1-2 Irrational Numbers and Square Roots

Check Skills You'll Need

1. **Vocabulary Review** How is a *terminating decimal* different from a *repeating decimal*?

Classify each decimal as *terminating* or *repeating*.

2. 1.276 **3.** $3.\overline{35}$

4. $8.1\overline{6}$ **5.** 12.1212

GO for Help Lesson 1-1

CONTENT STANDARDS

8.NS.1, 8.NS.2, 8.EE.2

What You'll Learn

To find and estimate square roots and to classify numbers as rational or irrational

New Vocabulary perfect square, square root, irrational numbers, real numbers

Why Learn This?

Not every situation can be modeled using the four basic operations. For example, you need square roots to relate the time and distance a skydiver falls.

A number that is the square of a whole number is a **perfect square.** The **square root** of a number is another number that when multiplied by itself is equal to the given number.

In the diagram at the right, 16 square tiles form a square with 4 tiles on each side. Since $4 \cdot 4 = 16$ and $-4 \cdot (-4) = 16$, 16 has two square roots, 4 and -4. Since $4^2 = 16$, 16 is a perfect square.

$4^2 = 16$

Perfect Squares

n	n^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144

EXAMPLE 1 Finding Square Roots of Perfect Squares

Find the two square roots of 25.

$5 \cdot 5 = 25$ and $-5 \cdot (-5) = 25$

The square roots of 25 are 5 and -5.

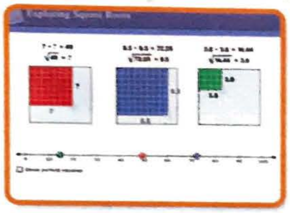
Quick Check

1. Find the square roots of each number.

a. 36 **b.** 1 **c.** $\frac{1}{16}$

The symbol $\sqrt{}$ means the square root of a number. In this book, $\sqrt{}$ means the nonnegative square root, unless stated otherwise. So $\sqrt{9}$ means the nonnegative square root of 9, or 3, and $-\sqrt{9}$ means the opposite of the nonnegative square root of 9, or -3.

Online active math

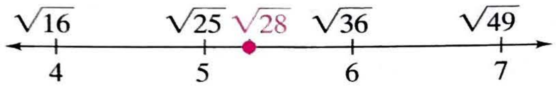


For: Square Roots Activity
Use: Interactive Textbook, 1-2

EXAMPLE Estimating a Square Root

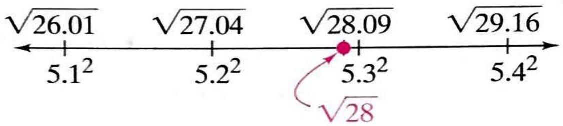
2 Estimate the value of $\sqrt{28}$ to the nearest integer and to the nearest tenth.

To estimate $\sqrt{28}$ to the nearest integer, find the closest perfect square greater than 28 and the closest perfect square less than 28.



The perfect squares closest to 28 are 25 and 36. Since 28 is closer to 25 than it is to 36, $\sqrt{28}$ must be closer to 5 than to 6. So $\sqrt{28} \approx 5$.

To estimate $\sqrt{28}$ to the nearest tenth, find two squares between 5 and 6 that are closest to 28. Start with $5.1^2, 5.2^2$, and so on.



The squares closest to 28 are 27.04 and 28.09. Since 28 is closer to 28.09 than it is to 27.04, $\sqrt{28}$ must be closer to 5.3 than to 5.2. So, $\sqrt{28} \approx 5.3$.

Quick Check

2. Estimate the value of $\sqrt{38}$ to the nearest integer and to the nearest tenth.

You can compare square roots in the same way you compare integers.

EXAMPLE Comparing Square Roots

3 Which is greater, $\sqrt{18}$ or 4.4?

First, estimate the value of the square root to the nearest tenth.

$\sqrt{18} \approx 4.2$

Since $4.2 < 4.4, \sqrt{18} < 4.4$.

Quick Check

3. Which is greater $\sqrt{20}$ or 4.7?

Finding a number's square root is the inverse of finding its square. So $\sqrt{3^2} = 3$.

EXAMPLE Application: Surface Area of a Sphere

4 The formula S.A. $= 13r^2$ approximates the surface area of a sphere with radius r. Find the radius of a sphere with S.A. 780 square units.

$\text{S.A.} = 13r^2$ ← **Use the formula for the surface area of a sphere.**

$780 = 13r^2$ ← **Substitute 780 for S.A.**

$\frac{780}{13} = r^2$ ← **Divide each side by 13 to isolate *r*.**

$$60 \approx r^2 \quad \leftarrow \text{Simplify.}$$
$$\sqrt{60} \approx \sqrt{r^2} \quad \leftarrow \text{Find the positive square root of each side. Use a calculator.}$$
$$7.7 \approx r \quad \leftarrow \text{Round to the nearest tenth.}$$

The radius of a sphere with surface area 780 square units is about 7.7 units.

Calculator Tip

To find $\sqrt{60}$ on your calculator, press [√]. Enter 60. Then press [=].

Quick Check

4. Use the formula S.A. = $13r^2$ to find the radius of a sphere with a surface area of 520 square units. Round to the nearest tenth of a unit.

Irrational numbers are numbers that cannot be written in the form $\frac{a}{b}$, where a is any integer and b is any nonzero integer. Rational and irrational numbers form the set of **real numbers.** The diagram below shows the relationships among sets of numbers.

Vocabulary Tip

The word *rational* has the word *ratio* in it.

The word *irrational* means "not rational."

