9-8 Systems of Linear and Quadratic Equations

Common Core State Standards

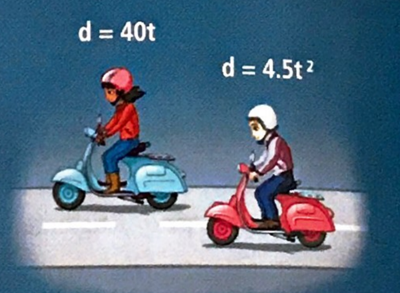
A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically . . . **Also A-CED.A.3, A-REI.D.11**

MP 1, MP 3, MP 4, MP 5

Objective To solve systems of linear and quadratic equations

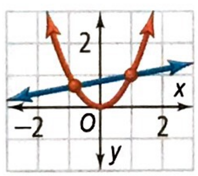
SOLVE IT! **Getting Ready!**

Two scooters leave a stoplight at the same time. The blue scooter accelerates and then travels at a constant speed, and the red scooter accelerates at a constant rate. The distance d, in feet, each scooter travels after t seconds is shown. When does the red scooter catch up to the blue scooter? Explain.

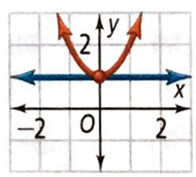


MATHEMATICAL PRACTICES

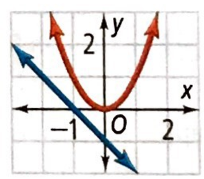
Essential Understanding You can solve systems of linear and quadratic equations graphically and algebraically. This type of system can have two solutions, one solution, or no solutions.



Two solutions



One solution



No solutions

Problem 1 Solving by Graphing

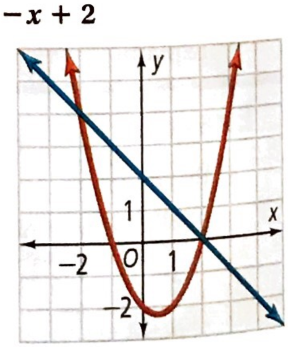
Plan

How can you solve this system by graphing?
The points where the two graphs intersect are the solutions of the system.

What are the solutions of the system? Solve by graphing. $y = x^2 - x - 2$
$y = -x + 2$

Step 1 Graph both equations in the same coordinate plane.

Step 2 Identify the point(s) of intersection, if any. The points of intersection are $(-2, 4)$ and $(2, 0)$.



The solutions of the system are $(-2, 4)$ and $(2, 0)$.

Got It? 1. What are the solutions of each system? Solve by graphing.

a. $y = 2x^2 + 1$
$y = -2x + 5$

b. $y = x^2 + x + 3$
$y = -x$

In Lesson 6-3, you solved linear systems using elimination. The same technique can be applied to systems of linear and quadratic equations.

Problem 2 Using Elimination

Recreation **Since opening day, attendance at Pool A has increased steadily, while attendance at Pool B first rose and then fell. Equations modeling the daily attendance *y* at each pool are shown below, where *x* is the number of days since opening day. On what day(s) was the attendance the same at both pools? What was the attendance?**

Pool A: $y = 28x + 4$
Pool B: $y = -x^2 + 39x + 64$

Know	Need	Plan
Equations giving the attendance at each pool	The day(s) when the attendance was the same	Use elimination to solve the system formed by the equations.

Step 1 Eliminate *y*.

$$\begin{array}{rl} y = -x^2 + 39x + 64 & \\ -\,(y = \qquad\quad 28x + 4) & \text{Subtract the two equations.} \\ \hline 0 = -x^2 + 11x + 60 & \text{Subtraction Property of Equality} \end{array}$$

Step 2 Factor and solve for *x*.

$$\begin{array}{ll} 0 = -x^2 + 11x + 60 & \\ 0 = -(x^2 - 11x - 60) & \text{Factor out } -1. \\ 0 = -(x + 4)(x - 15) & \text{Factor.} \\ x + 4 = 0 \quad \text{or} \quad x - 15 = 0 & \text{Zero-Product Property} \\ x = -4 \quad \text{or} \quad x = 15 & \text{Solve for } x. \end{array}$$

Step 3 Eliminate the impossible solution. The pools cannot be open a negative number of days, so $x \neq -4$.

Step 4 Use the viable solution to find the corresponding *y*-value. Use either equation.

$$\begin{array}{ll} y = -x^2 + 39x + 64 & y = 28x + 4 \\ y = -(15)^2 + 39(15) + 64 & y = 28(15) + 4 \\ y = -225 + 585 + 64 & y = 424 \\ y = 424 & \end{array}$$

The pools had the same attendance on Day 15 with 424 people.

Got It? 2. In Problem 2, suppose the daily attendance *y* at Pool A can be modeled by the equation $y = 36x + 54$. On what day(s) was the attendance the same at both pools? What was the attendance?

Substitution is another method you have used to solve linear systems. This method also works with systems of linear and quadratic equations.

