# 8-6 Factoring $ax^2 + bx + c$

**Common Core State Standards**

**A-SSE.A.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MP 1, MP 2, MP 3, MP 4**

**Objective** To factor trinomials of the form $ax^2 + bx + c$

You did this for one panel in Lesson 8-5—now there are more.

SOLVE IT!

**Getting Ready!**

An array of three rectangular solar panels has area $3x^2 + 21x + 36$. The height of the array is $x + 3$. What is the length of the array? Explain your reasoning.


MATHEMATICAL PRACTICES

**Essential Understanding** You can write some trinomials of the form $ax^2 + bx + c$ as the product of two binomials.

Consider the trinomial $6x^2 + 23x + 7$. To factor it, think of $23x$ as $2x + 21x$.

$$\begin{aligned} 6x^2 + 23x + 7 &= 6x^2 + 2x + 21x + 7 && \text{Rewrite } 23x \text{ as } 2x + 21x. \\ &= 2x(3x + 1) + 7(3x + 1) && \text{Factor out the GCF of each pair of terms.} \\ &= (2x + 7)(3x + 1) && \text{Distributive Property} \end{aligned}$$

How do you know to rewrite $23x$ as $2x + 21x$? Notice that multiplying 2 and 21 gives 42, which is the product of the $x^2$-coefficient 6 and the constant term 7. This example suggests that, to factor a trinomial of the form $ax^2 + bx + c$, you should look for factors of the product $ac$ that have a sum of $b$.

## Problem 1 Factoring When $ac$ Is Positive

**What is the factored form of $5x^2 + 11x + 2$?**

**Step 1** Find factors of $ac$ that have sum $b$. Since $ac = 10$ and $b = 11$, find positive factors of 10 that have sum 11.

| Factors of 10 | 1, 10 | 2, 5 |
|---|---|---|
| Sum of Factors | 11 ✔ | 7 |

**Step 2** To factor the trinomial, use the factors you found to rewrite $bx$.

$$\begin{aligned} 5x^2 + 11x + 2 &= 5x^2 + 1x + 10x + 2 && \text{Rewrite } bx\text{: } 11x = 1x + 10x. \\ &= x(5x + 1) + 2(5x + 1) && \text{Factor out the GCF of each pair of terms.} \\ &= (x + 2)(5x + 1) && \text{Distributive Property} \end{aligned}$$

**Think**

**Will the process still work if you write $5x^2 + 10x + x + 2$?**

Yes. You can rewrite this alternate expression as $5x(x + 2) + (x + 2)$, which equals $(5x + 1)(x + 2)$.

**Got It?** **1. a.** What is the factored form of $6x^2 + 13x + 5$?

**b. Reasoning** In $ax^2 + bx + c$, suppose $ac$ is positive and $b$ is negative. What do you know about the factors of $ac$? Explain.

## Problem 2 Factoring When *ac* Is Negative

**What is the factored form of $3x^2 + 4x - 15$?**

**Plan**

**Can you apply the steps for Problem 1 to this problem?**
Yes. Your goal is still to find factors of $ac$ that have sum $b$. Because $ac < 0$, the factors must have different signs.

**Step 1** Find factors of $ac$ that have sum $b$. Since $ac = -45$ and $b = 4$, find factors of $-45$ that have sum 4.

| Factors of −45 | 1, −45 | −1, 45 | 3, −15 | −3, 15 | 5, −9 | −5, 9 |
|---|---|---|---|---|---|---|
| Sum of Factors | −44 | 44 | −12 | 12 | −4 | 4 ✔ |

**Step 2** To factor the trinomial, use the factors you found to rewrite $bx$.

$3x^2 + 4x - 15 = 3x^2 - 5x + 9x - 15$ Rewrite $bx$: $4x = -5x + 9x$.

$= x(3x - 5) + 3(3x - 5)$ Factor out the GCF of each pair of terms.

$= (3x - 5)(x + 3)$ Distributive Property

**Got It?** **2.** What is the factored form of $10x^2 + 31x - 14$?

## Problem 3 Applying Trinomial Factoring

**Geometry** **The area of a rectangle is $2x^2 - 13x - 7$. What are the possible dimensions of the rectangle? Use factoring.**

**Plan**

**How can you find the dimensions of the rectangle?**
Factor the rectangle's area as the product of two binomials, one of which is the width. The other must be the length since area = length · width.

