

8-8

Factoring by Grouping

Common Core State Standards

A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients. Also A-SSE.A.1b, A-SSE.A.2

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To factor higher-degree polynomials by grouping

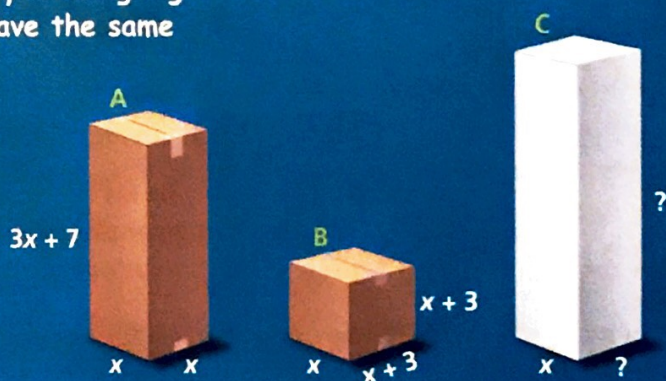


What information do you need before you can find the other two side lengths?



Getting Ready!

A packaging company sells two kinds of boxes, Box A and Box B. The company is designing a new box, Box C, that will have the same volume as Boxes A and B combined. Suppose one dimension of Box C is x . What could be the other two dimensions? Explain your reasoning.



Lesson Vocabulary

- factoring by grouping

Essential Understanding Some polynomials of a degree greater than 2 can be factored.

In Lesson 8-6, you factored trinomials of the form $ax^2 + bx + c$ by rewriting bx as a sum of two monomials. You then grouped the terms in pairs, factored the GCF from each pair, and looked for a common binomial factor. This process is called **factoring by grouping**. You can extend this technique to higher-degree polynomials.

Problem 1 Factoring a Cubic Polynomial

What is the factored form of $3n^3 - 12n^2 + 2n - 8$?

$$3n^3 - 12n^2 + 2n - 8 = 3n^2(n - 4) + 2(n - 4)$$

Factor out the GCF of each group of two terms.

$$= (3n^2 + 2)(n - 4)$$

Factor out the common factor $n - 4$.

Check $(3n^2 + 2)(n - 4) = 3n^3 - 12n^2 + 2n - 8$ ✓



Got It? 1. a. What is the factored form of $8t^3 + 14t^2 + 20t + 35$?

b. **Reasoning** How is the factoring method used in Problem 1 like the method used in Lesson 8-6? How is it different?

Plan

How should you group the terms of the polynomial?

First group the two terms with the highest degrees. If that doesn't work, try another grouping. Your goal is to find a common binomial factor.

Before factoring by grouping, you may need to factor out the GCF of all the terms.

Problem 2 Factoring a Polynomial Completely

What is the factored form of $4q^4 - 8q^3 + 12q^2 - 24q$? Factor completely.

$$\begin{aligned} 4q^4 - 8q^3 + 12q^2 - 24q &= 4q(q^3 - 2q^2 + 3q - 6) && \text{Factor out the GCF.} \\ &= 4q[q^2(q - 2) + 3(q - 2)] && \text{Factor by grouping.} \\ &= 4q(q^2 + 3)(q - 2) && \text{Factor again.} \end{aligned}$$

Got It? 2. What is the factored form of $6h^4 + 9h^3 + 12h^2 + 18h$? Factor completely.

