9-4 Factoring to Solve Quadratic Equations

Common Core State Standards
A-REI.B.4b Solve quadratic equations by . . . factoring . . . **Also A-SSE.B.3a, A-CED.A.1, F-IF.C.8a**
MP 1, MP 2, MP 3, MP 4, MP 7

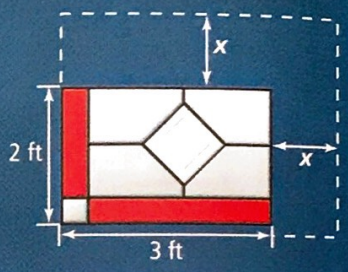
Objective To solve quadratic equations by factoring

Analyze the situation. What expressions can you use for the length and for the width?

SOLVE IT!

Getting Ready!

You are finishing a stained glass hanging that your friend has started. You have enough supplies to add 6 ft^2 to the hanging. You are planning to add the same amount to the length and width. What will be the dimensions of the hanging when you are finished? How do you know?



MATHEMATICAL PRACTICES

In the previous lesson, you solved quadratic equations $ax^2 + bx + c = 0$ by finding square roots. This method works if $b = 0$.

Lesson Vocabulary
- Zero-Product Property

Essential Understanding You can solve some quadratic equations, including equations where $b \neq 0$, by using the *Zero-Product Property*.

The Multiplication Property of Zero states that for any real number a, $a \cdot 0 = 0$. This is equivalent to the following statement: For any real numbers a and b, if $a = 0$ or $b = 0$, then $ab = 0$. The Zero-Product Property reverses this statement.

take note

Property Zero-Product Property

For any real numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$.

Example If $(x + 3)(x + 2) = 0$, then $x + 3 = 0$ or $x + 2 = 0$.

Problem 1 Using the Zero-Product Property

What are the solutions of the equation $(4t + 1)(t - 2) = 0$?

$(4t + 1)(t - 2) = 0$

$4t + 1 = 0$	or	$t - 2 = 0$	Use the Zero-Product Property.
$4t = -1$	or	$t = 2$	Solve for t.
$t = -\frac{1}{4}$	or	$t = 2$	

Think

How else can you write the solutions?
You can write the solutions as a set in roster form: $\{-\frac{1}{4}, 2\}$.

Got It? 1. What are the solutions of each equation?

a. $(x + 1)(x - 5) = 0$

b. $(2x + 3)(x - 4) = 0$

c. $(2y + 1)(y + 14) = 0$

d. $(7n - 2)(5n - 4) = 0$

You can also use the Zero-Product Property to solve equations of the form $ax^2 + bx + c = 0$ if the quadratic expression $ax^2 + bx + c$ can be factored.

Problem 2 Solving by Factoring

Multiple Choice **What are the solutions of the equation $x^2 + 8x + 15 = 0$?**

(A) $-5, -3$

(B) $-5, 3$

(C) $-3, 5$

(D) $3, 5$

Plan

How can you factor $x^2 + 8x + 15$?
Find two integers with a product of 15 and a sum of 8.

$x^2 + 8x + 15 = 0$

$(x + 3)(x + 5) = 0$ Factor $x^2 + 8x + 15$.

$x + 3 = 0$ or $x + 5 = 0$ Use the Zero-Product Property.

$x = -3$ or $x = -5$ Solve for x.

The solutions are -3 and -5. The correct answer is A.

Got It? 2. What are the solutions of each equation?

a. $m^2 - 5m - 14 = 0$

b. $p^2 + p - 20 = 0$

c. $2a^2 - 15a + 18 = 0$

Before solving a quadratic equation, you may need to add or subtract terms from each side in order to write the equation in standard form. Then factor the quadratic expression.

Problem 3 Writing in Standard Form First

What are the solutions of $4x^2 - 21x = 18$?

Think

Why do you need to subtract 18 from each side before you factor?
To use the Zero-Product Property, one side of the equation must be zero.

