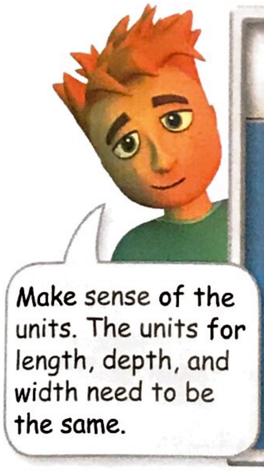
9-3 Solving Quadratic Equations

Common Core State Standards

A-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots . . . **Also A-APR.B.3, A-CED.A.1, A-CED.A.4**

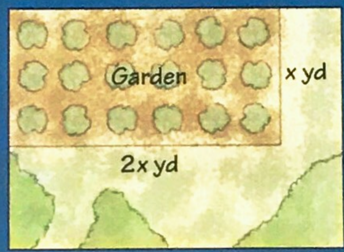
MP 1, MP 2, MP 3, MP 4, MP 7

Objective To solve quadratic equations by graphing and using square roots



SOLVE IT! Getting Ready!

The diagram shows a plan for your new garden. You want to use only 1.5 yd³ of topsoil and plan to spread a layer 4 in. deep. What are the dimensions of the largest garden you can build? How do you know?



MATHEMATICAL PRACTICES The situation in the Solve It can be modeled by a *quadratic equation*.

Lesson Vocabulary

- quadratic equation
- standard form of a quadratic equation
- root of an equation
- zero of a function

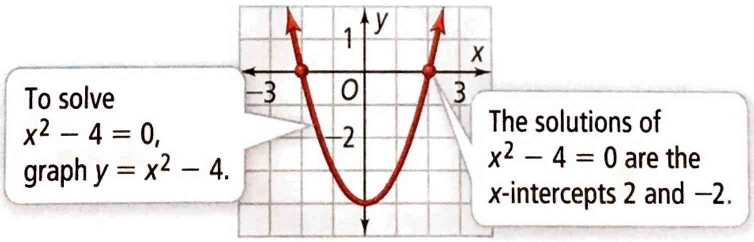
take note

Key Concept Standard Form of a Quadratic Equation

A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This form is called the **standard form of a quadratic equation.**

Essential Understanding Quadratic equations can be solved by a variety of methods, including graphing and finding square roots.

One way to solve a quadratic equation $ax^2 + bx + c = 0$ is to graph the related quadratic function $y = ax^2 + bx + c$. The solutions of the equation are the x-intercepts of the related function.



A quadratic equation can have two, one, or no real-number solutions. In a future course you will learn about solutions of quadratic equations that are not real numbers. In this course, *solutions* refers to real-number solutions.

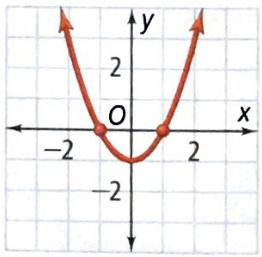
The solutions of a quadratic equation and the x-intercepts of the graph of the related function are often called **roots of the equation** or **zeros of the function.**

Problem 1 Solving by Graphing

What are the solutions of each equation? Use a graph of the related function.

A $x^2 - 1 = 0$

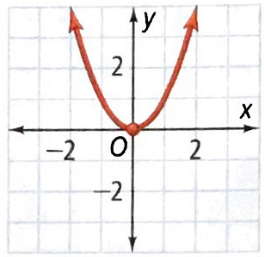
Graph $y = x^2 - 1$.



There are two solutions, ± 1.

B $x^2 = 0$

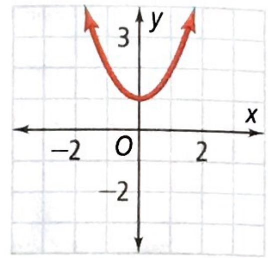
Graph $y = x^2$.



There is one solution, 0.

C $x^2 + 1 = 0$

Graph $y = x^2 + 1$.



There is no real-number solution.

Think

What feature of the graph shows the solutions of the equation?
The *x*-intercepts show the solutions of the equation.

