9-7 Linear, Quadratic, and Exponential Models

Common Core State Standards

F-LE.A.1a Prove that linear functions grow by equal differences . . . and that exponential functions grow by equal factors over equal intervals. **Also F-LE.A.2, F-LE.A.3**

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To choose a linear, quadratic, or exponential model for data

Do you see the pattern? You can model it with a function.

SOLVE IT!

Getting Ready!

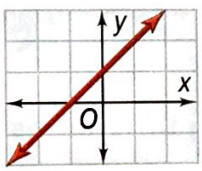
How many small triangles will be in Stage 9? Explain your reasoning.

MATHEMATICAL PRACTICES **Essential Understanding** You can use the linear, quadratic, or exponential functions you have studied to model some sets of data.

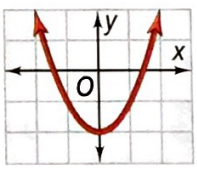
take note

Concept Summary Linear, Quadratic, and Exponential Functions

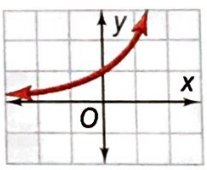
Linear: $y = mx + b$



Quadratic: $y = ax^2 + bx + c$



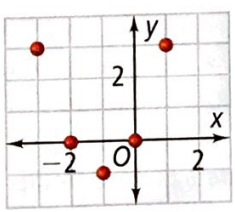
Exponential: $y = a \cdot b^x$



Problem 1 Choosing a Model by Graphing

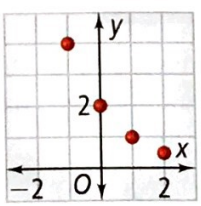
Graph each set of points. Which model is most appropriate for each set?

A $(1, 3), (0, 0), (-3, 3), (-1, -1), (-2, 0)$



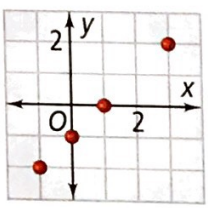
Quadratic model

B $(0, 2), (-1, 4), (1, 1), (2, 0.5)$



Exponential model

C $(-1, -2), (0, -1), (1, 0), (3, 2)$



Linear model

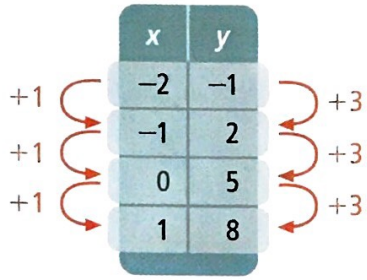
Think

Can you eliminate possibilities?

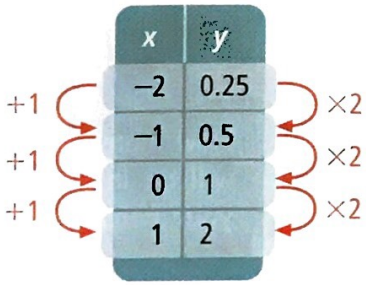
Yes. For example, you know that a linear model isn't appropriate in parts (A) and (B) because the slope between any two points is not constant.

Got It? 1. Graph each set of points. Which model is most appropriate for each set?
a. $(0, 0), (1, 1), (-1, -0.5), (2, 3)$ **b.** $(-2, 11), (-1, 5), (0, 3), (1, 5)$

When the x-values in a set of data pairs have a common difference, you can analyze data numerically to find the best model. You can use a linear function to model data pairs with y-values that have a common difference. You can use an exponential function to model data pairs with y-values that have a common ratio.