$4x^2 - 21x = 18$

$4x^2 - 21x - 18 = 0$ Subtract 18 from each side.

$(4x + 3)(x - 6) = 0$ Factor $4x^2 - 21x - 18$.

$4x + 3 = 0$ or $x - 6 = 0$ Use the Zero-Product Property.

$4x = -3$ or $x = 6$ Solve for x.

$x = -\frac{3}{4}$ or $x = 6$

The solutions are $-\frac{3}{4}$ and 6.

Got It? 3. a. What are the solutions of $x^2 + 14x = -49$?

b. **Reasoning** Why do quadratic equations of the form $x^2 + 2ax + a^2 = 0$ or $x^2 - 2ax + a^2 = 0$ have only one real-number solution?

Problem 4 Using Factoring to Solve a Real-World Problem

Photography You are constructing a frame for the rectangular photo shown. You want the frame to be the same width all the way around and the total area of the frame and photo to be 315 in.^{2}. What should the outer dimensions of the frame be?

Know
The size of the photo is 11 in. by 17 in. The total area is 315 in.^{2}.

Need
The outer dimensions of the frame

Plan
Write the frame's outer dimensions in terms of its width x. Use these dimensions to write an equation for the area of the frame and photo.

$(2x + 11)(2x + 17) = 315$ — Width × Length = Area

$4x^2 + 56x + 187 = 315$ — Find the product $(2x + 11)(2x + 17)$.

$4x^2 + 56x - 128 = 0$ — Subtract 315 from each side.

$4(x^2 + 14x - 32) = 0$ — Factor out 4.

$4(x + 16)(x - 2) = 0$ — Factor $x^2 + 14x - 32$.

$x + 16 = 0$ or $x - 2 = 0$ — Use the Zero-Product Property.

$x = -16$ or $x = 2$ — Solve for x.

The only reasonable solution is 2. So the outer dimensions of the frame are $2(2) + 11$ in. by $2(2) + 17$ in., or 15 in. by 21 in.

Think
Why can you ignore the factor of 4?
By the Zero-Product Property, one of the factors, 4, $x + 16$, or $x - 2$, must equal 0. Since $4 \neq 0$, either $x + 16$ or $x - 2$ equals 0.

Got It? **4.** In Problem 4, suppose the total area of the frame and photo were 391 in.^{2}. What would the outer dimensions of the frame be?

Lesson Check

Do you know HOW?

Solve each equation.

1. $(v - 4)(v - 7) = 0$
2. $t^2 + 3t - 54 = 0$
3. $3y^2 - 17y + 24 = 0$

STEM **4. Carpentry** You are making a rectangular table. The area of the table should be 10 ft^2. You want the length of the table to be 1 ft shorter than twice its width. What should the dimensions of the table be?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Give an example of how the Zero-Product Property can be used to solve a quadratic equation.

6. Compare and Contrast How is factoring the expression $x^2 - 6x + 8$ similar to solving the equation $x^2 - 6x + 8 = 0$? How is it different?

7. Reasoning Can you extend the Zero-Product Property to nonzero products of numbers? For example, if $ab = 8$, is it always true that $a = 8$ or $b = 8$? Explain.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the Zero-Product Property to solve each equation. See Problem 1.

8. $(x - 9)(x - 8) = 0$

9. $(4k + 5)(k + 7) = 0$

10. $n(n + 2) = 0$

11. $-3n(2n - 5) = 0$

12. $(7x + 2)(5x - 4) = 0$

13. $(4a - 7)(3a + 8) = 0$

Solve by factoring. See Problems 2 and 3.