**Step 1** Find factors of $ac$ that have sum $b$. Since $ac = -14$ and $b = -13$, find factors of $-14$ that have sum $-13$.

| Factors of −14 | 1, −14 | −1, 14 | 2, −7 | −2, 7 |
|---|---|---|---|---|
| Sum of Factors | −13 ✔ | 13 | −5 | 5 |

**Step 2** To factor the trinomial, use the factors you found to rewrite $bx$.

$2x^2 - 13x - 7 = 2x^2 + x - 14x - 7$ Rewrite $bx$: $-13x = x - 14x$.

$= x(2x + 1) - 7(2x + 1)$ Factor out the GCF of each pair of terms.

$= (2x + 1)(x - 7)$ Distributive Property

The possible dimensions of the rectangle are $2x + 1$ and $x - 7$.

**Got It?** **3.** The area of a rectangle is $8x^2 + 22x + 15$. What are the possible dimensions of the rectangle? Use factoring.

To factor a polynomial completely, first factor out the GCF of the polynomial's terms. Then factor the remaining polynomial until it is written as the product of polynomials that cannot be factored further.

### Problem 4 Factoring Out a Monomial First

**What is the factored form of $18x^2 - 33x + 12$?**

**Plan**

**How can you simplify this problem?**
Factor out the GCF of the trinomial's terms. The trinomial that remains is similar to those in Problems 1–3.

**Think**

Factor out the GCF.

Factor $6x^2 - 11x + 4$. Since $ac = 24$ and $b = -11$, find negative factors of 24 that have sum $-11$.

| Factors of 24 | $-1, -24$ | $-2, -12$ | $-3, -8$ | $-4, -6$ |
|---|---|---|---|---|
| Sum of Factors | $-25$ | $-14$ | $-11$ ✔ | $-10$ |

Rewrite the term $bx$. Then use the Distributive Property to finish factoring.

**Write**

$$18x^2 - 33x + 12 = 3(6x^2 - 11x + 4)$$

$$3(6x^2 - 3x - 8x + 4)$$
$$3[3x(2x - 1) - 4(2x - 1)]$$
$$3(3x - 4)(2x - 1)$$

**Got It?** **4.** What is the factored form of $8x^2 - 36x - 20$?

## Lesson Check

### Do you know HOW?

**Factor each expression.**

1. $3x^2 + 16x + 5$
2. $10q^2 + 9q + 2$
3. $4w^2 + 4w - 3$
4. The area of a rectangle is $6x^2 - 11x - 72$. What are the possible dimensions of the rectangle? Use factoring.

### Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Reasoning** Explain why you cannot factor the trinomial $2x^2 + 7x + 10$.
6. **Reasoning** To factor $8x^2 + bx + 3$, a student correctly rewrites the trinomial as $8x^2 + px + qx + 3$. What is the value of $pq$?
7. **Compare and Contrast** How is factoring a trinomial $ax^2 + bx + c$ when $a \neq 1$ different from factoring a trinomial when $a = 1$? How is it similar?

## Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

**A Practice** **Factor each expression.** See Problem 1.

8. $2x^2 + 13x + 6$
9. $3d^2 + 23d + 14$
10. $4n^2 - 8n + 3$
11. $4p^2 + 7p + 3$
12. $6r^2 - 23r + 20$
13. $8g^2 - 14g + 3$

**Factor each expression.**

See Problem 2.

**14.** $5z^2 + 19z - 4$

**15.** $2k^2 - 13k - 24$

**16.** $6t^2 + 7t - 5$

**17.** $3x^2 + 23x - 36$

**18.** $4w^2 - 5w - 6$

**19.** $4d^2 - 4d - 35$

**20. Interior Design** The area of a rectangular kitchen tile is $8x^2 + 30x + 7$. What are the possible dimensions of the tile? Use factoring.

See Problem 3.

**21. Crafts** The area of a rectangular knitted blanket is $15x^2 - 14x - 8$. What are the possible dimensions of the blanket? Use factoring.

**Factor each expression completely.**

See Problem 4.