Problem 3 Using Substitution

What are the solutions of the system? $y = x^2 - 6x + 10$
$y = 4 - x$

Plan

Which variable should you substitute for?
Substitute for *y* since both equations are already solved for *y*.

Step 1 Write a single equation containing only one variable.

$$y = x^2 - 6x + 10$$

$$4 - x = x^2 - 6x + 10 \quad \text{Substitute } 4 - x \text{ for } y.$$

$$4 - x - (4 - x) = x^2 - 6x + 10 - (4 - x) \quad \text{Subtract } 4 - x \text{ from each side.}$$

$$0 = x^2 - 5x + 6 \quad \text{Write in standard form.}$$

Step 2 Factor and solve for x.

$$0 = (x - 2)(x - 3) \quad \text{Factor.}$$

$$x - 2 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{Zero-Product Property}$$

$$x = 2 \quad \text{or} \quad x = 3 \quad \text{Solve for } x.$$

Step 3 Find corresponding y-values. Use either original equation.

$$y = 4 - x = 4 - 2 = 2 \qquad y = 4 - x = 4 - 3 = 1$$

The solutions of the system are (2, 2) and (3, 1).

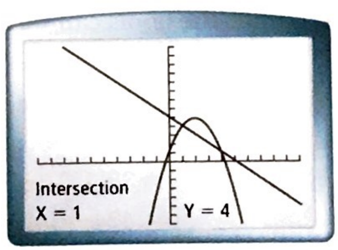
Got It? 3. What are the solutions of the system? $y - 30 = 12x$
$y = x^2 + 11x - 12$

Problem 4 Solving With a Graphing Calculator

What are the solutions of the system? $y = -x + 5$
Use a graphing calculator. $y = -x^2 + 4x + 1$

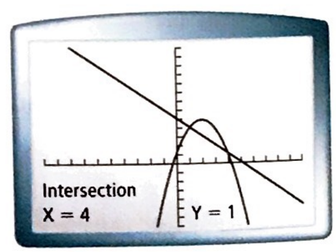
Step 1 Enter the equations on the **Y=** screen. Press **graph** to display the system.

Step 2



Use the **CALC** feature. Select **INTERSECT.** Move the cursor close to a point of intersection. Press **enter** three times to find the point of intersection.

Step 3



Repeat Step 2 to find the second intersection point.

Think

How can you check your solutions?
Substitute them into the original equations and simplify.

The solutions are (1, 4) and (4, 1).

Got It? 4. a. What are the solutions of the system? Use a graphing calculator.

$y = x^2 - 2$
$y = -x$

b. **Reasoning** How else can you solve the system in part (a)? Explain.

Lesson Check

Do you know HOW?

1. Use a graph to solve the system $y = x^2 + x - 2$ and $y = x + 2$.
2. Use elimination to solve the system $y = x^2 - 13x + 52$ and $y = -14x + 94$.
3. Use substitution to solve the system $y = x^2 - 6x + 9$ and $y + x = 5$.
4. Use a graphing calculator to solve the system $y = -x^2 + 4x + 1$ and $y = 2x + 2$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Use two different methods to solve the system $y = x$ and $y = 2x^2 + 10x + 9$. Which method do you prefer? Explain.
6. **Open-Ended** Write a system of linear and quadratic equations with the given number of solutions.
   a. two   b. exactly one   c. none
7. **Compare and Contrast** How are solving systems of linear equations and solving systems of linear and quadratic equations alike? How are they different?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each system by graphing. See Problem 1.

8. $y = x^2 + 1$
   $y = x + 1$
9. $y = x^2 + 4$
   $y = 4x$
10. $y = x^2 - 5x - 4$
    $y = -2x$
11. $y = x^2 + 2x + 1$
    $y = x + 1$
12. $y = x^2 + 2x + 5$
    $y = -2x + 1$
13. $y = 3x + 4$
    $y = -x^2 + 4$

Solve each system using elimination. See Problem 2.

14. $y = -x + 3$
    $y = x^2 + 1$
15. $y = x^2$
    $y = x + 2$
16. $y = -x - 7$
    $y = x^2 - 4x - 5$

17. **Sales** The equations at the right model the numbers y of two portable music players sold x days after both players were introduced. On what day(s) did the company sell the same number of each player? How many players of each type were sold?

Music Player A: $y = 191x - 32$
Music Player B: $y = -x^2 + 200x + 20$

Solve each system using substitution. See Problem 3.

18. $y = x^2 - 2x - 6$
    $y = 4x + 10$
19. $y = 3x - 20$
    $y = -x^2 + 34$
20. $y = x^2 + 7x + 100$
    $y + 10x = 30$
21. $-x^2 - x + 19 = y$
    $x = y + 80$
22. $3x - y = -2$
    $2x^2 = y$
23. $y = 3x^2 + 21x - 5$
    $-10x + y = -1$

Graphing Calculator Solve each system using a graphing calculator. See Problem 4.

