

## 9-2

## Quadratic Functions



**F-IF.C.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima. **Also A-CED.A.2, F-IF.B.4, F-IF.C.8a, F-IF.C.9**

**MP 1, MP 2, MP 3, MP 4, MP 7**

**Objective** To graph quadratic functions of the form  $y = ax^2 + bx + c$

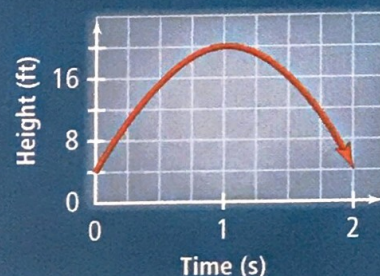


Be careful! The graph shows the height of the ball, not the path of the ball.



## Getting Ready!

You throw a ball straight up into the air and catch it at the same height you released it. The parabola at the right shows the height  $h$  of the ball in feet after  $t$  seconds. What is the total distance the ball travels? For how long does the ball travel up? Explain your reasoning.

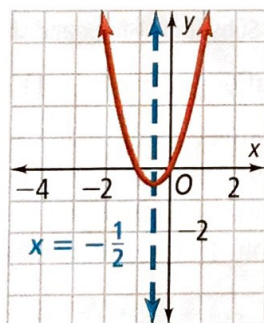


The parabola in the Solve It has the equation  $h = -16t^2 + 32t + 4$ . Unlike the quadratic functions you saw in previous lessons, this function has a linear term,  $32t$ .

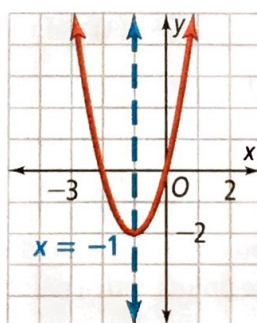
**Essential Understanding** In the quadratic function  $y = ax^2 + bx + c$ , the value of  $b$  affects the position of the axis of symmetry.

Consider the graphs of the following functions.

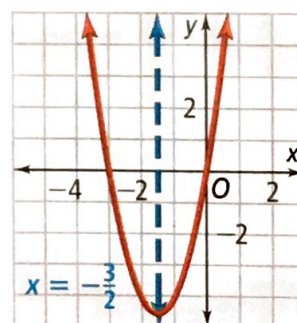
$$y = 2x^2 + 2x$$



$$y = 2x^2 + 4x$$



$$y = 2x^2 + 6x$$



Notice that the axis of symmetry changes with each change in the  $b$ -value. The equation of the axis of symmetry is related to the ratio  $\frac{b}{a}$ .

equation:  $y = 2x^2 + 2x$

$\frac{b}{a}$ :  $\frac{2}{2} = 1$

axis of symmetry:  $x = -\frac{1}{2}$

equation:  $y = 2x^2 + 4x$

$\frac{b}{a}$ :  $\frac{4}{2} = 2$

axis of symmetry:  $x = -1$ , or  $-\frac{2}{2}$

equation:  $y = 2x^2 + 6x$

$\frac{b}{a}$ :  $\frac{6}{2} = 3$

axis of symmetry:  $x = -\frac{3}{2}$

The equation of the axis of symmetry is  $x = -\frac{1}{2}\left(\frac{b}{a}\right)$ , or  $x = -\frac{b}{2a}$ .

## Key Concept Graph of a Quadratic Function

The graph of  $y = ax^2 + bx + c$ , where  $a \neq 0$ , has the line  $x = \frac{-b}{2a}$  as its axis of symmetry. The  $x$ -coordinate of the vertex is  $\frac{-b}{2a}$ .

When you substitute  $x = 0$  into the equation  $y = ax^2 + bx + c$ , you get  $y = c$ . So the  $y$ -intercept of a quadratic function is  $c$ . You can use the axis of symmetry and the  $y$ -intercept to help you graph a quadratic function.



### Problem 1 Graphing $y = ax^2 + bx + c$

What is the graph of the function  $y = x^2 - 6x + 4$ ?