Got It? **1.** What are the solutions of each equation? Use a graph of the related function.

a. $x^2 - 16 = 0$ **b.** $3x^2 + 6 = 0$ **c.** $x^2 - 25 = -25$

You can solve equations of the form $x^2 = k$ by finding the square roots of each side. For example, the solutions of $x^2 = 81$ are $\pm\sqrt{81}$, or ± 9.

Problem 2 Solving Using Square Roots

What are the solutions of $3x^2 - 75 = 0$?

Think	Write
Write the original equation.	$3x^2 - 75 = 0$
Isolate x^2 on one side of the equation.	$3x^2 = 75$ $x^2 = 25$
Find the square roots of each side and simplify.	$x = \pm\sqrt{25}$ $x = \pm 5$

Plan

How do you know you can solve using square roots?
The equation has an x^2-term and a constant term, but no *x*-term. So, you can write the equation in the form $x^2 = k$ and then find the square roots of each side.

Got It? **2.** What are the solutions of each equation?

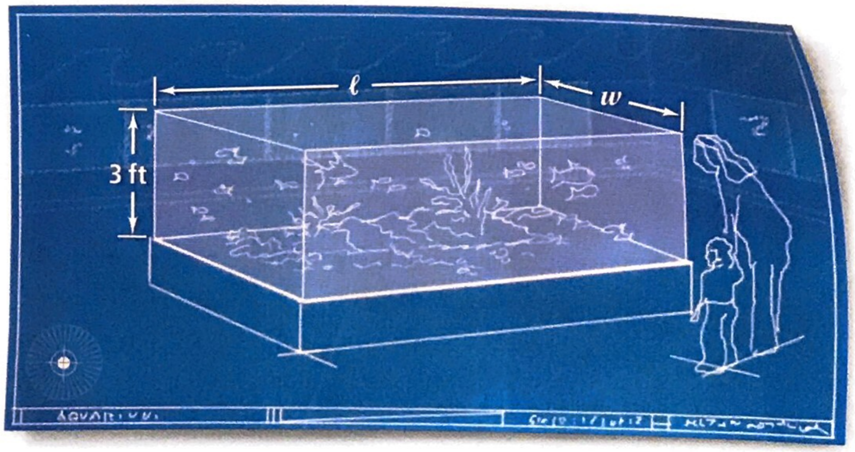
a. $m^2 - 36 = 0$ **b.** $3x^2 + 15 = 0$ **c.** $4d^2 + 16 = 16$

You can solve some quadratic equations that model real-world problems by finding square roots. In many cases, the negative square root may not be a reasonable solution.

Problem 3 Choosing a Reasonable Solution

GRIDDED RESPONSE

Aquarium An aquarium is designing a new exhibit to showcase tropical fish. The exhibit will include a tank that is a rectangular prism with a length ℓ that is twice the width w. The volume of the tank is 420 ft^3. What is the width of the tank to the nearest tenth of a foot?

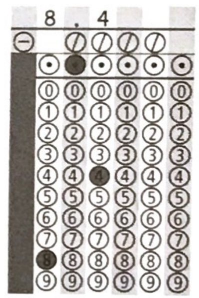


Plan

How can you write the length of the tank?

The length ℓ is twice the width w, so write the length as $2w$.

$V = \ell wh$	Use the formula for volume of a rectangular prism.
$420 = (2w)w(3)$	Substitute 420 for V, $2w$ for ℓ, and 3 for h.
$420 = 6w^2$	Simplify.
$70 = w^2$	Divide each side by 6.
$\pm\sqrt{70} = w$	Find the square roots of each side.
$\pm 8.366600265 \approx w$	Use a calculator.



A tank cannot have a negative width, so only the positive square root makes sense. The tank will have a width of about 8.4 ft.

Got It? **3. a.** Suppose the tank in Problem 3 will have a height of 4 ft and a volume of 500 ft^3. What is the width of the tank to the nearest tenth of a foot?

b. Reasoning What are the disadvantages of using a graph to approximate the solution to Problem 3? Explain.