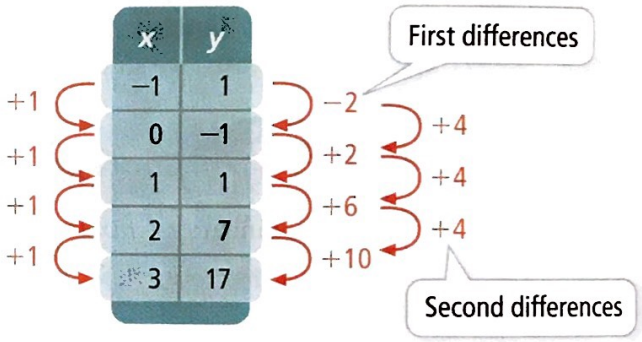
The y-values have a common difference of 3. A linear model fits the data.



The y-values have a common ratio of 2. An exponential model fits the data.

For quadratic functions, the second differences are constant.

In the table at the right, the second differences of the y-values are all 4, so a quadratic model fits the data.



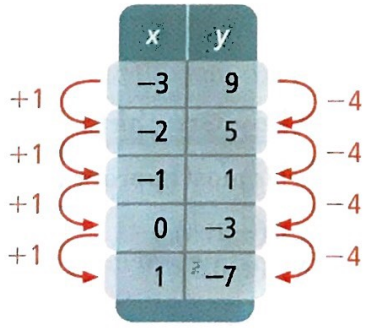
Problem 2 Choosing a Model Using Differences or Ratios

Which type of function best models the data? Use differences or ratios.

Plan

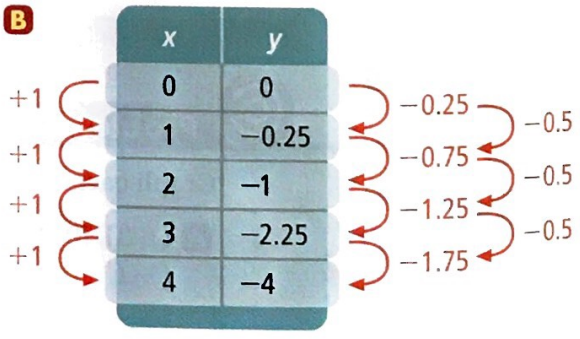
How can you get started?
Begin by checking the first differences of the y-values. Then check the second differences and ratios, if necessary.

A



The first differences are constant, so a linear function models the data.

B



The second differences are constant, so a quadratic function models the data.

Got It? 2. Which type of function best models the ordered pairs $(-1, 0.5), (0, 1), (1, 2), (2, 4)$, and $(3, 8)$? Use differences or ratios.

Real-world data seldom fall exactly into linear, exponential, or quadratic patterns. However, you can determine which type of function represents the best possible model for the data.

Problem 3 Modeling Real-World Data

Transportation **The data at the right give the value of a used car over time. Which type of function best models the data? Write an equation to model the data.**

Value of Used Car

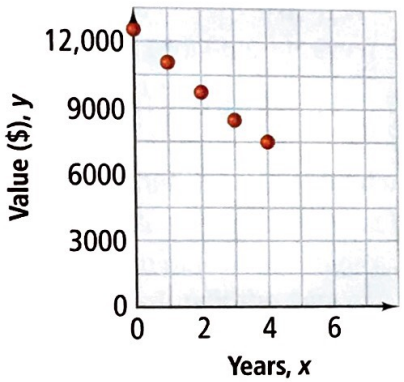
Years	Value ($)
0	12,575
1	11,065
2	9750
3	8520
4	7540

Know The value of a used car over time

Need The most appropriate model for the data

Plan Graph the data and then use differences or ratios to find a model for the situation.

Step 1
Graph the data.



The graph curves and does not look quadratic. It may be exponential.

Step 2
Test for a common ratio.

Years	Value ($)		
0	12,575		
1	11,065	+1	$\frac{11,065}{12,575} \approx 0.88$
2	9750	+1	$\frac{9750}{11,065} \approx 0.88$
3	8520	+1	$\frac{8520}{9750} \approx 0.87$
4	7540	+1	$\frac{7540}{8520} \approx 0.88$

The value of the car is roughly 0.88 times its value the previous year.

Step 3
Write an exponential model.

Relate $y = a \cdot b^x$

Define Let a = the initial value, 12,575.
Let b = the decay factor, 0.88.