**Step 1** Find the axis of symmetry and the coordinates of the vertex.

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3 \quad \text{Find the equation of the axis of symmetry.}$$

The axis of symmetry is  $x = 3$ . So the  $x$ -coordinate of the vertex is 3.

$$\begin{aligned} y &= x^2 - 6x + 4 \\ &= 3^2 - 6(3) + 4 \quad \text{Substitute 3 for } x \text{ to find the } y\text{-coordinate of the vertex.} \\ &= -5 \quad \text{Simplify.} \end{aligned}$$

The vertex is  $(3, -5)$ .

**Step 2** Find two other points on the graph.

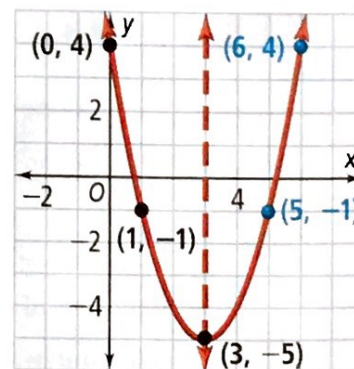
Find the  $y$ -intercept. When  $x = 0$ ,  $y = 4$ , so one point is  $(0, 4)$ .

Find another point by choosing a value for  $x$  on the same side of the vertex as the  $y$ -intercept. Let  $x = 1$ .

$$\begin{aligned} y &= x^2 - 6x + 4 \\ &= 1^2 - 6(1) + 4 = -1 \quad \text{Substitute 1 for } x \text{ and simplify.} \end{aligned}$$

When  $x = 1$ ,  $y = -1$ , so another point is  $(1, -1)$ .

**Step 3** Graph the vertex and the points you found in Step 2,  $(0, 4)$  and  $(1, -1)$ . Reflect the points from Step 2 across the axis of symmetry to get two more points on the graph. Then connect the points with a parabola.



## Think

**How are the vertex and the axis of symmetry related?**

The vertex is on the axis of symmetry. You can use the equation for the axis of symmetry to find the  $x$ -coordinate of the vertex.



**Got It?** 1. a. What is the graph of the function  $y = -x^2 + 4x - 2$ ?

b. **Reasoning** In Step 2 of Problem 1, why do you think it was useful to use the  $y$ -intercept as one point on the graph?



In Lesson 9-1, you used  $h = -16t^2 + c$  to find the height  $h$  above the ground of an object falling from an initial height  $c$  at time  $t$ . If an object projected into the air given an initial upward velocity  $v$  continues with no additional force acting on it, the formula  $h = -16t^2 + vt + c$  gives its approximate height above the ground.

## Problem 2 Using the Vertical Motion Model

**Entertainment** During halftime of a basketball game, a slingshot launches T-shirts at the crowd. A T-shirt is launched with an initial upward velocity of 72 ft/s. The T-shirt is caught 35 ft above the court. How long will it take the T-shirt to reach its maximum height? What is its maximum height? What is the range of the function that models the height of the T-shirt over time?



### Plan

What are the values of  $v$  and  $c$ ?

The T-shirt is launched from a height of 5 ft, so  $c = 5$ . The T-shirt has an initial upward velocity of 72 ft/s, so  $v = 72$ .

The function  $h = -16t^2 + 72t + 5$  gives the T-shirt's height  $h$ , in feet, after  $t$  seconds. Since the coefficient of  $t^2$  is negative, the parabola opens downward, and the vertex is the maximum point.

**Method 1** Use a formula.

$$t = \frac{-b}{2a} = \frac{-72}{2(-16)} = 2.25$$

Find the  $t$ -coordinate of the vertex.

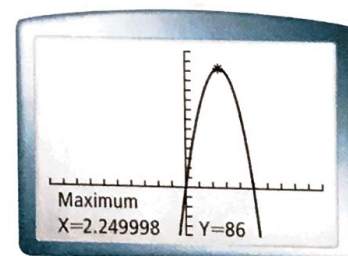
$$h = -16(2.25)^2 + 72(2.25) + 5 = 86 \quad \text{Find the } h\text{-coordinate of the vertex.}$$

The T-shirt will reach its maximum height of 86 ft after 2.25 s. The range describes the height of the T-shirt during its flight. The T-shirt starts at 5 ft, peaks at 86 ft, and then is caught at 35 ft. The height of the T-shirt at any time is between 5 ft and 86 ft, inclusive, so the range is  $5 \leq h \leq 86$ .

