

8-4

Multiplying Special Cases

Common Core State Standards

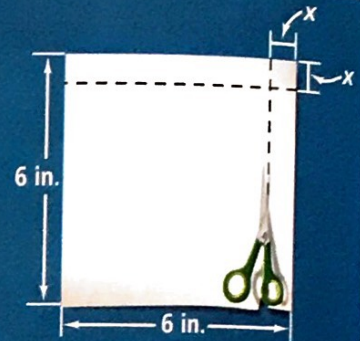
A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MP 1, MP 2, MP 3, MP 4, MP 7, MP 8

Objectives To find the square of a binomial and to find the product of a sum and difference



Getting Ready!

You are making square invitations for a party. You start with a square piece of paper with 6-in. sides. You reduce both its length and its width by x , as shown. What is the area of the invitation? Justify your reasoning.



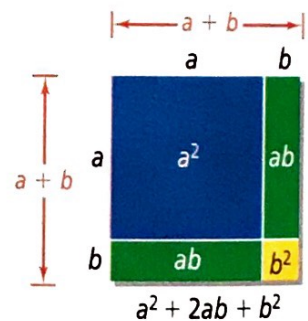
Essential Understanding There are special rules you can use to simplify the square of a binomial or the product of a sum and difference.

Squares of binomials have the form $(a + b)^2$ or $(a - b)^2$. You can algebraically simplify the product or you can use an area model to discover the rule for simplifying $(a + b)^2$, as shown below.

Simplify the product.

$$\begin{aligned} (a + b)^2 &= (a + b)(a + b) \\ &= a^2 + ab + ba + b^2 && \text{Multiply the binomials.} \\ &= a^2 + 2ab + b^2 && \text{Simplify.} \end{aligned}$$

Area Model



Take Note

Key Concept The Square of a Binomial

Words The square of a binomial is the square of the first term plus twice the product of the two terms plus the square of the last term.

Algebra

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Examples

$$(x + 4)^2 = x^2 + 8x + 16$$

$$(x - 3)^2 = x^2 - 6x + 9$$



Got It? 3. a. What is 85^2 ? Use mental math.

b. **Reasoning** Is there more than one way to find 85^2 using mental math? Explain your reasoning.

The product of the sum and difference of the same two terms also produces a pattern.

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ba - b^2 \\ &= a^2 - b^2\end{aligned}$$

Notice that the sum of $-ab$ and ba is 0, leaving $a^2 - b^2$.

Take note

Key Concept The Product of a Sum and Difference

Words The product of the sum and difference of the same two terms is the difference of their squares.

Algebra

$$(a + b)(a - b) = a^2 - b^2$$

Examples

$$(x + 2)(x - 2) = x^2 - 2^2 = x^2 - 4$$



Problem 4 Finding the Product of a Sum and Difference

What is a simpler form of $(x^3 + 8)(x^3 - 8)$?

Plan

How do you choose which rule to use?

The first factor in the product is the sum of x^3 and 8. The second factor is the difference of x^3 and 8. So, use the rule for the product of a sum and difference.

Think

Write the original product.

$$(x^3 + 8)(x^3 - 8)$$

Identify which terms correspond to a and b in the rule for the product of a sum and difference.

$$a = x^3; b = 8$$

Substitute for a and b in the rule.

$$(x^3 + 8)(x^3 - 8) = (x^3)^2 - (8)^2$$

Simplify.

$$= x^6 - 64$$

Write



Got It? 4. What is a simpler form of each product?

a. $(x + 9)(x - 9)$

b. $(6 + m^2)(6 - m^2)$

c. $(3c - 4)(3c + 4)$

You can use the rule for the product of a sum and difference to calculate products using mental math.