Write $y = 12{,}575 \cdot 0.88^x$

Step 4
Test two points other than (0, 12,575).

Test (2, 9750):

$y = 12{,}575 \cdot 0.88^2$

$y \approx 9738$

Test (4, 7540):

$y = 12{,}575 \cdot 0.88^4$

$y \approx 7541$

The point (2, 9738) is close to the data point (2, 9750). The point (4, 7541) is close to the data point (4, 7540). The equation $y = 12{,}575 \cdot 0.88^x$ models the data.

Got It? **3.** The table shows the annual income of a small theater company. Which type of function best models the data? Write an equation to model the data.

Theater Company Annual Income

Year	0	1	2	3	4
Income ($)	18,254	18,730	19,215	19,695	20,175

Lesson Check

Do you know HOW?

Which type of function best models each set of data points?

1. $(0, 11), (1, 5), (2, 3), (3, 5), (4, 11)$
2. $(-4, -10), (-2, -7), (0, -4), (2, -1), (4, 2)$
3. $(-1, 8), (0, 4), (2, 1), (3, 0.5)$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Reasoning** Can the y-values in a set of data pairs have both a common ratio and a common difference? Explain why or why not.
5. **Writing** Explain how to decide whether a linear, exponential, or quadratic function is the most appropriate model for a set of data.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Graph each set of points. Which model is most appropriate for each set? See Problem 1.

6. $(-2, -3), (-1, 0), (0, 1), (1, 0), (2, -3)$
7. $(-2, -8), (0, -4), (3, 2), (5, 6)$
8. $(-3, 6), (-1, 0), (0, -1), (1, -1.5)$
9. $(-2, 5), (-1, -1), (0, -3), (1, -1), (2, 5)$
10. $(-1, -5\frac{2}{3}), (0, -5), (2, 3), (3, 27)$
11. $(-3, 8), (-1, 6), (0, 5), (2, 3), (3, 2)$

Which type of function best models the data in each table? Use differences or ratios. See Problem 2.

12.

x	y
0	0
1	1.5
2	6
3	13.5
4	24

13.

x	y
0	−5
1	−3
2	−1
3	1
4	3

14.

x	y
0	1
1	1.2
2	1.44
3	1.728
4	2.0736

Which type of function best models the data in each table? Write an equation to model the data. See Problem 3.

15.

x	y
0	0
1	3
2	11.3
3	24.7
4	43.3

16.

x	y
0	5
1	2
2	0.79
3	0.32
4	0.128

17.

x	y
0	2
1	1.52
2	1
3	0.49
4	0

18. Sports The number of people attending a school's first five football games is shown in the table below. Which type of function best models the data? Write an equation to model the data.

Game	1	2	3	4	5
Attendance	248	307	366	425	484

19. Banking The average monthly balance of a savings account is shown in the table at the right. Which type of function best models the data? Write an equation to model the data.

Month	Balance ($)
0	540
1	556.20
2	572.89
3	590.07
4	607.77

B Apply

20. Error Analysis Tom claims that, because the data pairs (1, 4), (2, 6), (3, 9), and (4, 13.5) have y-value with a common ratio, they are best modeled by a quadratic function. What is his error?

21. **a.** Make a table of five ordered pairs for each function using consecutive x-values. Find the common second difference.

i. $f(x) = x^2 - 3$ **ii.** $f(x) = 3x^2$ **iii.** $f(x) = 4x^2 - 5x$

b. What is the relationship between the common second difference and the coefficient of the x^2-term?

c. Reasoning Explain how you could use this relationship to model data.

22. Think About a Plan The number of visitors at a Web site over several days is shown in the table at the right. What is an equation that models the data?

- Does the graph of the data suggest a type of function to use?
- Will your equation fit the data exactly? How do you know?