**Method 2** Use a graphing calculator.

Enter the function  $h = -16t^2 + 72t + 5$  as  $y = -16x^2 + 72x + 5$  on the **Y=** screen and graph the function.

Use the **CALC** feature and select **MAXIMUM**. Set left and right bounds on the maximum point and calculate the point's coordinates. The coordinates of the maximum point are (2.25, 86).



The T-shirt will reach its maximum height of 86 ft after 2.25 s. The range of the function is  $5 \leq h \leq 86$ .

**Got It?**

2. In Problem 2, suppose a T-shirt is launched with an initial upward velocity of 64 ft/s and is caught 35 ft above the court. How long will it take the T-shirt to reach its maximum height? How far above court level will it be? What is the range of the function that models the height of the T-shirt over time?

**Lesson Check****Do you know HOW?**

Graph each function.

1.  $y = x^2 - 4x + 1$
2.  $y = -2x^2 - 8x - 3$
3.  $y = 3x^2 + 6x + 2$
4.  $f(x) = -x^2 + 2x - 5$

**Do you UNDERSTAND?****MATHEMATICAL PRACTICES**

5. **Reasoning** How does each of the numbers  $a$ ,  $b$ , and  $c$  affect the graph of a quadratic function  $y = ax^2 + bx + c$ ?
6. **Writing** Explain how you can use the  $y$ -intercept, vertex, and axis of symmetry to graph a quadratic function. Assume the vertex is not on the  $y$ -axis.

**Practice and Problem-Solving Exercises****MATHEMATICAL PRACTICES****Practice**

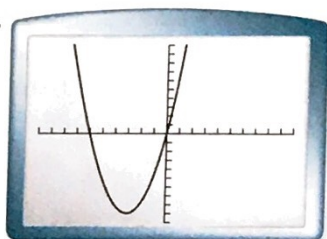
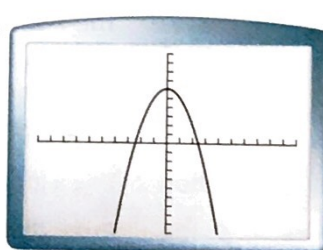
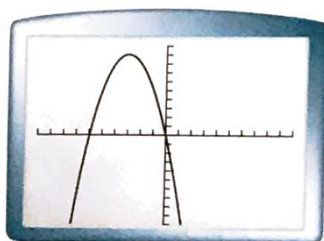
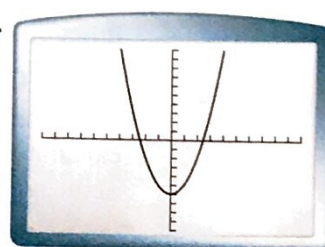
Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of each function.

**See Problem 1.**

- |                             |                            |                            |
|-----------------------------|----------------------------|----------------------------|
| 7. $y = 2x^2 + 3$           | 8. $y = -3x^2 + 12x + 1$   | 9. $f(x) = 2x^2 + 4x - 1$  |
| 10. $y = x^2 - 8x - 7$      | 11. $f(x) = 3x^2 - 9x + 2$ | 12. $y = -4x^2 + 11$       |
| 13. $f(x) = -5x^2 + 3x + 2$ | 14. $y = -4x^2 - 16x - 3$  | 15. $f(x) = 6x^2 + 6x - 5$ |

Match each function with its graph.

- |                     |                    |                   |                    |
|---------------------|--------------------|-------------------|--------------------|
| 16. $y = -x^2 - 6x$ | 17. $y = -x^2 + 6$ | 18. $y = x^2 - 6$ | 19. $y = x^2 + 6x$ |
|---------------------|--------------------|-------------------|--------------------|

**A.****B.****C.****D.**



Graph each function. Label the axis of symmetry and the vertex.