Lesson Check

Do you know HOW?

Solve each equation by graphing the related function or by finding square roots.

1. $x^2 - 25 = 0$
2. $2x^2 - 8 = 0$
3. $t^2 = 144$
4. $y^2 - 225 = 0$

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. **Vocabulary** What are the zeros of a function? Give an example of a quadratic function and its zeros.
6. **Compare and Contrast** When is it easier to solve a quadratic equation of the form $ax^2 + c = 0$ using square roots than to solve it using a graph?
7. **Reasoning** Consider the equation $ax^2 + c = 0$, where $a \neq 0$. What is true of a and c if the equation has two solutions? Only one solution? No solutions?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Solve each equation by graphing the related function. If the equation has no real-number solution, write *no solution*.

See Problem 1.

8. $x^2 - 9 = 0$
9. $x^2 + 7 = 0$
10. $3x^2 = 0$
11. $3x^2 - 12 = 0$
12. $x^2 + 4 = 0$
13. $\frac{1}{3}x^2 - 3 = 0$
14. $\frac{1}{2}x^2 + 1 = 0$
15. $x^2 + 5 = 5$
16. $\frac{1}{4}x^2 - 1 = 0$
17. $x^2 + 25 = 0$
18. $x^2 - 10 = -10$
19. $2x^2 - 18 = 0$

Solve each equation by finding square roots. If the equation has no real-number solution, write *no solution*.

See Problem 2.

20. $n^2 = 81$
21. $a^2 = 324$
22. $k^2 - 196 = 0$
23. $r^2 + 49 = 49$
24. $w^2 - 36 = -64$
25. $4g^2 = 25$
26. $64b^2 = 16$
27. $5q^2 - 20 = 0$
28. $144 - p^2 = 0$
29. $2r^2 - 32 = 0$
30. $3a^2 + 12 = 0$
31. $5z^2 - 45 = 0$

Model each problem with a quadratic equation. Then solve. If necessary, round to the nearest tenth.

See Problem 3.

32. Find the length of a side of a square with an area of 169 m^2.
33. Find the length of a side of a square with an area of 75 ft^2.
34. Find the radius of a circle with an area of 90 cm^2.

35. **Painting** You have enough paint to cover an area of 50 ft^2. What is the side length of the largest square that you could paint? Round your answer to the nearest tenth of a foot.

36. **Gardening** You have enough shrubs to cover an area of 100 ft^2. What is the radius of the largest circular region you can plant with these shrubs? Round your answer to the nearest tenth of a foot.

B Apply

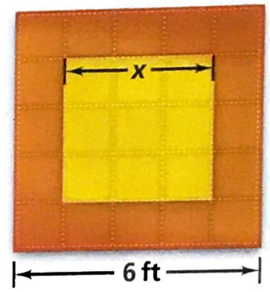
Mental Math Tell how many solutions each equation has.

37. $h^2 = -49$
38. $c^2 - 18 = 9$
39. $s^2 - 35 = -35$

40. **Think About a Plan** A circular above-ground pool has a height of 52 in. and a volume of 1100 ft^3. What is the radius of the pool to the nearest tenth of a foot? Use the equation $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the height.
    - How can drawing a diagram help you solve this problem?
    - Do you need to convert any of the given measurements to different units?

41. **Reasoning** For what values of n will the equation $x^2 = n$ have two solutions? Exactly one solution? No solution?

42. Quilting You are making a square quilt with the design shown at the right. Find the side length x of the inner square that would make its area equal to 50% of the total area of the quilt. Round to the nearest tenth of a foot.



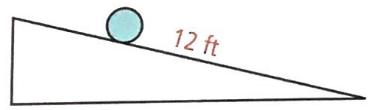
Solve each equation by finding square roots. If the equation has no real-number solution, write *no solution*. If a solution is irrational, round to the nearest tenth.