Think

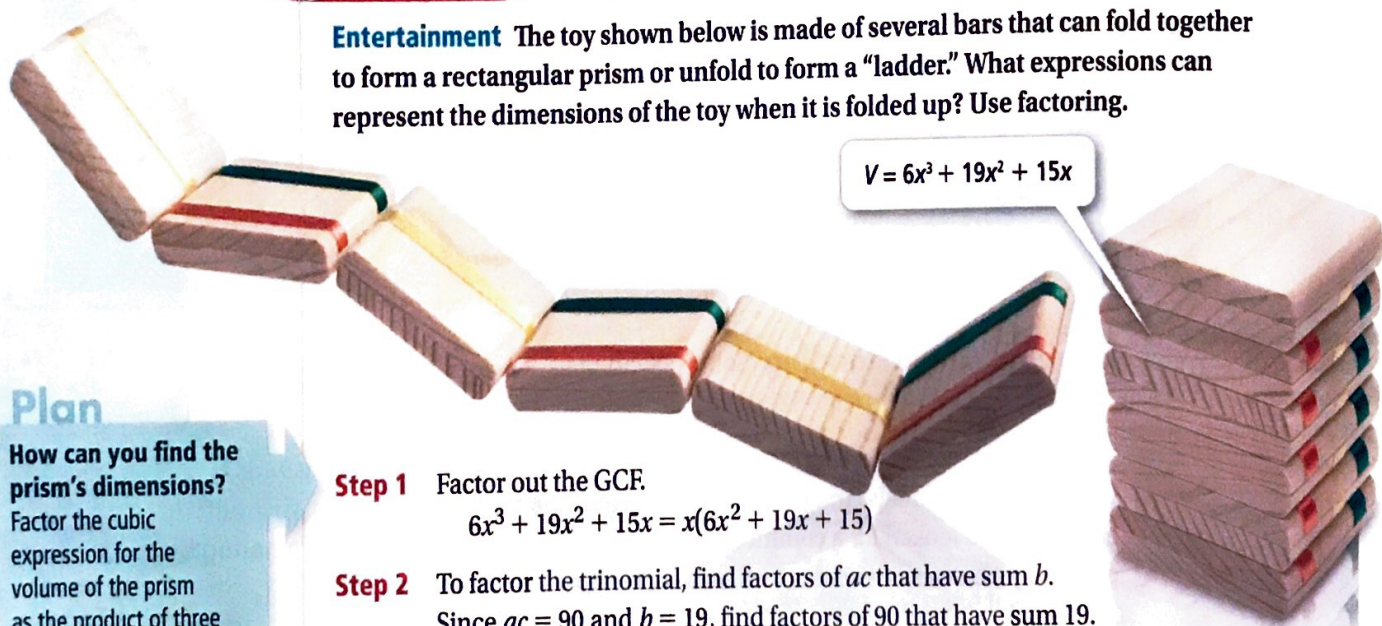
Do the terms share any numerical or variable factors?

Yes. The terms have a common numerical factor of 4 and a common variable factor of q . The GCF is $4q$.

You can sometimes factor to find possible expressions for the length, width, and height of a rectangular prism.

Problem 3 Finding the Dimensions of a Rectangular Prism

Entertainment The toy shown below is made of several bars that can fold together to form a rectangular prism or unfold to form a “ladder.” What expressions can represent the dimensions of the toy when it is folded up? Use factoring.



$$V = 6x^3 + 19x^2 + 15x$$

Plan

How can you find the prism's dimensions?

Factor the cubic expression for the volume of the prism as the product of three linear expressions. Each linear expression is a dimension.

Step 1 Factor out the GCF.

$$6x^3 + 19x^2 + 15x = x(6x^2 + 19x + 15)$$


Step 2 To factor the trinomial, find factors of ac that have sum b . Since $ac = 90$ and $b = 19$, find factors of 90 that have sum 19.

Factors of 90	1, 90	2, 45	3, 30	5, 18	6, 15	9, 10
Sum of Factors	91	47	33	23	21	19 ✓

Step 3 To factor the trinomial, use the factors you found to rewrite bx .

$$\begin{aligned} x(6x^2 + 19x + 15) &= x(6x^2 + 9x + 10x + 15) && \text{Rewrite } bx: 19x = 9x + 10x. \\ &= x[3x(2x + 3) + 5(2x + 3)] && \text{Factor by grouping.} \\ &= x(3x + 5)(2x + 3) && \text{Distributive Property} \end{aligned}$$

The possible dimensions are x , $3x + 5$, and $2x + 3$.

-  **Got It?** 3. **Geometry** A rectangular prism has volume $60x^3 + 34x^2 + 4x$. What expressions can represent the dimensions of the prism? Use factoring.

Here is a summary of what to remember as you factor polynomials.