20.  $f(x) = x^2 + 4x - 5$

21.  $y = 3x^2 - 20x$

22.  $y = -2x^2 + 8x + 9$

23.  $f(x) = -x^2 + 4x + 3$

24.  $y = -2x^2 - 10x$

25.  $y = 2x^2 - 6x + 1$

26. **Sports** A baseball is thrown into the air with an upward velocity of 30 ft/s. Its height  $h$ , in feet, after  $t$  seconds is given by the function  $h = -16t^2 + 30t + 6$ . How long will it take the ball to reach its maximum height? What is the ball's maximum height? What is the range of the function?

See Problem 2.

27. **School Fair** Suppose you have 100 ft of string to rope off a rectangular section for a bake sale at a school fair. The function  $A = -x^2 + 50x$  gives the area of the section in square feet, where  $x$  is the width in feet. What width gives you the maximum area you can rope off? What is the maximum area? What is the range of the function?

28. a. What is the vertex of the function  $y = x^2 + 4$ ?

- b. What is the vertex of the function given in the table?

$x$	-2	-1	0	1	2
$y$	-14	-8	-6	-8	-14

29. a. What is the vertex of the function  $y = 5x^2 + 10x + 24$ ?

- b. What is the vertex of the function given in the table?

$x$	-4	-3	-2	-1	0
$y$	3	-3	-5	-3	3

30. **Think About a Plan** The Riverside Geyser in Yellowstone National Park erupts about every 6.25 h. When the geyser erupts, the water has an initial upward velocity of 69 ft/s. What is the maximum height of the geyser? Round your answer to the nearest foot.

- What is the initial height of the geyser?
- What function gives the geyser's height  $h$  (in feet)  $t$  seconds after it starts erupting?

31. **Business** A cell phone company sells about 500 phones each week when it charges \$75 per phone. It sells about 20 more phones per week for each \$1 decrease in price. The company's revenue is the product of the number of phones sold and the price of each phone. What price should the company charge to maximize its revenue?

32. Graph the function  $f(x) = x^2 + 2x - 3$ . Then graph the following transformations of the function. Describe how the parent function changes with each transformation.

a.  $f(x) + 3$

b.  $2[f(x)]$

c.  $f(4x)$

d.  $f(x + 5)$

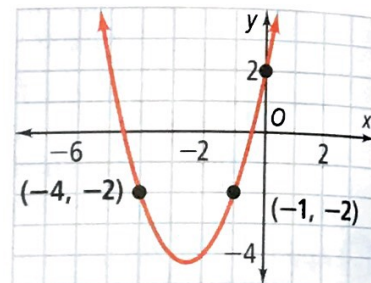
33. **Error Analysis** Describe and correct the error made in finding the axis of symmetry for the graph of  $y = -x^2 - 6x + 2$ .

$$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$$

34. **Reasoning** What do you know about the value of  $b$  in the function  $y = ax^2 + bx + c$  when the  $x$ -coordinate of the vertex is an integer?

## Challenge

35. **Sports** Suppose a tennis player hits a ball over the net. The ball leaves the racket 0.5 m above the ground. The equation  $h = -4.9t^2 + 3.8t + 0.5$  gives the ball's height  $h$  in meters after  $t$  seconds.
- When will the ball be at the highest point in its path? Round to the nearest tenth of a second.
36. The parabola at the right is of the form  $y = x^2 + bx + c$ .
- Use the graph to find the  $y$ -intercept.
  - Use the graph to find the equation of the axis of symmetry.
  - Use the formula  $x = \frac{-b}{2a}$  to find  $b$ .
  - Write the equation of the parabola.
  - Test one point using your equation from part (d).
  - Reasoning** Would this method work if the value of  $a$  were not known? Explain.



## Apply What You've Learned



Look back at the information on page 545 about the sale sign on the wall of the Ski Barn.

- Copy the figure from page 545. Label the width of the rectangular sign  $x$ , and label the height  $y$ .
- The triangle above the sign is an isosceles triangle that is similar to the triangular wall. Use the similar triangles to write a proportion. Solve your proportion for  $y$  in terms of  $x$  to find a function that models the height of the rectangular sign.
- Using the formula for the area of a rectangle and your height function from part (b), determine an equation for the function  $A(x)$  that represents the area of the rectangular sign.
- What kind of function is the function you found in part (c)? Explain.