43. $1.2z^2 - 7 = -34$ **44.** $49p^2 - 16 = -7$ **45.** $3m^2 - \frac{1}{12} = 0$

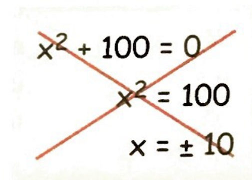
46. $\frac{1}{2}t^2 - 4 = 0$ **47.** $7y^2 + 0.12 = 1.24$ **48.** $-\frac{1}{4}x^2 + 3 = 0$

49. Find the value of c such that the equation $x^2 - c = 0$ has 12 and -12 as solutions.

STEM **50. Physics** The equation $d = \frac{1}{2}at^2$ gives the distance d that an object starting at rest travels given acceleration a and time t. Suppose a ball rolls down the ramp shown at the right with acceleration $a = 2\text{ ft/s}^2$. Find the time it will take the ball to roll from the top of the ramp to the bottom. Round to the nearest tenth of a second.



51. Error Analysis Describe and correct the error made in solving the equation.



52. Open-Ended Write and solve an equation in the form $ax^2 + c = 0$, where $a \neq 0$, that satisfies the given condition.

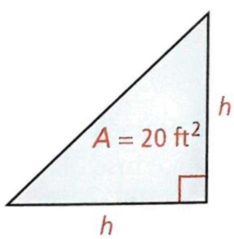
a. The equation has no solution.

b. The equation has exactly one solution.

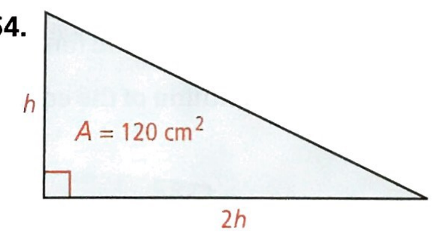
c. The equation has two solutions.

Geometry **Find the value of h for each triangle. If necessary, round to the nearest tenth.**

53.



54.



55. You can use a spreadsheet like the one at the right to solve a quadratic equation.

a. What spreadsheet formula would you use to find the value in cell B2?

b. Use a spreadsheet to find the solutions of the quadratic equation $6x^2 - 24 = 0$. Explain how you used the spreadsheet to find the solutions.

c. Reasoning Suppose a quadratic equation has solutions that are not integers. How could you use a spreadsheet to approximate the solutions?

	A	B	
1	x	6x^2 − 24 = 0	
2	−3		
3	−2		
4	−1		
5	0		
6	1		
7	2		
8	3		

Challenge

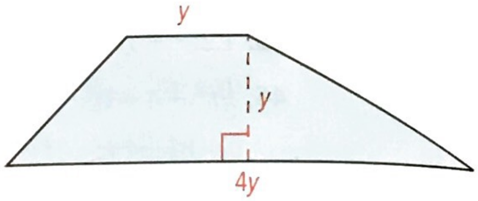
56. a. Solve the equation $(x - 7)^2 = 0$.

b. Find the vertex of the graph of the related function $y = (x - 7)^2$.

c. **Open-Ended** Choose a value for h and repeat parts (a) and (b) using $(x - h)^2 = 0$ and $y = (x - h)^2$.

d. Where would you expect to find the vertex of the graph of $y = (x + 4)^2$? Explain.

57. **Geometry** The trapezoid has an area of 1960 cm^2. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$, where A represents the area of the trapezoid, h represents its height, and b_1 and b_2 represent its bases, to find the value of y.



Apply What You've Learned

MATHEMATICAL PRACTICES

MP 5

In the Apply What You've Learned in Lesson 9-2, you found a function that represents the area of the rectangular sign described on page 545. Use a graphing calculator or other graphing utility to graph the function. Select all of the following that are true. Explain your reasoning.

A. The graph of the function is a parabola that has a maximum point.

B. The graph of the function is a line with a positive slope.

C. The axis of symmetry of the graph is $x = 4.5$.

D. The axis of symmetry of the graph is $x = 9$.

E. The x-intercepts of the function are 0 and 18.

F. The only solution of the equation $0 = -\frac{2}{3}x^2 + 12x$ is 0.