Day	Visitors
1	52
2	197
3	447
4	805
5	1270

23. Open-Ended Write a set of data pairs that you could model with a quadratic function.

STEM

24. Zoology A conservation organization collected the data on the number of frogs in a local wetland, shown in the table at the right. Which type of function best models the data? Write an equation to model the data.

Year	Number of Frogs
0	120
1	101
2	86
3	72
4	60

25. The table below shows the projected population of a small town. Let $t = 0$ correspond to the year 2020.

a. Graph the data. Does the graph suggest a linear, exponential, or quadratic model?

b. Find the rate of change in population with respect to time from one data pair to the next. How do the results support your answer to part (a)?

c. Write a function that models the data shown in the table.

d. Use the function from part (c) to predict the town's population in 2050.

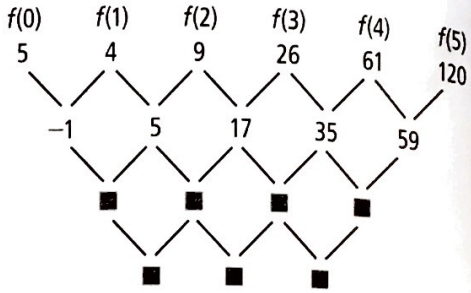
e. Suppose the projected population s of another small town is represented by the function $s = 50t + 1300$. Let $t = 0$ correspond to the year 2020. Write an expression that can be used to find the difference in population of the two towns.

Year, t	0	5	10	15
Population, p	5100	5700	6300	6900

Challenge

26. Reasoning Write a quadratic function $y = ax^2 + bx + c$ whose graph passes through the points (0, 7), (2, 13), and (4, 35).

27. Reasoning The diagram at the right shows the differences for the cubic function $f(x) = x^3 - 2x + 5$ for the x-values 0, 1, 2, 3, 4, and 5.



a. Write the second and third differences in the appropriate locations in the diagram.

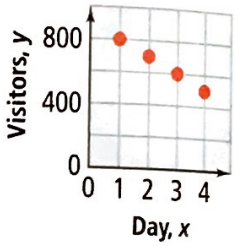
b. What do you predict the third difference would be if $f(6)$ were added to the diagram?

c. Do you think that the third differences will be constant for other cubic functions? Explain why or why not.

Standardized Test Prep

SAT/ACT

28. The graph at the right shows the number y of visitors to a museum over x days. Which function models the number of visitors?



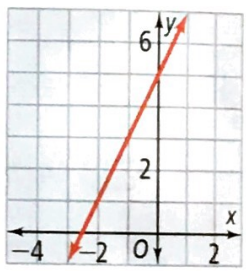
(A) $y = -100x + 900$ (C) $y = -100x + 800$

(B) $y = 900(0.875)^x$ (D) $y = -50x^2 - 400x + 1300$

29. Which expression is equivalent to $(4x^3 + 2x^2 + 1) + (3x^2 + 8x + 2)$?

(F) $7x^2 + 10x + 3$ (G) $7x^3 + 10x^2 + 3x$ (H) $4x^3 + 5x^2 + 3$ (I) $4x^3 + 5x^2 + 8x + 3$

30. Which line passes through the point (1, 3) and is parallel to the line graphed at the right?



(A) $y = 2x + 1$ (C) $y = 2x - 5$

(B) $y = 2x + 3$ (D) $y = -5x + 8$

Short Response

31. What are the factors of $10x^2 - x - 2$? Show your work.

Mixed Review

Use the quadratic formula to solve each equation. If necessary, round to the nearest hundredth. See Lesson 9-6.

32. $4x^2 + 4x - 3 = 0$ **33.** $x^2 + 2x - 7 = 0$ **34.** $3x^2 - 8x = -1$

Get Ready! **To prepare for Lesson 9-8, do Exercises 35–37.**

Solve by elimination. See Lesson 6-3.

35. $x + y = 10$
$x - y = 2$

36. $5x - 6y = -32$
$3x + 6y = 48$

37. $-2x + 15y = -32$
$7x - 5y = 17$