Problem 5 Using Mental Math

GRIDDED RESPONSE

What is $64 \cdot 56$?

$$\begin{aligned}
 64 \cdot 56 &= (60 + 4)(60 - 4) && \text{Write as a product of a sum and a difference.} \\
 &= 60^2 - 4^2 && \text{Use } (a + b)(a - b) = a^2 - b^2. \\
 &= 3600 - 16 && \text{Simplify powers.} \\
 &= 3584 && \text{Simplify.}
 \end{aligned}$$



Got It? 5. What is $52 \cdot 48$? Use mental math.

Lesson Check

Do you know **HOW**?

Simplify each product.

- $(c + 3)(c + 3)$
- $(g - 4)^2$
- $(2r - 3)(2r + 3)$
- A square has side length $(2x + 3)$ in. What is the area of the square?

Do you **UNDERSTAND**? **MATHEMATICAL PRACTICES**

What rule would you use to find each product? Why?

- $(3x - 1)^2$
- $(4x - 9)(4x + 9)$
- $(7x + 2)(7x + 2)$
- Reasoning** How do you know whether it is convenient to use the rule for the product of a sum and difference to mentally multiply two numbers?

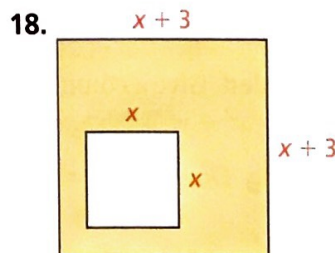
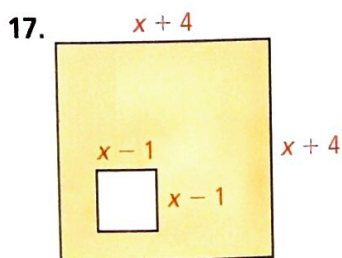
Practice and Problem-Solving Exercises **MATHEMATICAL PRACTICES**

Practice

Simplify each expression.

- $(w + 5)^2$
- $(h + 2)^2$
- $(3s + 9)^2$
- $(2n + 7)^2$
- $(a - 8)^2$
- $(k - 11)^2$
- $(5m - 2)^2$
- $(4x - 6)^2$

Geometry The figures below are squares. Find an expression for the area of each shaded region. Write your answers in standard form.



See Problem 1.

See Problem 2.

19. **Interior Design** A square green rug has a blue square in the center. The side length of the blue square is x inches. The width of the green band that surrounds the blue square is 6 in. What is the area of the green band?

Mental Math Simplify each product.

20. 61^2

21. 79^2

22. 48^2

23. 403^2

24. 302^2

See Problem 3.

Simplify each product.

25. $(v + 6)(v - 6)$

26. $(b + 1)(b - 1)$

27. $(z - 5)(z + 5)$

28. $(x - 3)(x + 3)$

29. $(10 + y)(10 - y)$

30. $(t - 13)(t + 13)$

See Problem 4.

Mental Math Simplify each product.

31. $42 \cdot 38$

32. $79 \cdot 81$

33. $63 \cdot 57$

34. $399 \cdot 401$

35. $303 \cdot 297$

See Problem 5.

B Apply

Simplify each product.

36. $(m + 3n)^2$

37. $(2a + b)^2$

38. $(4s - t)^2$

39. $(g - 7h)^2$

40. $(9k + 2q)^2$

41. $(8r - 5s)^2$

42. $(s + 6t^2)^2$

43. $(p^4 - 9q^2)^2$

44. $(4x + 7y)(4x - 7y)$

45. $(a - 6b)(a + 6b)$

46. $(2g + 9h)(2g - 9h)$

47. $(r^2 + 3s)(r^2 - 3s)$

48. $(2p^2 + 7q)(2p^2 - 7q)$

49. $(3w^3 - z^2)(3w^3 + z^2)$

50. **Error Analysis** Describe and correct the error made in simplifying the product.

$$\cancel{(3a - 7)^2 = 9a^2 - 21a + 49}$$

51. **Think About a Plan** A company logo is a white square inside a red square. The side length of the white square is $x + 2$. The side length of the red square is three times the side length of the white square. What is the area of the red part of the logo? Write your answer in standard form.

