & \underline{\qquad\qquad 2x - 7} \\ & -21x^2 - 7x + 35 \quad \text{Multiply by } -7. \\ 6x^3 + 2x^2 - 10x & \quad \text{Multiply by } 2x. \\ \hline 6x^3 - 19x^2 - 17x + 35 & \quad \text{Add like terms.} \end{array}$$

The product is $6x^3 - 19x^2 - 17x + 35$.

Got It? **5. a.** What is a simpler form of $(2x^2 - 3x + 1)(x - 3)$?

b. Reasoning How can you use the Distributive Property to find the product of a trinomial and a binomial?

Lesson Check

Do you know HOW?

Simplify each product.

1. $(x + 3)(x + 6)$
2. $(2x - 5)(x + 3)$
3. $(x + 2)(x^2 + 3x - 4)$
4. A rectangle has length $x + 5$ and width $x - 3$. What is the area of the rectangle? Write your answer as a polynomial in standard form.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Reasoning** Explain how to use the FOIL method to find the product of two binomials.
6. **Compare and Contrast** Simplify $(3x + 8)(x + 1)$ using a table, the Distributive Property, and the FOIL method. Which method is most efficient? Explain.
7. **Writing** How is the degree of the product of two polynomials $p(x)$ and $q(x)$ related to the degrees of $p(x)$ and $q(x)$?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each product using the Distributive Property. See Problem 1.

8. $(x + 7)(x + 4)$
9. $(y - 3)(y + 8)$
10. $(m + 6)(m - 7)$
11. $(c - 10)(c - 5)$
12. $(2r - 3)(r + 1)$
13. $(2x + 7)(3x - 4)$

Simplify each product using a table. See Problem 2.

14. $(x + 5)(x - 4)$
15. $(a - 1)(a - 11)$
16. $(w - 2)(w + 6)$
17. $(2h - 7)(h + 9)$
18. $(x - 8)(3x + 1)$
19. $(3p + 4)(2p + 5)$

Simplify each product using the FOIL method.

See Problem 3.

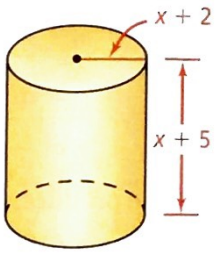
20. $(a + 8)(a - 2)$ **21.** $(x + 4)(4x - 5)$ **22.** $(k - 6)(k + 8)$

23. $(b - 3)(b - 9)$ **24.** $(5m - 2)(m + 3)$ **25.** $(9z + 4)(5z - 3)$

26. $(3h + 2)(6h - 5)$ **27.** $(4w + 13)(w + 2)$ **28.** $(8c - 1)(6c - 7)$

See Problem 4.

29. Geometry What is the total surface area of the cylinder? Write your answer as a polynomial in standard form.



30. Design The radius of a cylindrical gift box is $(2x + 3)$ in. The height of the gift box is twice the radius. What is the surface area of the cylinder? Write your answer as a polynomial in standard form.

Simplify each product.

See Problem 5.

31. $(x + 5)(x^2 - 3x + 1)$ **32.** $(k^2 - 4k + 3)(k - 2)$

33. $(2a^2 + 4a + 5)(5a - 4)$ **34.** $(2g + 7)(3g^2 - 5g + 2)$

35. Sports A school's rectangular athletic fields currently have a length of 125 yd and a width of 75 yd. The school plans to expand both the length and the width of the fields by x yards. What polynomial in standard form represents the area of the expanded athletic field?

B Apply

Simplify each product. Write in standard form.

36. $(x^2 + 1)(x - 3)$ **37.** $(-n^2 - 1)(n + 3)$ **38.** $(b^2 - 1)(b^2 + 3)$

39. $(2m^2 + 1)(m + 5)$ **40.** $(c^2 - 4)(2c + 3)$ **41.** $(4z^2 + 1)(z + 3z^2)$

42. Error Analysis Describe and correct the error made in finding the product.

$$(x - 2)(3x + 4) = x(3x) + x(4) - 2(4)$$
$$= 3x^2 + 4x - 8$$

43. Reasoning Is the product of two polynomials always a polynomial? Explain.

44. Think About a Plan You are planning a rectangular dining pavilion. Its length is three times its width x. You want a stone walkway that is 3 ft wide around the pavilion. You have enough stones to cover 396 ft^2 and want to use them all in the walkway. What should the dimensions of the pavilion be?

- Can you draw a diagram that represents this situation?
- How can you write a variable expression for the area of the walkway?

45. a. Simplify each pair of products.

i. $(x + 1)(x + 1)$	ii. $(x + 1)(x + 2)$	iii. $(x + 1)(x + 3)$
$11 \cdot 11$	$11 \cdot 12$	$11 \cdot 13$

b. Reasoning What are the similarities between your two answers in each pair of products?

46. Geometry The dimensions of a rectangular prism are n, $n + 7$, and $n + 8$. Use the formula $V = \ell wh$ to write a polynomial in standard form for the volume of the prism.

Challenge **For Exercises 47–49, each expression represents the side length of a cube. Write a polynomial in standard form for the surface area of each cube.**

47. $x + 2$

48. $3a + 1$

49. $2c^2 + 3$

50. Financial Planning Suppose you deposit \$1500 for college in a savings account that has an annual interest rate r (expressed as a decimal). At the end of 3 years, the value of your account will be $1500(1 + r)^3$ dollars.

a. Rewrite the expression $1500(1 + r)^3$ by finding the product $1500(1 + r)(1 + r)(1 + r)$. Write your answer in standard form.

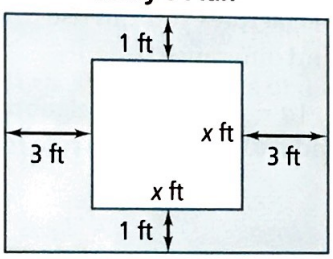
b. How much money is in the account after 3 yr if the interest rate is 3% per year?

Apply What You've Learned

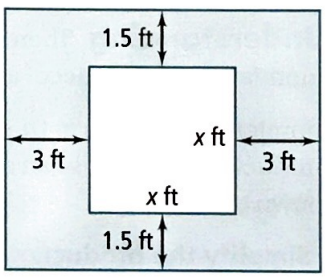
MATHEMATICAL PRACTICES
MP 2, MP 3

Look back at the information on page 485 about a community garden. Kelly's and Roberto's plans for their original plots are shown again below. In the Apply What You've Learned in Lesson 8-1, you wrote binomials for the length and width of Kelly's plot.

Kelly's Plan



Roberto's Plan



a. Use a product of two binomials to find a polynomial that represents the area of Kelly's plot.

b. Use a product of two binomials to find a polynomial that represents the area of Roberto's plot.

c. Explain how you can use a specific value of x to check that you found the products in parts (a) and (b) correctly.

d. Based on the polynomials you wrote in parts (a) and (b), can you conclude that the area of Roberto's plot is greater than the area of Kelly's plot for any value of x? Give an argument to support your answer.