- Reals
  - Rationals
    - Integers: $-3, 8, 0, -120$
    - Fractions: $-\frac{1}{2}, \frac{2}{3}, \frac{7}{8}, \frac{12}{5}$
    - Terminating and repeating decimals: $1.25, -0.13, 0.\overline{2}$
  - Irrationals: $\sqrt{2}, 0.1010010001\ldots, 2+\pi, \sqrt{\frac{1}{5}}, \sqrt{11}$

The decimal expansion of irrational numbers do not terminate or repeat. The decimal digits of $\pi = 3.14159265359\ldots$ do not terminate or repeat, so π is an irrational number. Irrational numbers can also include decimals that have a pattern in their digits, like $0.02022022202222\ldots$

For any integer n that is not a perfect square, $\sqrt{n}$ is irrational.

EXAMPLE Classifying Real Numbers

5 Is each number *rational* or *irrational*? Explain.

a. $0.818118111\ldots$ Irrational; the decimal does not terminate or repeat.

b. $-0.\overline{81}$ Rational; the decimal repeats.

c. $1\frac{2}{9}$ Rational; the number can be written as the ratio $\frac{11}{9}$.

d. $\sqrt{2}$ Irrational; 2 is not a perfect square.

GO for Help

For help with terminating and repeating decimals, go to Lesson 1-1, Example 3.

Quick Check

5. Is $0.\overline{6}$ *rational* or *irrational*? Explain.

Check Your Understanding

Vocabulary Write all the possible names for each number. Choose from the terms *rational number, irrational number, real number,* and *perfect square.*

1. $\sqrt{6}$ 2. $-0.\overline{6}$ 3. $\frac{1}{6}$ 4. 25

Find the positive and negative square roots of each number.

5. 4 6. $\frac{1}{4}$ 7. 100 8. $\frac{1}{100}$

Homework Exercises

For more exercises, see Extra Skills and Word Problems.

GO for Help

For Exercises	See Examples
9–13	1
14–17	2
18–21	3
22–25	4
26–29	5

Find the square roots of each number.

9. 49 10. 900 11. $\frac{1}{36}$ 12. $\frac{1}{121}$ 13. $\frac{4}{25}$

Estimate the value of each expression to the nearest integer.

14. $\sqrt{3}$ 15. $\sqrt{10}$ 16. $-\sqrt{22}$ 17. $-\sqrt{120}$

Which number is greater?

18. $\sqrt{26}, 5.2$ 19. $7.3, \sqrt{55}$ 20. $\sqrt{44}, 6.4$ 21. $10.3, \sqrt{104}$

Use $s = 20\sqrt{273 + T}$ to estimate the speed of sound s in meters per second for each Celsius temperature T. Estimate square roots to the nearest tenth.

22. 0°C 23. 20°C 24. −10°C 25. 70°C

Is each number *rational* or *irrational*? Explain.

26. -0.6 27. $\sqrt{40}$ 28. 0.606606660... 29. $-\sqrt{144}$

GPS

30. **Guided Problem Solving** The area of a square postage stamp is $\frac{81}{100}$ in.^{2}. What is the side length of the stamp?
 - What is the formula for the area of a square?
 - How can you use the formula to find the side length of a square?

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31. **Boxing** The area of a square boxing ring is 484 ft^2. What is the perimeter of the boxing ring?

32. **Open-Ended** Give an example of an irrational number that is less than 2 and greater than 1.5. Explain how you know the number is irrational.

33. **Writing in Math** Explain how you can approximate $\sqrt{30}$.

Find the value of each expression.

34. $(\sqrt{36})^2$

35. $\sqrt{(10)^2}$

36. $\sqrt{(3.2)^2}$

37. $(\sqrt{a})^2$

38. The area of a square is $\frac{25}{36}$ in.^{2}. What is the length of its side?

39. **Ferris Wheels** The formula $d = 1.23\sqrt{h}$ represents the distance in miles d you can see from h feet above ground. On the London Eye Ferris Wheel, you are 450 ft above ground. To the nearest tenth of a mile, how far can you see?

40. **Number Sense** For what values of n is $\sqrt{n}$ a rational number?

41. **Error Analysis** A student evaluated the expression $\sqrt{4 + 9}$ and got the answer 5. What error did the student make?

42. **Challenge** Explain how you know that the number 123,456,789,101,112 cannot be a perfect square. (*Hint:* What is the units digit?)

Test Prep and Mixed Review

Practice

Multiple Choice

43. The area of a square is 150 square centimeters. Which best represents the side length of the square?

Ⓐ 11.7 cm Ⓒ 2.9 cm
Ⓑ 12.2 cm Ⓓ 13 cm

44. The formula $d = 16t^2$ represents the approximate distance d in feet a skydiver falls in t seconds before opening the parachute. The formula assumes there is no air resistance. How long does a skydiver take to fall 816 feet before opening the parachute?

Ⓕ 2.7 seconds Ⓗ 51 seconds
Ⓖ 7.1 seconds Ⓙ 2,601 seconds

45. Which of the following inequalities correctly compares two numbers?

Ⓐ $\sqrt{97} < 9.9$ Ⓒ $\sqrt{97} < 9.5$
Ⓑ $\sqrt{97} \leq 9.7$ Ⓓ $\sqrt{97} \leq 9.3$

GO for Help

For Exercises	See Lesson
46–49	1-1

Write the decimal expansion of each fraction.

46. $\frac{27}{36}$

47. $\frac{20}{64}$

48. $\frac{88}{121}$

49. $\frac{196}{144}$