24. $y = x^2 - 2x - 2$
$y = -2x + 2$

25. $y = -x^2 + 2$
$y = 4 - 0.5x$

26. $y = x - 5$
$y = x^2 - 6x + 5$

27. $y = -0.5x^2 - 2x + 1$
$y + 3 = -x$

28. $y = 2x^2 - 24x + 76$
$y + 7 = 11$

29. $-x^2 - 8x - 15 = y$
$-x + y = 3$

B Apply

30. The equation $x^2 + y^2 = 25$ defines a circle with center at the origin and radius 5. The line $y = x + 1$ passes through the circle. Using the substitution method, find the point(s) at which the circle and the line intersect.

31. **Think About a Plan** A company's logo consists of a parabola and a line. The parabola in the logo can be modeled by the function $y = 3x^2 - 4x + 2$. The line intersects the parabola when $x = 0$ and when $x = 2$. What is an equation of the line?
- How can you find the coordinates of the points of intersection?
- Can you write an equation of the line given the points of intersection?

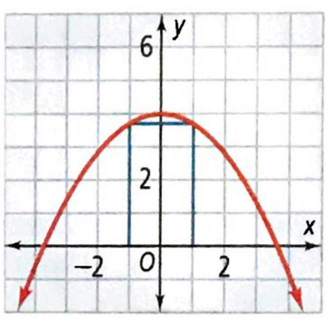
32. **Business** The daily number of customers y at a coffee shop can be modeled by the function $y = 0.25x^2 - 5x + 80$, where x is the number of days since the beginning of the month. The daily number of customers at a second shop can be modeled by a linear function. Both shops have the same number of customers on days 10 and 20. What function models the number of customers at the second shop?

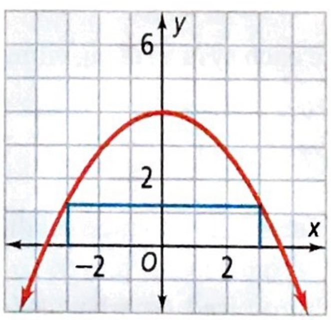
33. **Error Analysis** A classmate says that the system $y = x^2 + 2x + 4$ and $y = x + 1$ has one solution. Explain the classmate's error.

34. **Writing** Explain why a system of linear and quadratic equations cannot have an infinite number of solutions.

C Challenge

35. **Geometry** The figures below show rectangles that are centered on the y-axis with bases on the x-axis and upper vertices defined by the function $y = -0.3x^2 + 4$. Find the area of each rectangle.

a.


b.


c. Find the coordinates of the vertices of the square constructed in the same manner. Round to the nearest hundredth.

d. Find the area of the square. Round to the nearest hundredth.

36. What are the solutions of the system $y = x^2 + x + 6$ and $y = 2x^2 - x + 3$? Explain how you solved the system.

Standardized Test Prep

SAT/ACT

37. A designer sketches a design for a tabletop on graph paper. The table is bounded by a parabola and a line. The parabola can be modeled by the function $y = 2x^2 - 3x + 2$. The line intersects the parabola when $x = -1$ and when $x = 3$. What is an equation of the line?

   (A) $y = -x + 8$
   (B) $y = x + 8$
   (C) $y = 2x + 5$
   (D) $y = -2x + 5$

38. Which equation illustrates the Distributive Property?

   (F) $4(x + 2) = 4x + 8$
   (G) $4(x + 2) = (x + 2)4$
   (H) $4(x + 2) = 4(2 + x)$
   (I) $4(x + 2) = 4(x + 2)$

39. Which rate is equivalent to 30 m/s?

   (A) 3 km/h
   (B) 108 km/h
   (C) 3000 km/h
   (D) 108,000 km/h

40. Which of the following is equivalent to 0.05%?

   (F) 0.00005
   (G) 0.0005
   (H) 0.005
   (I) 0.05

Short Response

41. A box with 4 balls weighs 5 lb. The same box with 10 balls weighs 11 lb. Write an equation in slope-intercept form for the weight y of a box containing x balls. Then rewrite the equation in standard form using integer coefficients.

Mixed Review

Which type of function best models the data in each table? Write an equation to model the data.

See Lesson 9-7.

42.

x	y
−1	0.2
0	0
1	0.2
2	0.8
3	1.8
4	3.2

43.

x	y
−1	1.6
0	4
1	10
2	25
3	62.5
4	156.25

44.

x	y
−1	11.2
0	7
1	2.8
2	−1.4
3	−5.6
4	−9.8

Get Ready! **To prepare for Lesson 10-1, do Exercises 45–50.**

Simplify each expression.

See Lesson 1-3.

45. $\sqrt{196}$

46. $\sqrt{\frac{25}{49}}$

47. $\sqrt{1.44}$

48. $\sqrt{81}$

49. $\sqrt{0.36}$

50. $\sqrt{400}$