14. $x^2 + 11x + 10 = 0$

15. $g^2 + 4g - 32 = 0$

16. $s^2 - 14s + 45 = 0$

17. $2z^2 - 21z - 36 = 0$

18. $3q^2 + q - 14 = 0$

19. $4m^2 - 27m - 40 = 0$

20. $x^2 + 13x = -42$

21. $p^2 - 4p = 21$

22. $c^2 = 5c$

23. $2w^2 - 11w = -12$

24. $3h^2 + 17h = -10$

25. $9b^2 = 16$

26. Geometry A box shaped like a rectangular prism has a volume of 280 in.^{3}. Its dimensions are 4 in. by $(n + 2)$ in. by $(n + 5)$ in. Find n. See Problem 4.

27. Knitting You are knitting a blanket. You want the area of the blanket to be 24 ft^2. You want the length of the blanket to be 2 ft longer than its width. What should the dimensions of the blanket be?

STEM **28. Construction** You are building a rectangular deck. The area of the deck should be 250 ft^2. You want the length of the deck to be 5 ft longer than twice its width. What should the dimensions of the deck be?

B Apply

Use the Zero-Product Property to solve each equation. Write your solutions as a set in roster form.

29. $x^2 + 6x + 8 = 0$

30. $a^2 + 8a + 12 = 0$

31. $k^2 + 7k + 10 = 0$

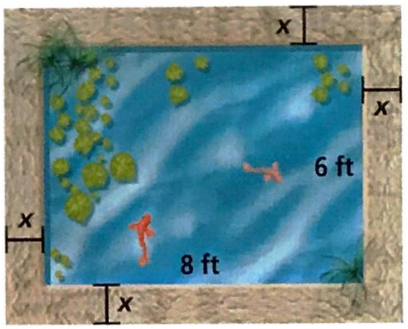
Write each equation in standard form. Then solve.

32. $7n^2 + 16n + 15 = 2n^2 + 3$

33. $4q^2 + 3q = 3q^2 - 4q + 18$

34. Think About a Plan You have a rectangular koi pond that measures 6 ft by 8 ft. You have enough concrete to cover 72 ft^2 for a walkway, as shown in the diagram. What should the width of the walkway be?

- How can you write the outer dimensions of the walkway?
- How can you represent the total area of the walkway and pond in two ways?



35. Reasoning Find the zeros of the function $f(x) = x^2 - 3x + 2$ by factoring. How can you verify the zeros of the function are correct by looking at the graph?

36. Error Analysis Describe and correct the error made in solving the equation.

$2x^2 + 3x = 20$
$x(2x + 3) = 20$
$x = 0$ or $2x + 3 = 0$
$x = 0$ or $x = -\frac{3}{2}$

37. Reasoning How many solutions does an equation of the form $x^2 - k^2 = 0$ have? Explain.

38. Sports You throw a softball into the air with an initial upward velocity of 38 ft/s and an initial height of 5 ft.

a. Use the vertical motion model to write an equation that gives the ball's height h, in feet, at time t, in seconds.

b. The ball's height is 0 ft when it is on the ground. Solve the equation you wrote in part (a) for $h = 0$ to find when the ball lands.

Solve each cubic equation by factoring out the GCF first.

39. $x^3 - 10x^2 + 24x = 0$ **40.** $x^3 - 5x^2 + 4x = 0$ **41.** $3x^3 - 9x^2 = 0$

Challenge

42. Find an equation that has the given numbers as solutions. For example, 4 and -3 are solutions of $x^2 - x - 12 = 0$.

a. $-5, 8$ **b.** $3, -2$ **c.** $\frac{1}{2}, -10$ **d.** $\frac{2}{3}, -\frac{5}{7}$

Solve. Factor by grouping.

43. $x^3 + 5x^2 - x - 5 = 0$ **44.** $x^3 + x^2 - 4x - 4 = 0$ **45.** $x^3 + 2x^2 - 9x - 18 = 0$

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2

In the Apply What You've Learned in Lesson 9-2, you found a function that represents the area of the rectangular sign described on page 545.

a. Write the function for the area of the rectangular sign in factored form.

b. Find the zeros of the function.

c. Describe how you can use the zeros to help you determine the maximum area of the rectangle.