**22.** $12p^2 + 20p - 8$

**23.** $8v^2 + 34v - 30$

**24.** $6s^2 + 57s + 72$

**25.** $20w^2 - 45w + 10$

**26.** $12x^2 - 46x - 8$

**27.** $9r^2 + 3r - 30$

Apply

**Open-Ended** **Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.**

**28.** $4s^2 + \blacksquare s + 10$

**29.** $15v^2 + \blacksquare v - 24$

**30.** $35m^2 + \blacksquare m - 16$

**31.** $9g^2 + \blacksquare g + 4$

**32.** $6n^2 + \blacksquare n + 28$

**33.** $8r^2 + \blacksquare r - 42$

**34. Error Analysis** Describe and correct the error made in factoring the expression at the right.

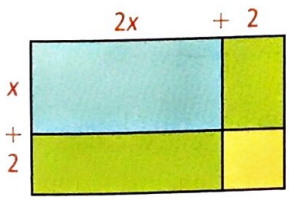
$$3x^2 - 16x - 12 = 3x^2 + 4x - 20x - 12$$
$$= x(3x + 4) - 4(5x + 3)$$
$$= (x - 4)(3x + 4)(5x + 3)$$

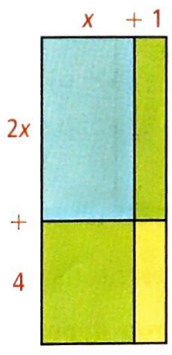
**35. Think About a Plan** A triangle has area $9x^2 - 9x - 10$. The base of the triangle is $3x - 5$. What is the height of the triangle?

- What is the formula for the area of a triangle?
- How does factoring the given trinomial help you solve the problem?

STEM **36. Carpentry** The top of a rectangular table has an area of $18x^2 + 69x + 60$. The width of the table is $3x + 4$. What is the length of the table?

**37. a.** Write each area as a product of two binomials.




**b.** Are the products equal?

**c. Writing** Explain how the two products you found in part (a) can equal the same trinomial.

**Factor each expression.**

**38.** $54x^2 + 87x + 28$

**39.** $66k^2 + 57k + 12$

**40.** $14z^2 - 53z + 14$

**41.** $28h^2 + 28h - 56$

**42.** $21y^2 + 72y - 48$

**43.** $55n^2 - 52n + 12$

**44.** $36p^2 + 114p - 20$

**45.** $63g^2 - 89g + 30$

**46.** $99v^2 - 92v + 9$

**47. Reasoning** If $a$ and $c$ in $ax^2 + bx + c$ are prime numbers and the trinomial is factorable, how many positive values are possible for $b$? Explain your reasoning.

## Challenge

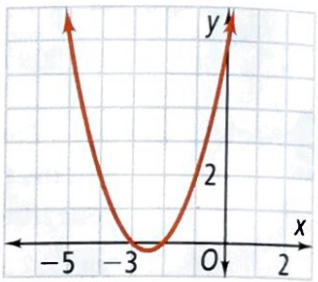
**Factor each expression.**

**48.** $56x^3 + 43x^2 + 5x$

**49.** $49p^2 + 63pq - 36q^2$

**50.** $108g^2h - 162gh + 54h$

**51.** The graph of the function $y = x^2 + 5x + 6$ is shown at the right.




**a.** What are the $x$-intercepts?

**b.** Factor $x^2 + 5x + 6$.

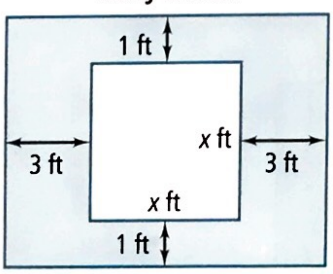
**c. Reasoning** Describe the relationship between the binomial factors you found in part (b) and the $x$-intercepts.

## Apply What You've Learned

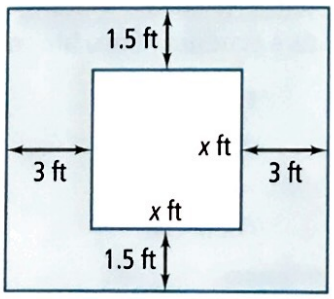
MATHEMATICAL PRACTICES
MP 7

Look back at the information on page 485 about Kelly's and Roberto's plan to combine their plots in a community garden. Kelly's and Roberto's plans for their original plots are shown again below. In the Apply What You've Learned in Lesson 8-3, you wrote trinomials for the area of Kelly's original plot and the area of Roberto's original plot.

**Kelly's Plan**




**Roberto's Plan**




**a.** Write a trinomial that represents the total area of the two original plots.

**b.** Factor the trinomial you wrote in part (a).

**c.** What do the factors in your answer to part (b) represent in relation to the new plot? Explain how you know.