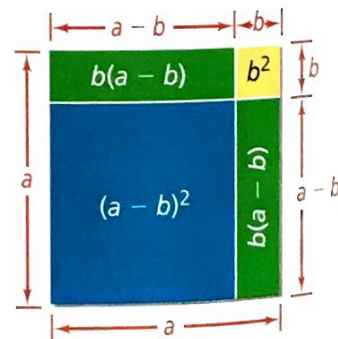
- How can drawing a diagram help you solve the problem?
- How can you express the area of the red part of the logo as a difference of areas?

52. **STEM Construction** A square deck has a side length of $x + 5$. You are expanding the deck so that each side is four times as long as the side length of the original deck. What is the area of the new deck? Write your answer in standard form.

53. **Reasoning** Use the area model at the right to write a second expression for the area of the square labeled $(a - b)^2$. Then simplify the expression to derive the rule for the square of a binomial of the form $a - b$.

54. **Open-Ended** Give a counterexample to show that $(x + y)^2 = x^2 + y^2$ is false.

55. **Reasoning** Does $(3\frac{1}{2})^2 = 9\frac{1}{4}$? Explain.



Challenge 56. Simplify $(a + b + c)^2$.

57. Number Theory You can use factoring to show that the sum of two multiples of 3 is also a multiple of 3.

If m and n are integers, then $3m$ and $3n$ are multiples of three.

$$3m + 3n = 3(m + n)$$

Since $m + n$ is an integer, $3(m + n)$ is a multiple of three.

a. Show that if an integer is one more than a multiple of 3, then its square is also one more than a multiple of 3.

b. **Reasoning** If an integer is two more than a multiple of 3, is its square also two more than a multiple of 3? Explain.

58. The formula $V = \frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r . Find the volume of a sphere with radius $x + 3$. Write your answer in standard form.

Standardized Test Prep

SAT/ACT 59. What is a simpler form of $(2x + 5)(2x - 5)$?

(A) $4x^2 - 20x - 25$

(B) $4x^2 + 20x + 25$

(C) $4x^2 - 25$

(D) $2x^2 - 5$

60. Sara and Nick sold tickets to a play. Sara sold 20 student tickets and 3 adult tickets for more than \$60. Nick sold 15 student tickets and 5 adult tickets for less than \$75. This information can be represented by $20x + 3y > 60$ and $15x + 5y < 75$, where x is the price of a student ticket and y is the price of an adult ticket. The inequalities are graphed at the right. Which could be the price of a student ticket?

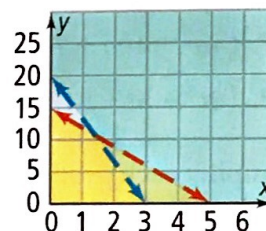
(F) \$1

(H) \$5.50

(G) \$2.75

(I) \$6

Ticket Sales



Short Response 61. Graph the solutions of the system.

$$5x + 4y \geq 20$$

$$5x + 4y \leq 20$$

Mixed Review

Simplify each product.

62. $(3x + 2)(2x - 5)$

63. $(4m - 1)(6m - 7)$

64. $(x + 9)(5x + 8)$

◀ See Lesson 8-3.

Find each percent change. Describe the percent change as an *increase* or *decrease*. If necessary, round to the nearest tenth.

65. \$4 to \$3

66. 4 ft to 5 ft

67. 12 lb to 15 lb

68. \$40 to \$35

◀ See Lesson 2-10.

Get Ready! To prepare for Lesson 8-5, do Exercises 69-71.

Factor each polynomial.

69. $12x^4 + 30x^3 + 42x$

70. $72x^3 + 54x^2 + 27$

71. $35x^3 + 7x^2 + 63x$

◀ See Lesson 8-2.