Take note

Summary Factoring Polynomials

1. Factor out the greatest common factor (GCF).
2. If the polynomial has two terms or three terms, look for a difference of two squares, a perfect-square trinomial, or a pair of binomial factors.
3. If the polynomial has four or more terms, group terms and factor to find common binomial factors.
4. As a final check, make sure there are no common factors other than 1.



Lesson Check

Do you know HOW?

Factor each expression.

1. $20r^3 + 8r^2 + 15r + 6$
2. $6d^3 + 3d^2 - 10d - 5$
3. $24x^3 + 60x^2 + 36x + 90$
4. A rectangular prism has a volume of $36x^3 + 36x^2 + 8x$. What expressions can represent the dimensions of the prism? Use factoring.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

-  **Vocabulary** Tell whether you would factor the polynomial by grouping. Explain your answer.
5. $x^2 - 6x + 9$
 6. $4w^2 + 23w + 15$
 7. $24t^3 - 42t^2 - 28t + 49$
-  **8. Reasoning** Can you factor the polynomial $6q^3 + 2q^2 + 12q - 3$ by grouping? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

- | | |
|------------------------------|-------------------------------|
| 9. $2z^3 + 6z^2 + 3z + 8$ | 10. $10g^3 - 25g^2 + 4g - 10$ |
| 11. $2r^3 + 12r^2 - 5r - 30$ | 12. $6p^3 + 3p^2 + 2p + 1$ |

Factor each expression.

- | | | |
|--------------------------------|-------------------------------|--------------------------------|
| 13. $15q^3 + 40q^2 + 3q + 8$ | 14. $14y^3 + 8y^2 + 7y + 4$ | 15. $14z^3 - 35z^2 + 16z - 40$ |
| 16. $11w^3 - 9w^2 + 11w - 9$ | 17. $8m^3 + 12m^2 - 2m - 3$ | 18. $12k^3 - 27k^2 - 40k + 90$ |
| 19. $20v^3 + 24v^2 - 25v - 30$ | 20. $18h^3 + 45h^2 - 8h - 20$ | 21. $12y^3 + 4y^2 - 9y - 3$ |

 **See Problem 1.**

Factor completely.

22. $8p^3 - 32p^2 + 28p - 112$

23. $3w^4 - 2w^3 + 18w^2 - 12w$

24. $5g^4 - 5g^3 + 20g^2 - 20g$

25. $6q^4 + 3q^3 - 24q^2 - 12q$

26. $36v^3 - 126v^2 + 48v - 168$

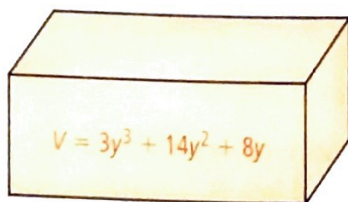
27. $4d^3 - 6d^2 + 16d - 24$

◀ See Problem 2.

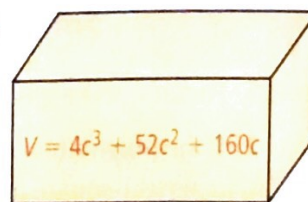
Find expressions for the possible dimensions of each rectangular prism.

◀ See Problem 3.

28.



29.



- STEM** 30. **Carpentry** A trunk in the shape of a rectangular prism has a volume of $6x^3 + 38x^2 - 28x$. What expressions can represent the dimensions of the trunk?

B Apply

Factor completely.

31. $9t^3 - 90t^2 + 144t$

32. $60y^4 - 300y^3 - 42y^2 + 210y$

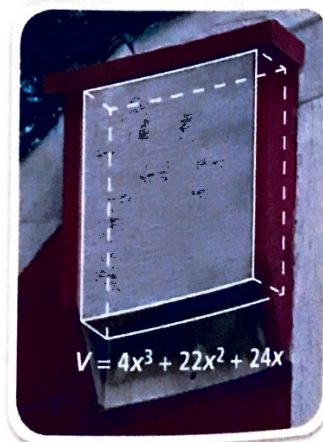
33. $8m^3 + 32m^2 + 40m + 160$

34. $10p^2 - 5pq - 180q^2$

- © 35. **Error Analysis** Describe and correct the error made in factoring completely.

$$\begin{aligned}
 4x^4 + 12x^3 + 8x^2 + 24x &= 4(x^4 + 3x^3 + 2x^2 + 6x) \\
 &= 4[x^3(x + 3) + 2x(x + 3)] \\
 &= 4(x^3 + 2x)(x + 3)
 \end{aligned}$$

- © 36. a. Factor $(20x^3 - 5x^2) + (44x - 11)$.
 b. Factor $(20x^3 + 44x) + (-5x^2 - 11)$.
 c. **Reasoning** Why can you factor the same polynomial using different pairs of terms?
- © 37. **Writing** Describe how to factor the expression $6x^5 + 4x^4 + 12x^3 + 8x^2 + 9x + 6$.
- © 38. **Think About a Plan** Bat houses, such as the one at the right, are large wooden structures that people mount on buildings to attract bats. What expressions can represent the dimensions of the bat house?
- Into how many factors should you factor the expression for the volume?
 - What is the first step in factoring this expression?
- © 39. **Open-Ended** Write a four-term polynomial that you can factor by grouping. Factor your polynomial.
40. **Art** The pedestal of a sculpture is a rectangular prism with a volume of $63x^3 - 28x$. What expressions can represent the dimensions of the pedestal? Use factoring.



Challenge Factor by grouping.

41. $y^3 + 11y^2 - 4y - 44$

42. $p^2m + p^2n^5 + qm + qn^5$

43. $30g^5 + 24g^3h - 35g^2h^2 - 28h^3$

44. **Geometry** The polynomial $2\pi x^3 + 12\pi x^2 + 18\pi x$ represents the volume of a cylinder. The formula for the volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.

a. Factor $2\pi x^3 + 12\pi x^2 + 18\pi x$.

b. Based on your answer to part (a), write an expression for a possible radius of the cylinder.

You can write the number 63 as $2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$. For Exercises 45 and 46, factor each expression by grouping. Then simplify the powers of 2 to write 63 as the product of two numbers.

45. $(2^5 + 2^4 + 2^3) + (2^2 + 2^1 + 2^0)$

46. $(2^5 + 2^4) + (2^3 + 2^2) + (2^1 + 2^0)$

Standardized Test Prep

47. What is $30z^3 - 12z^2 + 120z - 48$ factored completely?

A $2(15z^3 - 6z^2 + 60z - 24)$

C $6(5z^3 - 2z^2 + 20z - 8)$

B $(6z^2 + 24)(5z - 2)$

D $6(z^2 + 4)(5z - 2)$

48. What is the simplified form of $2x^3 \cdot x^8$?

F $2x^{11}$

G $8x^{11}$

H $2x^{24}$

I $8x^{24}$

49. Which equation represents the line with slope -3 that passes through $(2, 5)$?

A $y = -3x + 17$

B $y = -3x + 11$

C $y = 4x - 3$

D $y = x - 3$

50. What is the solution of the inequality $7 < -2x + 5$?

F $x > -1$

G $x < -1$

H $x > 1$

I $x < 1$

51. Factor $10r^4 + 30r^3 + 5r^2 + 15r$ completely. Show your work.

Mixed Review

Factor each expression.

52. $m^2 + 12m + 36$

53. $64x^2 - 144x + 81$

54. $49p^2 - 4$

See Lesson 8-7.

Use a mapping diagram to determine whether each relation is a function.

See Lesson 4-6.

55. $\{(4, 3), (3, 4), (4, 7), (7, 4)\}$

56. $\{(-1, 8), (1, 8), (3, 8), (5, 8)\}$

57. $\{(2, 7), (4, -7), (6, 7), (8, -7)\}$

Get Ready! To prepare for Lesson 9-1, do Exercises 58-61.

Use the slope and y-intercept to graph each equation.

See Lesson 5-3.

58. $y = \frac{1}{2}x + 3$

59. $y = -4x - 1$

60. $y = 2x - 3$

61. $y = -\frac{5}{3}x + 2$